Predictive Flavor Symmetries of the Neutrino Mass Matrix

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Here we propose an $A_4$ flavor symmetry model that implies a lower bound on the neutrinoless double beta decay rate, corresponding to an effective mass parameter $M_{ee} \approx 0.03$ eV, and a direct correlation between the expected magnitude of CP violation in neutrino oscillations and the value of $\sin^2\theta_{13}$, as well as a nearly maximal CP phase $\delta$.


Unless flavor symmetries are assumed, particle masses and mixings are generally undetermined in gauge theories. Understanding mass and mixing constitutes one of the biggest challenges in elementary particle physics. Current observations do not determine all elements of the effective neutrino mass matrix $M_\nu$ completely, and this will be a great challenge even for future experiments. Therefore theoretical ideas restricting the structure of $M_\nu$ are needed in order to guide future searches. One such input studied extensively is the assumption that some entries in the neutrino mass matrix vanish [1].

Here we propose a predictive flavor symmetry for leptons based on a relatively small and simple flavor group, namely $A_4$ or its $Z_3$ subgroup, and briefly analyze its phenomenological implications. We show how this provides a simple means of understanding some of the two-zero textures of $M_\nu$ studied earlier [2].

The discrete group $A_4$ is a 12 element group consisting of even permutations among four objects. The group is small enough to lead to a simple model but large enough to give interesting predictions. The distinguishing feature of $A_4$ compared to other smaller discrete groups is the presence of a three-dimensional irreducible representation appropriate to describe the three generations. This has been exploited in a number of variants. Originally, the $A_4$ was proposed [4,5] for understanding degenerate neutrino spectrum with nearly maximal atmospheric neutrino mixing angle. More recently, predictions for the solar neutrino mixing angle have also been incorporated in so-called tri-bi-maximal [6] neutrino mixing schemes [7–12]. There also exist attempts at unified $A_4$ models [13]. The resulting models, however, are not always simple and usually require many Higgs fields. Here we show that a very simple model based on $A_4$ leads to two-zero textures for $M_\nu$.

The lepton doublets $L_i$ are assigned as the triplet representation in all the $A_4$ models proposed so far. Here we propose the opposite assignment indicated in Table I, where the $L_i$ are assigned to the 1, 1', 1'' representations. The $l_{Ri}$ as well as the Higgs doublets responsible for lepton mass transform as $A_4$ triplets, while the (undisplayed) quarks and the $SU(2)$ Higgs doublet that gives their masses are all singlets under $A_4$. This leads to the following terms responsible for the lepton masses:

\[
-L = h_1 \bar{L}_1 (l_{R1} \Phi)_1 + h_2 \bar{L}_2 (l_{R2} \Phi')_1 + h_3 \bar{L}_3 (l_{R3} \Phi''^*)_1 + h_{1D} \bar{L}_1 (\nu_{R1} \Phi)_1 + h_{2D} \bar{L}_2 (\nu_{R2} \Phi')_1 + h_{3D} \bar{L}_3 (\nu_{R3} \Phi''^*)_1 + \frac{M}{2} \nu_{R1}^T C \nu_{R1} + \text{H.c.,}
\]

where the quantities in parenthesis denote products of two $A_4$-triplets $l_R$ (or $\nu_R$) and $\Phi$ forming the representations 1, 1', 1'', respectively. Note that Eq. (1) includes the most general terms allowed by the symmetry and field content in Table I. Hence, in contrast to many other $A_4$ models, here one does not need to impose any additional symmetry to forbid unwanted terms.

Earlier studies on $A_4$ have shown that it is possible to obtain a minimum of the Higgs potential with equal vacuum expectation values (VEVs) [4]

\[
\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \langle \Phi_3 \rangle = \frac{v}{\sqrt{3}}.
\]

This minimum leads to charged lepton and Dirac neutrino mass matrices $M_l$ and $m_D$ given by, respectively

<table>
<thead>
<tr>
<th>TABLE I. Lepton multiplet structure of the model.</th>
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<tbody>
<tr>
<td>$L_1$</td>
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<tr>
<td>$SU(2)$</td>
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<tr>
<td>$U(1)$</td>
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<tr>
<td>$A_4$</td>
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