Nonlinear Langmuir Waves in Two-Electron-Temperature Plasmas

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Abstract

We investigate the stationary propagation of nonlinear Langmuir waves in a bi-Maxwellian plasma consisting of two electron-species in the critical parameter regime (\( \Delta = 3 \)). It is shown that there exists a new class of Langmuir envelope waves which have qualitatively different structure from the solutions reported earlier. In particular, the Langmuir field envelope has two nodes, and accordingly the field intensity has a triple-hump structure. The low-frequency density perturbation which traps the Langmuir field has the usual single-dip structure. The coupled fields co-propagate with super-sonic speeds but are accompanied by density rarefactions.

1. Introduction

Collective processes such as wave propagation and shock formation in bi-Maxwellian plasmas consisting of two species of electrons characterized by different temperatures have recently received renewed interest due to various applications in space [1–3] and laboratory [4] plasmas. It is well known that the so-called two-electron-temperature plasmas exhibit unusual properties which are absent in the usual electron-ion plasmas having only one type of electron species. For example, the dispersion characteristics of ion-acoustic waves in the linear regime are drastically modified by the presence of an additional cold electron species [5].

On the other hand, in the nonlinear regime, the conditions for the existence of nonlinear structures such as the solitons, double layers as well as shocks are significantly modified in a two-electron-temperature plasma [6–9].

Recently, it has been shown [10] that the nonlinear propagation of coupled Langmuir and ion-acoustic waves in a two-electron-temperature plasma is governed by the Schrödinger-Boussinesq system of equations. The latter equation describes the low-frequency density depletion which traps the high-frequency Langmuir waves, whose envelope is governed by the Schrödinger-like equation. It was found that unlike in the case of the usual electron-ion plasmas, the sign of the leading order (quadratic) nonlinear term (the driven) Boussinesq equation crucially depends on a parameter (\( \Delta \)), which is a function of the electron number densities and temperatures. In particular, for \( \Delta = 3 \), the quadratic nonlinear term vanishes and, therefore, it is essential to retain the cubic nonlinearity which gives rise to the leading order nonlinear regime. While exact stationary solutions of the Schrödinger-Boussinesq system valid in different parameter regimes have been obtained [10] for the case when \( \Delta \neq 3 \), the nature of the coupled wave propagation for the critical case when \( \Delta = 3 \) has not been analyzed so far.

We show, in this Letter, that the coupling between Schrödinger-Boussinesq systems for \( \Delta = 3 \) gives rise to a new type of localized envelope solution which has qualitatively different structure than those reported earlier. In particular, the Langmuir field amplitude has two nodes. Accordingly, the field intensity has a triple-hump structure whereas the low-frequency density perturbation has the usual single-dip structure. The two coupled fields co-propagate at super-sonic speeds, accompanied by density rarefactions.

2. Governing equations

We consider the nonlinear propagation of amplitude-modulated Langmuir waves coupled to the low-frequency ion-acoustic waves in a three-component plasma consisting of ions and two species of electrons. For slow modulations, the low-frequency dynamics of the electron species is described by respective Boltzmann distributions modified by the ponderomotive potential, and characterized by their respective temperatures, namely, \( T_h \) and \( T_c \) where the subscripts “h” and “c” denote the hot and the cold species, respectively. On the other hand, the ion dynamics is governed by the full set of fluid equations including the Poisson equation which accounts for the charge separation effects.

The complex amplitude \( E \) of the modulated Langmuir waves is governed by the Schrödinger-like equation [10],

\[
\text{i} \left( \frac{\partial E}{\partial t} + V_s \frac{\partial E}{\partial x} \right) + \frac{3n_e}{2} \frac{\partial^2 E}{\partial x^2} = \frac{1}{2} \rho E, \tag{1}
\]

where \( \rho = (m_e/m_h)^{1/2} \) is the square root of the electron to ion mass ratio, \( n = n_e/n_i \) is the normalized electron number density perturbation, \( V_s = kD_0/C_s \) is the normalized Langmuir wave group velocity, \( D_0 \equiv 3v_{te}^2/\omega_{pe0}^2 \) is the Langmuir wave group dispersion coefficient, \( C_s = (T_{eff}/m_h)^{1/2} \) is the effective ion-acoustic speed, \( \lambda_D = (T_{eff}/4\pi n_0 e^2)^{1/2} \) is the effective Debye length, and \( T_{eff} \) is the effective temperature defined by,

\[
T_{eff} = \frac{T_h T_c}{N_h T_e + N_c T_h}, \tag{2}
\]

where, \( N_h = n_{h0}/n_0 \) and \( N_c = n_{c0}/n_0 \) are, respectively, the relative number densities of the hot and the cold components, and \( n_0 = n_{h0} + n_{c0} \) is the unperturbed electron number density. In eq. (1), we have defined, \( \eta = D_0/3\omega_{pe0}\lambda_D^2 \), \( \omega_{pe0} = 4\pi n_0 e^2/m_e \), and the variables \( E \), \( x \) and \( t \) are normalized with respect to \( (4\pi n_0 T_{eff})^{1/2} \), \( \lambda_D \) and \( \omega_p^{-1} \) where \( \omega_p = (4\pi n_0 e^2/m_i) \) is the ion plasma frequency. The quantities \( C_s \) and \( \lambda_D \) generalize thus the definitions of the ion-acoustic speed and the electron Debye length for the usual two-component plasmas.

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