Entanglement production in coupled chaotic systems: Case of the kicked tops

Jayendra N. Bandyopadhyay* and Arul Lakshminarayan1,4
Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India
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Entanglement production in coupled chaotic systems is studied with the help of kicked tops. Deriving the correct classical map, we have used the reduced Husimi function, the Husimi function of the reduced density matrix, to visualize the possible behaviors of a wave packet. We have studied a phase-space based measure of the complexity of a state and used random matrix theory (RMT) to model the strongly chaotic cases. Extensive numerical studies have been done for the entanglement production in coupled kicked tops corresponding to different underlying classical dynamics and different coupling strengths. An approximate formula, based on RMT, is derived for the entanglement production in coupled strongly chaotic systems. This formula, applicable for arbitrary coupling strengths and also valid for long time, complements and extends significantly recent perturbation theories for strongly chaotic weakly coupled systems.

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I. INTRODUCTION

A quantum mechanical system, which consists of at least two interacting subsystems, has an unique property called ‘entanglement’ [1]. This property is unique in the sense that even if we know the exact state of the system, it is in general not possible to assign any pure state to the subsystems. Entanglement is a nonclassical correlation among the subsystems which exists even between spatially well separated subsystems [2]. This unique property of a quantum system has been characterized as a quantum resource for quantum information theory and quantum computation [3]. Moreover, quantum entanglement has also been studied extensively from the decoherence point of view. It has been argued that a quantum system in the presence of an “environment” can loose its coherence and behave more like a classical system [4].

A quantum computer is a collection of many interacting particles. Such a many-particle structure may be prone to problems of decoherence and chaos. Decoherence can create some errors in the operation of a quantum computer, however, these errors, in principle, can be removed by quantum error correcting codes [3]. On the other hand, the problem due to chaos has recently attracted some attention. It has been shown that residual, uncontrolled interaction between the particles might induce quantum chaos in the quantum computer if the interaction strength crosses certain critical limits and consequently, it may destroy the operational condition of the quantum computer [5]. Besides quantum chaos can also emerge during the implementation of some quantum algorithms [6]. Obviously, a quantum algorithm which simulates a quantum chaotic system is by definition a unitary operation showing quantum chaos [7]. However, it has been shown that well-known algorithms, such as Grover’s search algorithm and the quantum Fourier transform algorithm give rise to some unusual combination of quantum signatures of chaos and of integrability [6]. The error due to the presence of chaos in a quantum computer can also be corrected by error correcting codes, but the presence of chaos enhances the complexity and hence much more error correction is needed [8]. Therefore, the knowledge of the presence and effects of chaos in a quantum computer is necessary to implement proper error correcting codes. Very recently, the behavior of quantum entanglement during the operation of an efficient algorithm for quantum chaos have been studied [9]. However, here we are interested at a more basic level to study the effect of the underlying classical dynamics on entanglement production.

Recently, several studies have explored this question [10–18]. The first one studied the entanglement production in an N-atom Jaynes-Cummings model [10], and they found that the entanglement rate was considerably enhanced if the initial wave packet was placed in a chaotic region. They also argued that their results support an earlier conjecture which predicted that the entanglement production in a chaotic system, coupled to an environment, would be more than the regular system [19]. According to that conjecture, the entanglement production rate would be higher for a chaotic system coupled to an environment. For the N-atom Jaynes-Cummings model, each atomic subsystem plays the role of an environment for the other. Later, it has been shown that large entanglement production rate is not the hallmark of a nonintegrable system [11]. Even in the integrable N-atom Jaynes-Cummings model some special initial coherent states exhibit strikingly similar entanglement production as corresponding to the chaotic case [12].

In another paper, the entanglement production rate has been related to the classical Lyapunov exponents with the help of a coupled kicked tops model [13]. They also justified their findings on the basis of the above mentioned conjecture [19]. However, the classical limit of the coupled kicked tops derived in this rather well-quoted work is incorrect, in fact it is not even canonical. However, they consider very weakly coupled tops and therefore their conclusions turn out to be qualitatively valid. In other work, one of us studied the entanglement in coupled standard maps and found that en-