Computing the multifractal spectrum from time series: An algorithmic approach

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We show that the existing methods for computing the \( f(\alpha) \) spectrum from a time series can be improved by using a new algorithmic scheme. The scheme relies on the basic idea that the smooth convex profile of a typical \( f(\alpha) \) spectrum can be fitted with an analytic function involving a set of four independent parameters. While the standard existing schemes [P. Grassberger et al., J. Stat. Phys. 51, 135 (1988); A. Chhabra and R. V. Jensen, Phys. Rev. Lett. 62, 1327 (1989)] generally compute only an incomplete \( f(\alpha) \) spectrum (usually the top portion), we show that this can be overcome by an algorithmic approach, which is automated to compute the \( D_q \) and \( f(\alpha) \) spectra from a time series for any embedding dimension. The scheme is first tested with the logistic attractor with known \( f(\alpha) \) curve and subsequently applied to higher-dimensional cases. We also show that the scheme can be effectively adapted for analyzing practical time series involving noise, with examples from two widely different real world systems. Moreover, some preliminary results indicating that the set of four independent parameters may be used as diagnostic measures are also included. © 2009 American Institute of Physics.

It is now well established that multifractal sets and objects abound in nature. A characteristic feature of these objects is the self-similarity since their formation is governed by subtle scaling laws. An important tool to analyze these sets is the \( f(\alpha) \) spectrum, which describes how the fractal dimensions of the interwoven sets with definite singularity strength are distributed. In the recent issue of Chaos, participating in the discussion “Is the normal heart rate chaotic?,” many authors12 stressed the importance of multifractality in the study of heart rate variability and suggested that it can provide a new observational window into the complexity mechanism of heart rate control. The study also highlights the need for evaluating new nonlinear parameters for a better physiological investigation and for finding new clinical applications. Here, we present a novel automated scheme to compute the \( f(\alpha) \) spectrum of a multifractal attractor from its time series. We show that the scheme can be applied to synthetic as well as practical time series involving noise. It also provides us with an additional set of two independent parameters apart from the conventional \( \alpha_{\text{min}} \) and \( \alpha_{\text{max}} \) to characterize any general \( f(\alpha) \) curve. The utility of these parameters from the point of view of diagnostic measures is also pursued by analyzing a few cases of physiological time series.

I. INTRODUCTION

Multifractal sets and objects form the supporting structure of nonlinear phenomena, prime examples are strange

\begin{align}
\alpha &= \frac{d}{dq}[(q-1)D_q], \\
f(\alpha) &= q\alpha - (q-1)D_q. 
\end{align}

However, such a procedure is generally considered to be very difficult when done subjectively as it involves first smoothing the \( D_q \) curve and then Legendre transforming. Moreover, the error bar from the smoothing procedure makes