Diffraction tomography using power extinction measurements

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We propose a new method for determining structures of semitransparent media from measurements of the extinguished power in scattering experiments. The method circumvents the problem of measuring the phase of the scattered field. We illustrate how this technique may be used to reconstruct both deterministic and random scatterers. © 1999 Optical Society of America [S0740-3232(99)00211-2]

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1. INTRODUCTION

In the short-wavelength limit, tomographic reconstruction of two- and three-dimensional media has long been carried out from intensity measurements. More accurate methods of reconstruction that take into account diffraction require knowledge of both the field amplitude and the phase of the scattered field. For rapidly varying fields such as optical fields the phase may be prohibitively difficult to measure and presents, at best, a technical challenge at lower frequencies. In this paper we propose a method to circumvent the phase problem. We will show that one can determine a function that is related to the scattering amplitude and makes it possible to reconstruct the scattering object for certain model media.

We begin by recalling a well-known result in scattering theory, the optical cross-section theorem. It relates the total power extinguished from a plane wave on scattering to the scattering amplitude in the forward (incident) direction. More explicitly, let

\[ \psi^{(s)}(r, t) = \psi^{(1)}(r) \exp(-i \omega t) \]  

be a monochromatic field incident on the scatterer. We assume that it is a plane wave that propagates in the direction of a unit vector \( \mathbf{s}_0 \),

\[ \psi^{(1)}(r) = a \exp(i kr \cdot \mathbf{s}_0), \]  

with \( k = \omega/c \), \( c \) being the speed of light in vacuum. Let

\[ \psi^{(s)}(r, t) = \psi^{(1)}(r) \exp(-i \omega t) \]

represent the scattered wave. The total field [with time dependence \( \exp(-i \omega t) \) being omitted from now on] is then given by the expression

\[ \psi(r) = \psi^{(1)}(r) + \psi^{(s)}(r). \]  

In the far zone in a direction specified by the unit vector \( \mathbf{s} \), the scattered field has the asymptotic form

\[ \psi^{(s)}(r) \sim \frac{\exp(i kr)}{r} f(s, s_0), \]  

\[ f(s, s_0) \] being the so-called scattering amplitude.

The total power extinguished from the incident field as a result of scattering and absorption is given by the formula

\[ P_e = |a|^2 \left( \frac{4\pi}{k} \right) \Im \{ f(s_0, s) \}, \]  

where \( \Im \) denotes the imaginary part. In general, one needs to know the scattering amplitude for all directions of incidence and scattering in order to reconstruct the low-pass-filtered version of the scattering object; however, equation (1.6) gives information only about the imaginary part of the scattering amplitude \( f(s, s_0) \) in the forward direction \( s = s_0 \). Within the accuracy of the first-order Born approximation, \( f(s_1, s_2) \) is related to the Fourier transform of the susceptibility \( \eta(r) \) of the medium by the formula

\[ f(s_1, s_2) = k^2 \int \eta(r) \exp(-i k r \cdot (s_2 - s_1)) d^3 r, \]  

and consequently Eq. (1.6) yields information only about the volume integral of the imaginary part of the susceptibility of the scattering object.

We will make use of a recent generalization of the optical cross-section theorem to introduce a method of determining a complex function that is related to the scattering amplitude of the object whose structure is to be determined. It is possible to determine this function experimentally from measurements of power alone. In many cases this function is simply related to the structure of the object.

2. THE DATA FUNCTION

Let us consider the power extinguished from a coherent beam consisting of two monochromatic plane waves,