Integrable parameter regimes and stationary states of nonlin-early coupled electromagnetic and ion-acoustic waves

N. N. Rao a)

Theoretical Physics Division, Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India

(Received 5 August 1997; accepted 13 October 1997)

A systematic analysis of the stationary propagation of nonlin-earlyly coupled electromagnetic and
ion-acoustic waves in an unmagnetized plasma via the ponderomotive force is carried out. For small
but finite amplitudes, the governing equations have a Hamiltonian structure, but with a kinetic
energy term that is not positive definite. The Hamiltonian is similar to the well-known Hénon–
Heiles Hamiltonian of nonlinear dynamics, and is completely integrable in three regimes of the
allowed parameter space. The corresponding second invariants of motion are also explicitly
obtained. The integrable parameter regimes correspond to supersonic values of the Mach number,
which characterizes the propagation speed of the coupled waves. On the other hand, in the sub- as
well as near-sonic regimes, the coupled mode equations admit different types of exact analytical
solutions, which represent nonlinear localized eigenstates of the electromagnetic field trapped in the
density cavity due to the ponderomotive potential. While the density cavity has always a single-dip
structure, for larger amplitudes it can support higher-order modes having a larger number of nodes
in the electromagnetic field. In particular, we show the existence of a new type of localized
electromagnetic wave whose field intensity has a triple-hump structure. For typical parameter
values, the triple-hump solitons propagate with larger Mach numbers that are closer to the sonic
limit than the single- as well as the double-hump solitons, but carry a lesser amount of the
electromagnetic field energy. A comparison between the different types of solutions is carried out.
The possibility of the existence of trapped electromagnetic modes having a larger number of humps
is also discussed. © 1998 American Institute of Physics. [S1070-664X(98)04101-9]

I. INTRODUCTION

Electromagnetic wave interaction with plasmas is a fun-
damental process that is of practical interest in the labora-
tory, as well as in space and astrophysical situations. Accord-
ing to the linear theory, electromagnetic waves with
frequencies less than the electron plasma frequency cannot
propagate into a plasma. However, for finite amplitudes,
nonlinear effects arising due to processes such as self-
interaction and relativistic electron-mass variation lead to a
downshift in the local plasma frequency. This has the impor-
tant consequence that the nonlinear wave can propagate into
the plasma, thus carrying the energy into denser regions.
Such a mechanism is important in understanding various
physical processes associated with laser energy deposition
onto the pellet,1,2 rf heating of magnetically confined plasmas,3
and ionospheric modification by powerful electromag-
netic waves.4,5 Large-amplitude electromagnetic waves are
also responsible for spontaneous generation of a mega-
Gauss magnetic field during laser–pellet interaction,6,7 par-
ticle acceleration by laser-driven wakefield structures,8–10
profile modifications, and shock waves.11

Three types of nonlinearities become important for large
laser intensities. First, the self-interaction of the waves leads to
a time-averaged, low-frequency nonlinear force, namely,
the ponderomotive force, which provides the coupling be-
tween the incident electromagnetic field and the low-
frequency dynamics of the plasma.12 The ponderomotive
force expels the plasma from localized regions, leading to
the formation of density cavities. Second, nonlinearities in
the ion dynamics driven by the ponderomotive force become
significant when the cavity amplitude is finite, particularly
for the case when the density fluctuations propagate in the
near-sonic regime.13,14 While both these nonlinearities are
usually the dominant ones, at very high field intensities the
directed component of the electron velocity can be relativis-
tic, in which case the mass variation should be taken into
account.15 Relativistic electron-mass variation typically leads to
frequency downshift, and can enhance the formation of
density cavities.

Over the years, a number of authors have analyzed the
problem of large-amplitude electromagnetic wave propaga-
tion by including one or the other of the nonlinear effects
mentioned above.16–19 These investigations show that non-
linearities can significantly affect wave propagation depend-
ing on the parameter regimes. For example, in the near-sonic
regime, ion nonlinearities together with the charge separation
effects arising from the Poisson’s equation are important.13,14
For moderate intensities in the small but finite-amplitude re-
gime, the nonlinearities due to the ponderomotive force
dominate over those from relativistic electron-mass
variation.20 A self-consistent general formulation of the
problem by including all the three types of nonlinearities,
namely those from the self-interaction, the electron-mass
variation, and the ion dynamics, together with the full Pois-
son equation, leads21 to a pair of governing equations for the
wave fields. These equations are highly nonlinear as well as
coupled, and there are hardly any exact analytical results
related to them. In particular, it has neither been possible to
obtain their exact analytical solutions nor discuss the exis-

a)Electronic mail: raonn@prl.ernet.in