Effects of spontaneously generated coherence on the pump-probe response of a \( \Lambda \) system

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Near-degenerate lower levels in a \( \Lambda \) system have an additional coherence term due to interaction with the vacuum of the radiation field. We report the effects of this \textit{spontaneously generated coherence} on the formation of a trapped state in the presence of two coherent fields of arbitrary intensity. We show that such coherence preserves both electromagnetically induced transparency and coherent population trapping (CPT) phenomena. However, it changes the time scales associated with the formation of the CPT state, and brings about quantitative changes in the line profiles. We present a clear analytical explanation for our numerical results. We also report the dependence of line shapes on the relative phase between the two applied fields.

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I. INTRODUCTION

It is now well understood how the decay of a system of closely lying states induced by interaction with a common bath leads to new types of coherences [1–8]. These coherences modify, among other things, the line shapes of spontaneous emission. An early study of a V system consisting of two degenerate levels showed the possibility of coherent trapping in the excited state [1]. This system and its generalizations have been the subject of recent studies in connection with the production of quantum beats and probe absorption [4–6]. Javanainen [8] discussed the possibility of spontaneously generated coherence (SGC) effects in a \( \Lambda \) system. In particular, he examined the response of the \( \Lambda \) system to an external pump of arbitrary intensity, and demonstrated that the dark state could disappear in the presence of a strong spontaneously generated coherence. The existence of these coherence effects depends on the nonorthogonality of the two dipole matrix elements.

More explicitly, the basic equations describing spontaneous emission from an excited state to two close-lying lower states are given by [8] (Fig. 1 with \( \vec{e}_1 = \vec{e}_2 = 0 \))

\[
\dot{\rho}_{11} = -2(\gamma_1 + \gamma_2)\rho_{11}, \quad \dot{\rho}_{12} = -(\gamma_1 + \gamma_2)\rho_{12}, \\
\dot{\rho}_{22} = 2\gamma_2\rho_{11}, \quad \dot{\rho}_{13} = -(\gamma_1 + \gamma_2)\rho_{13}, \\
\dot{\rho}_{33} = 2\gamma_1\rho_{11}, \quad \dot{\rho}_{23} = 2\sqrt{\gamma_1\gamma_2}\cos\theta\rho_{11}e^{i\Omega t}.
\]

Here \( 2\gamma \)'s represent the spontaneous emission rate, \( \theta \) is the angle between the two induced dipole moments \( \vec{d}_{12} \) and \( \vec{d}_{13} \), and \( \hbar\Omega \) is the energy spacing between the two ground levels. All the elements of \( \rho \) above, except \( \rho_{23} \), have their usual dependence. It should be noted that only if small \( \Omega \) are the effects of generated coherence between [2] and [3] important, as for large \( \Omega \) the rapid oscillations in \( \rho_{23} \) will average out any such effects. While Javanainen examined the effects of such coherences on the response of the system to a single field of arbitrary intensity, we examine its effect on electromagnetically induced transparency (EIT) [9,10] and coherent population trapping (CPT) [11] phenomena.

The organization of this paper is as follows: In Sec. II, we present the effects of SGC on the absorption and dispersion line shapes. We show the existence of CPT even in the presence of SGC. In Sec. III, we study the approach to CPT, i.e., we study dynamical effects. In Sec. IV, we develop an analysis which explains the numerical results of Secs. II and III in a transparent manner. In Sec. V, we show the dependence of the line shapes on the relative phase between the pump and probe fields. Finally in Sec. VI, we discuss the kind of situations where SGC will be nonzero and connect to other recent works on this subject.

II. ABSORPTION AND DISPERSION LINE SHAPES IN PRESENCE OF SGC

We consider a \( \Lambda \) system driven by two coherent fields with amplitudes \( \vec{e}_1 \) and \( \vec{e}_2 \). Since the dipole moments are not orthogonal, we have to consider an arrangement where each field (pump and probe) acts on one transition. This can be achieved by considering the case shown in Fig. 1, where the probe (pump) acts on the transition \( |1\rangle \rightarrow |3\rangle \) (\( |1\rangle \rightarrow |2\rangle \)). In such case the density-matrix equation in the rotating-wave approximation will be

\[
\dot{\rho}_{11} = -2(\gamma_1 + \gamma_2)\rho_{11} + ig\rho_{13} + iG\rho_{21} - iG^*\rho_{12} - ig^*\rho_{13}, \\
\dot{\rho}_{22} = 2\gamma_2\rho_{11} + iG^*\rho_{12} - iG\rho_{21}, \\
\dot{\rho}_{13} = 2\gamma_1\rho_{11} + ig^*\rho_{13} - ig\rho_{31}, \\
\dot{\rho}_{23} = -i(\Delta_1 - \Delta_2)\rho_{23} + 2\sqrt{\gamma_1\gamma_2}\cos\theta\rho_{11} + iG^*\rho_{13} - ig\rho_{21},
\]

where \( \eta \) will be zero (one) if the spontaneously generated coherence effect is ignored (included). Here the coupling coefficients are denoted as \( G = \vec{d}_{12}\vec{e}_2/h \) and \( g = \vec{d}_{13}\vec{e}_1/h \), i.e., \( G = G_0\sin\theta \) and \( g = g_0\sin\theta \), and the detunings are \( \Delta_2 = W_{12} - w \) and \( \Delta_1 = W_{13} - w \), where \( hW_{ij} \) is the energy separation between states \( |i\rangle \) and \( |j\rangle \). For simplicity we assume \( \vec{d} \cdot \vec{e}_i \) (\( i = 1,2 \)) to be real, and ignore the dephasing terms. The general steady-state analytical solution for \( \rho_{13} \) in all orders of probe and pump is

\[ \rho_{13} \text{ as a function of } \rho_{13} \text{ and } \Delta_2 \text{ and } \eta. \]