Coherence properties of sunlight

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The coherence properties of sunlight were first studied by Verdet around 1869 and were later examined by other scientists. However, all the previous calculations assumed that the Earth is in the far zone of the Sun, an assumption that is incorrect. An investigation of why Verdet's result is nevertheless correct reveals a surprising property of radiation from incoherent sources. © 2004 Optical Society of America

One of the basic problems related to the theory of optical coherence is the determination of the coherence properties of sunlight that is incident upon the surface of the Earth. The earliest determination was made by Verdet and is considered the first calculation of the coherence properties of light. Here is a key passage from his publication on the subject:

The points to which all the elements of the source transmit practically identical movements are contained in...a circle whose center is the point P and is of radius

$$\frac{R}{\rho} h \lambda$$

It is only in the interior of this circle that the vibrations can be considered as coherent on the sphere S.

In applying these results to the case where the luminous source is the Sun, one is surprised by the smallness of the region in which the movements can be considered coherent.

In the above formula, $\rho$ is the radius of the source, $R$ is the distance from the source to the observation point, $\lambda$ is the wavelength, and $h$ is a numerical factor “certainly less than 1/4.” Because the diameter of the circle of coherent movements is linear with respect to distance from the source, Verdet’s calculation suggests, within its scope of validity, that the angular diameter of the region of coherence is constant with respect to this distance.

Since Verdet’s time, this calculation of the coherence of sunlight has been performed more quantitatively (for instance, Ref. 3, Sect. 10.4.2, and Ref. 4, Sect. 4.2.2 and 4.4.4). However, such calculations assume that the Earth is in the far zone of the Sun, an assumption that does not hold, even approximately. For instance, the far zone of a source may be defined as the region at a distance $r$ from the source at which the Fresnel number that the source subtends at the observation point is much smaller than unity, i.e.,

$$\frac{\pi a^2}{\lambda r} \ll 1,$$

where $a$ is the radius of the source and $\lambda$ is the wavelength of the radiation. With filtered sunlight at the Earth’s surface, $\lambda \approx 500$ nm, $a = 6.96 \times 10^5$ km, and consequently the far zone of the Sun is at distance $r >> 3 \times 10^8$ km, a condition not even remotely satisfied by the distance between the Earth and the Sun, $r = 1.5 \times 10^8$ km. It is therefore of interest to examine whether Verdet’s estimate, and other estimates, of the coherence area of sunlight on the Earth’s surface are correct.

We consider the properties of a scalar wave field radiated by the surface of a spherical source of radius $a$ centered at the origin (see Fig. 1). The second-order statistical properties of the field at frequency $\omega$ outside the source may be characterized by the cross-spectral density function, defined as (Ref. 4, Sect. 4.7.2)

$$W(r_1, r_2, \omega) = \langle U^*(r_1, \omega) U(r_2, \omega) \rangle,$$

where $U(r, \omega)$ is a monochromatic realization of the (statistical) field at frequency $\omega$ and the angle brackets denote ensemble averaging over these realizations. The field $U(r, \omega)$, which is a solution of the scalar Helmholtz equation, may be represented everywhere outside the source domain in a series of the form

$$U(r, \omega) = \sum_{lm} c_{lm} h_{l}^{(1)}(kr) Y_{lm}(\theta, \phi),$$

where $h_{l}^{(1)}$ is the spherical Hankel function of the first kind and order $l$, $Y_{lm}$ is the spherical harmonic of order $l$, $m$, and $c_{lm}$ are random coefficients that depend upon the statistical properties of the field on the surface of the source. On substituting from Eq. (3) into Eq. (2),