Implementing Deutsch-Jozsa algorithm using light shifts and atomic ensembles

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We present an optical scheme to implement the Deutsch-Jozsa algorithm using ac Stark shifts. The scheme uses an atomic ensemble consisting of four-level atoms interacting dispersively with a field. This leads to a Hamiltonian in the atom-field basis which is quite suitable for quantum computation. We show how one can implement the algorithm by performing proper one- and two-qubit operations. We emphasize that in our model the decoherence is expected to be minimal due to our usage of atomic ground states and freely propagating photon.

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I. INTRODUCTION

Quantum computers are expected to be much faster than the classical ones, especially in performing some specific jobs, like factorization, searching, etc. For example, any function of $N$ binary variables (bits) can be identified as a constant (single valued) or balanced (bivalent) by a single enquiry through the well-known Deutsch-Jozsa algorithm [1], whereas classically it needs up to $(2^{N-1}+1)$ enquiries [2]. Let us consider $N=2$ bits. There are $2^N=4$ possible states 00, 01, 10, 11, which we can designate as four values of a function $f(x)$ to determine its characteristic, whereas this algorithm needs only one evaluation for the same. Further, in Grover’s search algorithm [3], searching for a specific state from an unsorted database of $N$ states requires $O(\sqrt{N})$ repetitions of a certain unitary operation, whereas any classical computer would need $O(N)$ attempts. All these faster algorithms have been made possible by the special property of a quantum state, which enables one to apply the same unitary operations simultaneously on a number of basis states (quantum parallelism).

These algorithms have been implemented using nuclear magnetic resonance technologies, trapped ions, etc. Bulk nuclear magnetic resonance has been used to implement scalable versions of the Deutsch-Jozsa algorithm [4] and Grover’s algorithm [5]. The Deutsch-Jozsa algorithm using selective pulses [6] and quadrupolar nuclei [7] in nuclear magnetic resonance systems has been demonstrated. Guide et al. [8] have implemented the Deutsch-Jozsa algorithm in an ion trap system. Theoretical proposals to implement Grover’s algorithm using trapped ions [9] and cavity quantum electrodynamics [10] have been reported.

In this paper, we propose an optical system to implement the Deutsch-Jozsa algorithm. We use an ensemble of four-level atoms interacting dispersively with a field to obtain an effective Hamiltonian like the one on which a typical nuclear magnetic resonance experiment is based [11]. This enables us to implement various two-qubit and single-qubit operations to realize the Deutsch-Jozsa algorithm. The entire algorithm can be implemented by using, for example, the clock transitions in the Cs atomic ensemble.

The structure of the paper is as follows. In Sec. II, we briefly discuss the basic method to implement the Deutsch-Jozsa algorithm. In Sec. III, we describe the model and the relevant Hamiltonian. In Sec. IV, we provide the required pulse sequence to implement the algorithm using our model.

II. BASIC REQUIREMENTS OF THE DEUTSCH-JOZSA ALGORITHM

To start with, let us recall the main features of this algorithm. This algorithm is used to determine whether a given function is constant or balanced. In the simplest case, we consider a one-bit output function $f(x)$ (which can take only the value 0 or 1) of a one-bit input $x$ ($x=0$ or 1). If $f_1(x)=0$ and $f_2(x)=1$, then these functions are constant. On the other hand, if $f_3(x)=x$ and $f_4(x)=\overline{x}$, then they are balanced functions. In Table I, we show the above functions $f_n(x)$. The one-bit version of the Deutsch-Jozsa algorithm ($N=1$) [8] determines the characteristics of the function by a single function call, contrary to its classical counterpart, which requires $2^{N-1}+1=2$ function calls to check whether the function is constant or balanced. In fact, in this algorithm, one calculates the value $f(0)\oplus f(1)$ (where $\oplus$ denotes addition modulo 2). This yields 0 (or 1) for a constant (or balanced) function.

In Fig. 1, we show the basic circuit [2] to perform one-bit version of the Deutsch-Jozsa algorithm. We start with the initial state of the two qubits as

\[
\begin{array}{c|c|c}
\text{Input} & f_1(x) & f_2(x) \\
\hline
0 & 0 & 1 \\
1 & 0 & 1 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
\text{Input} & f_3(x) & f_4(x) \\
\hline
0 & 0 & 1 \\
1 & 0 & 0 \\
\end{array}
\]

TABLE I. Different one-bit functions showing their characterizations.

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