Coherent states for exactly solvable potentials

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(Received 3 September 2003; published 12 January 2004)

A general algebraic procedure for constructing coherent states of a wide class of exactly solvable potentials, e.g., Morse and Pöschl-Teller, is given. The method, a priori, is potential independent and connects with earlier developed ones, including the oscillator-based approaches for coherent states and their generalizations. This approach can be straightforwardly extended to construct more general coherent states for the quantum-mechanical potential problems, such as the nonlinear coherent states for the oscillators. The time evolution properties of some of these coherent states show revival and fractional revival, as manifested in the autocorrelation functions, as well as, in the quantum carpet structures.

DOI: 10.1103/PhysRevA.69.012102 PACS number(s): 03.65.Fd

I. INTRODUCTION

Coherent states (CS) and their generalizations, in the context of harmonic oscillators, are well studied in the literature [1–4]. Algebraic approaches have been particularly useful for providing a unified treatment of these states and their interrelationships. For example, in Ref. [5], not only a general procedure for constructing a large class of oscillator-based CS has been provided, but it has also been shown that some of these states are dual to each other in a well-defined manner. The algebraic approaches straightforwardly lead to the construction of squeezed and other states showing interesting nonclassical features. These elegant and powerful algebraic procedures of construction owe their origin, partly, to the simplicity of the Heisenberg-Weyl algebra; \([a, a^\dagger] = 1\), characterizing the harmonic oscillator. Based on the symmetries and keeping in mind the desired requirements, various procedures have been developed for constructing CS for Morse [6–9], hydrogen atom [10], Pöschl-Teller [11–14], and other potentials [15]. The role of Morse potential in molecular physics is well known [16]. The study of CS for hydrogenic atoms has assumed increasing importance in light of its relevance to Rydberg states [17], which may find potential application for quantum information processing [18].

It is known that all the criteria desired of a coherent state and found in the oscillator-based CS, e.g., minimum uncertainty product, eigenstate of the annihilation operator (AO), and displacement operator states are not simultaneously achievable in other potential based CS. Hence, a number of CS, diluting one or more of the above criteria, have been constructed in the literature.

In the context of algebraic approaches, supersymmetric (SUSY) quantum mechanics [19] based raising and lowering operators have found significant application. In particular, eigenstates of the lowering operator for Morse [6] and Pöschl-Teller [14] potentials have been found and their properties studied. Recently Antoine et al. [13] have constructed Klauder type CS for the Pöschl-Teller potential, using a matrix realization of ladder operators, their motivation being the temporal stability of the CS. It is well known that the SUSY ladder operators act on the Hilbert space of different Hamiltonians except for the case of the harmonic oscillator. Establishing a precise connection between the complete set of states, describing the above CS, and the symmetries of these potential problems has faced difficulties [8,20]. To be specific, in case of the Barut-Girardello CS for the Morse potential, the ladder operators are taken to be functions of quantum numbers, which has led to problems in defining a proper algebraic structure. To resolve the same, some authors [8] have resorted to the introduction of additional angular coordinates [21].

In this work, we provide an algebraic construction of the CS for a wide class of potentials, belonging to the confluent hypergeometric (CHG) and hypergeometric (HG) classes. The procedure is based on a simple method of solving linear differential equations (DEs) [22], which enables one to express the solutions in terms of monomials. In the space of monomials, it is straightforward to identify various types of ladder operators, their underlying algebraic structures [23], and construct lowering operator eigenstate in a transparent manner. The fact that the monomials and the quantum-mechanical eigenfunctions are connected through similarity transformations, enables one to preserve these algebraic structures at the level of the wave functions and simultaneously obtain the desired CS. Thus the coherent state is initially potential independent. The information about a specific potential is then incorporated by fixing the parameters of the series and also the ground-state wave function of the potential under study. The known results for CS are obtained in specific limits. In addition, our procedure demonstrates the construction of more general CS, similar to the nonlinear CS [24] in the oscillator example. The origin of confusion in identifying the algebraic structure in SUSY based approaches is then pointed out and subsequently resolved in a natural manner. It is shown that the recently found CS for various potentials are related to the two different realizations of su(1,1) algebra.

The paper is organized as follows. Keeping in mind the