Long-range interactions in the quantum many-body problem in one dimension: Ground state

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(Received 17 June 2003; revised manuscript received 8 January 2004; published 30 March 2004; publisher error corrected 7 April 2004)

We investigate the ground state properties of a family of \( N \)-body systems in one dimension, trapped in a polynomial potential and having long-range two-body interaction in addition to the inverse square potential studied in the Calogero-Sutherland model (CSM). We show that for such a Hamiltonian, the ground state energy is similar to that of free fermions in a harmonic well with a displacement that depends on the number of particles and depth of the well. We obtain the ground state wave function and using random matrix results, study the particle density and pair correlation function (PCF). We observe that the particles are arranged in bands. Due to the presence of long-range interaction, the PCF shows a departure from the CSM.

DOI: 10.1103/PhysRevE.69.036118

PACS number(s): 05.30.Jp, 03.65.Ge, 03.75.Kk

Theoretical understanding of the ground state properties of complex many-body systems has received considerable attention in recent years [1–13]. In this context, we study rigorously a wide class of one-dimensional \( N \)-body systems having different densities, nature, and strength of a two-body interaction. We obtain the ground state properties of these systems having a two-body potential \( V_2 = g/r_{12}^2 + \Phi_1 \), where \( r_{12} \) is the interparticle spacing and \( \Phi_1 \) contains the long-range two-body interaction. The system is trapped in a polynomial potential. This may be relevant in understanding the various aspects of the Bose-Einstein condensates, where a wide variety of potentials under a controlled environment is possible.

We derive the ground state eigenvalues and eigenfunctions for such systems and extract several interesting properties by identifying the square of the wave function with the joint probability distribution (JPD) of eigenvalues of non-Gaussian ensembles of random matrices. Using the polynomial method developed by Ghosh and Pandey in the context of random matrix theory (RMT) [14,15], we observe band structure [16] in the particle density, which in turn corresponds to the density of zeros of the corresponding polynomials [7,14,15]. For a given value of the interaction strength, we study the pair correlation function (PCF) for different interparticle spacings. Due to the presence of the long-range interaction, we observe a deviation from the Calogero-Sutherland model (CSM) [1,2].

We shall consider the ground state of a system of \( N \) particles satisfying the Schrödinger equation

\[
-\sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} + \sum_{i<j} \frac{g}{(x_i-x_j)^2} + \Phi_1(x_i,x_j) + V_1(x_i) - E_n \psi_n = 0, \tag{1}
\]

where terms corresponding to long-range interaction derive from the relation

\[
\Phi_1(x_i,x_j) = \frac{h}{r_0} \sum_{i \neq j} \frac{P(x_i)}{(x_i-x_j)^2}, \tag{2}
\]

\( P(x) \) being any analytic function having a power expansion in \( x \), while \( g \) and \( h \) are the interaction strengths. The system is trapped in a potential

\[
V_1(x_i) = \sum_{i=1}^{N} [P^2(x_i) - P^4(x_i)]. \tag{3}
\]

In this paper, we consider the case where \( P(x) \) is a polynomial of order \( 2m + 1 \) and is represented by

\[
P(x) = \gamma \sum_{k=0}^{m} a_{2k+1} x^{2k+1}. \tag{4}
\]

For convenience, we take \( a_{2m+1} = 1 \). The parameter \( \gamma \) will determine the depth of the well. The case where \( m = 0 \) corresponds to the CSM [1], where the two-body interaction is purely of the inverse square type. For \( m > 0 \), we will encounter the long-range interaction.

For such a Hamiltonian, the ground state wave function can be written as

\[
\psi_0 = \phi \Psi = \prod_{i<j} |x_i - x_j|^\beta \exp \left[ -\sum_{i} \int_{0}^{x_i} P(t_i)dt_i \right]. \tag{5}
\]

It should be noted that if the particles are confined to a configuration space \( x_1 > x_2 > \cdots > x_N \), the \( g/r^2 \) interaction in one dimension does not allow them to cross, thereby respecting the ordering. This is valid provided the potential is not too attractive, which leads to the restriction \( g \geq -1/2 \). For such an ordering, the modulus becomes irrelevant and hence depending on the value of \( \lambda \) (odd or even) the system is bosonic or fermionic in nature. Also, for such a configuration, \( \psi_0 \) is nodeless (apart from the trivial ones at the points of coincidence) and hence corresponds to the ground state of the system. Here

\[
\phi = \prod_{i<j} |x_i - x_j|^\lambda, \tag{6}
\]

\[
\varphi = \exp \left[ -\sum_{i} \int_{0}^{x_i} P(t_i)dt_i \right], \tag{7}
\]