Scaling in the neutrino mass matrix, $\mu-\tau$ symmetry and the see-saw mechanism

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1. Introduction

There have been many approaches to explain the peculiar (both measured and possible) features of neutrino masses and lepton mixing, for recent reviews see e.g. [1,2]. Very often, the see-saw mechanism [3], and mostly the type I variant, is used to explain the smallness of neutrino masses. As additional motivation, the see-saw mechanism contains all necessary ingredients to produce the baryon asymmetry of the Universe via leptogenesis [4].

Unfortunately, the number of parameters in general see-saw scenarios exceeds the measurable ones, and full reconstruction of the see-saw parameters is at least very difficult [5]. Moreover, irrespective of their origin, it seems very unlikely that (ranges of) all the elements of the neutrino mass matrix can be determined purely from experiments. This has led to postulates of various ansätze or symmetries for the neutrino mass matrix in order to have more predictivity. Texture zeros or $\mu-\tau$ symmetry are popular examples for such strategies. Another ansatz, proposed in Refs. [6,7], is called scaling. In this Letter we wish to show that the scaling hypothesis is deeply connected to a generalized version of the $\mu-\tau$ symmetry and can follow from such symmetry. We also study yet unexplored implications of the scaling hypothesis on the see-saw structure and at the phenomenological level on leptogenesis and lepton flavor violation.

We will first, in Section 3, discuss properties of the Dirac mass matrix $m_D$ in general see-saw frameworks incorporating scaling. We show that the scaling hypothesis uniquely determines its structure. Moreover, scaling is shown to follow from a generalized $\mu-\tau$ invariance applied to the type I see-saw. As the possibility of leptogenesis is one important consequence of the see-saw mechanism, we will investigate leptogenesis in the context of the scaling hypothesis in Section 4. First we study a simple two right-handed neutrino case, followed by a three right-handed neutrino case generated by a $Z_{2L} \times Z_{2R}$ symmetry. In the case of two right-handed neutrinos we show that the baryon asymmetry for unflavored leptogenesis is proportional to the solar neutrino mass-squared difference. The three neutrino model is shown to lead to the same result. Details of the related case of $\mu-\tau$ symmetric see-saw with two heavy neutrinos are delegated to Appendix A, where we also summarize relevant formulae for flavored and unflavored leptogenesis. We conclude and summarize in Section 5. In the following, to set the stage, we will first summarize the properties and predictions of scaling.

2. Neutrino mixing, scaling and generalized $\mu-\tau$ symmetry

In the charged lepton basis, neutrino mass and lepton mixing originates at low energies from the following neutrino mass matrix appearing in the Lagrangian

$$\mathcal{L} = \frac{1}{2} \sum_{\nu} V_{\nu}^\dagger m_\nu V_{\nu} + \text{h.c.}$$  \hspace{1cm} (1)

Diagonalization of $m_\nu$ is achieved via $U^\dagger m_\nu \text{diag} U = m_\nu$, where $U$ is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, whose standard parametrization is

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1 Similar mass matrices with this form have been obtained in specific models in [8,9].