Teleportation in the presence of common bath decoherence at the transmitting station

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We investigate the effect of common bath decoherence on the qubits of Alice in the usual teleportation protocol. The system bath interaction is studied under the central spin model, where the qubits are coupled to the bath spins through isotropic Heisenberg interaction. We have given a more generalized representation of the protocol in terms of density matrices and calculated the average fidelity of the teleported state for different Bell state measurements performed by Alice. The common bath interaction differentiates the outcome of various Bell state measurements made by Alice. There will be a high fidelity teleportation for a singlet measurement made by Alice when both the qubits of Alice interact either ferromagnetically or antiferromagnetically with bath. In contrast if one of Alice's qubits interacts ferromagnetically and the other antiferromagnetically then measurement of Bell states belonging to the triplet sector will give better fidelity. We have also evaluated the average fidelity when Alice prefers nonmaximally entangled states as her basis for measurement.

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I. INTRODUCTION

Quantum protocols are mostly designed in the idealistic situation of a decoherence-free system. In practical implementation of these protocols the external environment can play a significant role in reducing the fidelity of the expected outcomes. The various environmental interactions of the quantum system are dealt using either a harmonic oscillator bath or a spin bath [1]. At low enough temperatures where only a few energy modes of environmental constituents contribute to the dynamics, modeling the bath by a set of spins (finite dimensional) seems to be a more natural choice [2]. On the other hand it was shown that spin environments always lead to non-Markovian decoherence of the quantum system, where the decoherence time scales and saturation value of qubit polarization will depend on the initial polarizations of the both the system and bath spins [3,4]. This dependence can have a considerable effect on quantum protocols, we shall discuss one such situation later in this paper.

In implementing quantum teleportation [5], Bob gets the same unknown state after a proper unitary transformation irrespective of the Bell state measurement made by Alice. The same may not be true in the presence of environmental interaction both at the transmitting and the receiving station. Most of the recent work has been devoted to studying the effect of local noises for both the Alice and Bob qubits [6,7]. It is important to note that Alice is in possession of two qubits where as Bob has only one. Hence the decoherence at the transmitting station can be quite different from that of the receiving station. If the qubits of Alice see different environments, there is nothing new to the dynamics as Bob gets the same decohered state irrespective of the Bell state shared with Alice. On the other hand if both the qubits of Alice see a common environment the situation will be quite different, as there will be bath mediated interaction between the two. This can lead to the bath induced entanglement between Alice's qubits. As the common environment for Alice's qubits seems to be a more natural choice to study the effects of decoherence on teleportation, the present study aims at explicating such effects through an exactly solvable model described below. The problem of common bath decoherence becomes more relevant in cases where Alice and Bob share an n qubit entangled state with n-k qubits at Alice possession and k with Bob [8]. The present work can be a starting point for all such generalized studies.

A. Model

In this paper we shall consider the central spin model to study the effect of spin bath on the qubits used for teleportation. This model can be more realistic in quantum dot systems where the dominant contribution to decoherence comes from the nuclear spins interacting through the homogeneous Heisenberg interaction with the system spins. Recently it was shown how quantum dots can be used as a resource for teleportation [9,10]. The present study can also be relevant for other solid state systems using spins as qubits.

The Hamiltonian describing the interaction between the qubits and the spin bath is given by

$$H = K_a \vec{S}_a \cdot \vec{I}_{\mathcal{E}a} + K_A \vec{S}_A \cdot \vec{I}_{\mathcal{E}A} + K_B \vec{S}_B \cdot \vec{I}_{\mathcal{E}B}, \tag{1}$$

where $\vec{S}_a, \vec{S}_A$ represents the spin operators of the two qubits at the transmitting site which are in possession of Alice and $\vec{S}_B$ the spin operator of Bob's qubit. The total spin of the environmental particles seen by each qubit is represented by $\vec{I}_{\mathcal{E}} = \Sigma_k \vec{I}_{\mathcal{E},k}$. The interaction strengths of the qubits with their respective baths are denoted by K_a, K_A, and K_B. Depending on the common set of environmental spins seen by Alice's qubits, one has either a common spin bath or separate baths. The decoherence at Bob's site is trivial since Bob has only one qubit, we set $K_B = 0$. The interest is on the transmitting side as there are two qubits. It is well-known from the earlier studies that a common bath can induce entanglement between two initially unentangled qubits [11,12]. The common bath interaction between the qubits can lead to a natural selection of two-qubit states which are less prone to decoherence [12]. Hence all these effects can show considerable effect on the fidelity of the transmitted state.

