Bivariate-$t$ distribution for transition matrix elements in Breit-Wigner to Gaussian domains of interacting particle systems

V. K. B. Kota,¹ N. D. Chavda,² and R. Sahu³

¹Physical Research Laboratory, Ahmedabad 380 009, India
²Department of Physics, Faculty of Science, M.S. University of Baroda, Vadodara 390 001, India
³Department of Physics, Berhampur University, Berhampur 760 007, India

(Received 11 August 2005; published 20 April 2006)

Interacting many-particle systems with a mean-field one-body part plus a chaos generating random two-body interaction having strength $\lambda$ exhibit Poisson to Gaussian orthogonal ensemble and Breit-Wigner (BW) to Gaussian transitions in level fluctuations and strength functions with transition points marked by $\lambda=\lambda_c$ and $\lambda=\lambda_F$, respectively; $\lambda_F \gg \lambda_c$. For these systems a theory for the matrix elements of one-body transition operators is available, as valid in the Gaussian domain, with $\lambda > \lambda_F$, in terms of orbital occupation numbers, level densities, and an integral involving a bivariate Gaussian in the initial and final energies. Here we show that, using a bivariate-$t$ distribution, the theory extends below from the Gaussian regime to the BW regime up to $\lambda=\lambda_c$. This is well tested in numerical calculations for 6 spinless fermions in 12 single-particle states.

DOI: 10.1103/PhysRevE.73.047203 PACS number(s): 05.45.Mt, 05.30.–d, 24.60.Lz, 32.70.–n

Two-body random matrix ensembles apply in a generic way to finite interacting many-fermion systems such as nuclei [1,2], atoms [3,4], quantum dots [5], small metallic grains [6], etc. A common feature of all these systems is that their Hamiltonian $H$ consists of a mean-field one-body $h(1)$ plus a complexity generating two-body $V(2)$ interaction. With this, one has the embedded Gaussian orthogonal ensemble of one- plus two-body interactions [EGOE(1+2)] operating in many-particle spaces [2]; for the definition of EGOE(1+2) for $m$ fermions in $N$ single-particle states, see [2,7]. The most significant aspect of EGOE(1+2) is that as $\lambda$, the strength of the random (represented by the GOE) two-body interaction in $H=h(1)+\lambda V(2)$, changes, in terms of state density, level fluctuations, strength functions, and entropy [8], the ensemble admits three chaos markers. First, it is well known that the state densities take their Hamiltonian $H$ and its $m$-particle eigenstates $|E\rangle$, the transition strengths generated by a one-body transition operator $O$ are denoted by $|\langle E_f|O|E_i\rangle|^2$; $O=\sum_{\alpha\beta} e_{\alpha\beta} a_{\alpha}^\dagger a_{\beta}$ where $e_{\alpha\beta}$ are single-particle matrix elements of the operator $O$, $a_{\alpha}^\dagger$ creates a particle in the single-particle state $\alpha$, and $a_{\beta}$ destroys a particle in the state $\beta$. Note that the one-body transition operators $O$ will not change $m$. Now the bivariate strength density $I_{bic,O}^{H,m}(E_i,E_f)$ is defined by

$$I_{bic,O}^{H,m}(E_i,E_f) = \langle \langle O^\dagger \delta(H-E_f)O \delta(H-E_i) \rangle \rangle^m$$

$$= \langle P^m(E_f) \langle |E_f|O|E_i\rangle|^2 \rangle^m$$

$$= \langle \langle O^\dagger O \rangle \rangle^m I_{bic,O}^{H,m}(E_i,E_f).$$

In Eq. (1), $\langle \langle \cdot \cdot \cdot \rangle \rangle$ denotes a square of the matrix elements of $O$ in $H$ eigenstates weighted by the state densities $P^m(E_i)$ and $P^m(E_f)$ at the initial and final energies.