Optical activity of a neutrino gas

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For photons that propagate through a gas of neutrinos with a non-zero chemical potential, the left-handed and right-handed polarization modes acquire different dispersion relations. This is due to the \( CP \) and \( CPT \)-odd terms induced by such a background on the photon self-energy. We present a detailed calculation of this effect, which does not depend on any physical assumptions beyond those of the standard electroweak model. Some possible cosmological and astrophysical implications of our results are considered in several contexts, including the recent discussions regarding the rotation of the plane of polarization of electromagnetic waves over cosmological distances. [S0556-2821(98)05021-8]

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I. INTRODUCTION

It was shown some time ago by two of the present authors [1] that, in an isotropic medium, the most general parametrization of the photon polarization tensor \( \Pi_{\mu \nu} \) admits three invariant form factors. This is due to the fact that, in addition to the photon momentum \( k^\mu \), \( \Pi_{\mu \nu} \) depends also on the velocity four vector of the center of mass of the background medium, which we denote by \( u^\mu \). Thus, the most general form of \( \Pi_{\mu \nu} \) consistent with the gauge invariance conditions

\[
k^\mu \Pi_{\mu \nu}(k) = 0, \quad k^\nu \Pi_{\mu \nu}(k) = 0, \tag{1.1}
\]

is

\[
\Pi_{\mu \nu}(k) = \Pi_{T} R_{\mu \nu} + \Pi_{T} Q_{\mu \nu} + \Pi_{P} P_{\mu \nu}, \tag{1.2}
\]

where

\[
R_{\mu \nu} = g_{\mu \nu} - \frac{k_\mu k_\nu}{k^2} - Q_{\mu \nu}, \tag{1.3}
\]

\[
Q_{\mu \nu} = \frac{\bar{u}_\mu \bar{u}_\nu}{u^2}, \tag{1.4}
\]

\[
P_{\mu \nu} = \frac{i}{k} \epsilon_{\mu \nu \lambda \rho} k_\lambda u^\rho, \tag{1.5}
\]

with

\[
\bar{u}_\mu = \left( g_{\mu \nu} - \frac{k_\mu k_\nu}{k^2} \right) u_\nu, \tag{1.6}
\]

and

\[
K = \sqrt{\omega^2 - k^2}, \quad \omega = k \cdot u. \tag{1.7}
\]

In Eq. (1.2), \( \Pi_{T,L,P} \) are scalar functions of the Lorentz invariants \( K \) and \( \omega \), which have the interpretation of being the momentum and energy of the photon in the frame in which the medium is at rest. The functions \( \Pi_{T,L} \) are related to the well-known dielectric and magnetic permeability functions of the medium [2], while \( \Pi_{P} \) is related to a third constant that is responsible for natural optical activity. This third constant, which was called the activity constant in Ref. [1], can arise only as a result of parity (\( P \)), \( CP \) as well as \( CPT \) asymmetric effects. This does not necessarily mean that these symmetries have to be violated at a fundamental level. The desired asymmetries may also come from the background medium. In particular, the \( P \) and \( CP \) breaking effects may arise either from the violation of these symmetries at the level of the fundamental interactions, or from \( P \) and \( CP \) asymmetries in the background. The \( CPT \) breaking effect on the other hand, must necessarily arise from the \( CPT \) asymmetries in the background. Thus, for example, since the weak interactions violate \( P \) and \( CP \) at some level, and since normal matter is \( CPT \)-asymmetric, the activity constant is present in all normal matter.

The activity constant must also have a non-zero value in an asymmetric background of neutrinos and antineutrinos. In this case, parity is broken by the neutrino interactions. In addition, the required \( CP \) and \( CPT \) asymmetries arise if the neutrino gas has a chemical potential, so that the number of neutrinos and antineutrinos in the background is not the same. For such a system, \( \Pi_{P} \) is then expected to depend on \( n_\nu - n_\bar{\nu} \).

This effect can have in principle a number of interesting physical implications. For example, the standard big bang model predicts a primordial neutrino background with a density comparable to that of the microwave background radiation. Light from all astrophysical sources travel through this background before reaching us and, if the background induces some optical activity, it could have interesting physical effects on this light.