

Generation, characterization and application of vector vortex beams

A thesis submitted in partial fulfilment of
the requirements for the degree of

Doctor of Philosophy

by

Subith Kumar P M

(Roll No. 17330032)

Under the supervision of

Dr. Goutam K. Samanta

Professor

Atomic, Molecular and Optical Physics Division

Physical Research Laboratory, Ahmedabad, India



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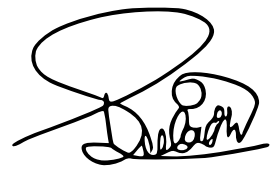
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Dedicated to

My Achamma Smt.Thankam

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Name: Subith Kumar P M

(Roll No: 17330032)

Date:

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It is certified that the work contained in the thesis titled **“Generation, characterization and application of vector vortex beams”** by Mr. Subith Kumar P M (Roll No. 17330032), has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

Dr. G. K. Samanta

Professor

Atomic, Molecular and Optical Physics Division,

Physical Research Laboratory,

Ahmedabad, India.

(Thesis Supervisor)

Date:

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Abstract

The study of “structuring light” entails the construction of optical beams according to their fundamental characteristics. These characteristics include but are not limited to intensity, phase, and polarization. Vector vortex beams are structured optical beams in which the light has been modulated in all these degrees of freedom, making it possible to exploit light’s inherent vectorial nature. Material processing, biomedical applications, and optical manipulation are just some of the many fields that have found a use for vector vortex beams. Furthermore, such beams have recently been used to visualize intricate mathematical concepts like the Hilbert hotel paradox, Möbius strips, and Lissajous figures.

In the current thesis, we have theoretically and experimentally studied three different classes of vector vortex beams having intriguing features that can be useful for various applications. First, we studied the Full Poincaré (FP) beams, a special class of structured beams that contain all possible polarization states on the surface of the Poincaré sphere. Such beams are readily created by linear optical techniques and, more recently, by nonlinear optical processes. An inherent limitation in the nonlinear generation of FP beams is the inability to achieve all polarization states coined as coverage, due to modal size, polarization, and modal weighting changes during the nonlinear conversion of the constituent modes. We have devised a simple technique to control the coverage of FP beams, using second harmonic generation as an example, from fully scalar (no coverage) to fully vectorial (full coverage). Our study confirms the vectorial characteristics of the FP beams. It also reveals a balancing act between mode order, modal nonlinear efficiency, and initial relative modal weights, all in close agreement with that theoretically predicted.

Using the same polarization coverage measurement technique, we have studied another special class of beam known Poincaré Bessel beams (PBB), carrying polarization singularities, nondiffraction, and self-healing properties. Using the mode transformation of FP beam in rectangular basis ideally carrying 100% polarization coverage of polarization states represented on the surface of the Poincaré sphere, we observe the PBB as the superposition of an infinite number of FP beams as each ring of PBB has polarization coverage $>75\%$. We also observe the resilience of PBB's degree of polarization to perturbation. The polarization-ellipse orientation map of PBBs shows infinite series of C-point singularity pairs. The number of such infinite series is decided by the number of C-point singularity pairs of FP beam. The dynamics of C-point singularity pairs in the self-healing process show the generation of new C-point singularity pairs due to the beam obstruction, which annihilates with each other and also with the intrinsic singularity pairs and eventually reconstitute the singularity pattern. Such dynamics show an optical analogy of Hilbert's Hotel type setting representing mathematics of infinite sets.

Further, we have experimentally demonstrated the exact Hilbert's Hotel transition establishing the optical analogy of mathematics of infinite sets using the new types of vector beams called fractional vector beams. Historically, infinity was long considered a vague concept – boundless, endless, larger than the largest – without any quantifiable mathematical foundation. This view changed in the 1800s through the pioneering work of Georg Cantor, showing that infinite sets follow their own seemingly paradoxical mathematical rules. In 1924, David Hilbert highlighted the strangeness of infinity through a thought experiment now referred to as the Hilbert Hotel paradox, or simply Hilbert's Hotel. The paradox describes an “fully” occupied imaginary hotel having an infinite number of single-occupancy rooms, the manager can always find a room for new guests by simply shifting current guests to the next highest room, leav-

ing the first room vacant. The investigation of wavefield singularities has uncovered the existence of a direct optical analogy to Hilbert's thought experiment. Since then, efforts have been made to investigate the properties of Hilbert's Hotel by controlling the dynamics of phase singularities in "fractional" order optical vortex beams. To take such proposals to the next level and experimentally demonstrate Hilbert's Hotel using both phase and polarization singularities of optical fields. Using a multi-ramped spiral-phase-plate and a supercontinuum source, we generated and controlled fractional order vortex beams for the practical implementation of Hilbert's Hotel in scalar and vector vortex beams. Using a multi-ramped spiral-phase-plate, we show the possibility for complicated transitions of the generalized Hilbert's Hotel.

Our findings will be of value to the communities interested in nonlinear structured light, particularly for vectorial nonlinear modal creators and detectors and control of quantum hybrid entangled states, imaging in the presence of depolarizing surroundings, studying turbulent atmospheric channels, and in visualizing unusual mathematical concepts and also for fundamental and applied research.

Keywords: Structured beams, Vector beams, Nonlinear optics, second harmonic generation, polarization optics, Mathematics of infinity, lasers, ultrafast lasers, light modulators.

List of Publications

Journals

1. Evolution of C-point singularities and polarization coverage of Poincare-Bessel beam in self-healing process, Subith Kumar, Anupam Pal, Arash Shiri, G. K. Samanta, and Greg Gbur, Scientific Reports 14 , 16647 (2024).
2. Simple experimental realization of optical Hilbert Hotel using scalar and vector fractional vortex beams (Cover page, Featured) , Subith Kumar, Anirban Ghosh, Chahat Kaushik, Arash Shiri, Greg Gbur, Sudhir Sharma, and G. K. Samanta APL Photonics 8, 066105 (2023).
3. Controlling the coverage of full Poincare beams through second-harmonic generation, Subith Kumar, Ravi K. Saripalli, Anirban Ghosh, Wagner T. Buono, Andrew Forbes, and G.K. Samanta, Physical Review Applied 19, 034082 (2023)

Conference proceedings

1. Hilbert hotel realization using Poincare Bessel beam, Subith Kumar, Anupam Pal, Arash Shiri, G. K. Samanta, and Greg Gbur, Frontiers in Optics, JTu5B. 15
2. Frequency-doubling characteristics of non-collinear Poincaré beams, Subith Ku-

mar, Ravi K. Saripalli, Anirban Ghosh, and G.K. Samanta, *Frontiers in Optics*, FM7C. 2

3. Generation of non-collinear Poincaré beam through the spin-orbit interaction in epsilon-near-zero film, Anirban Ghosh, RK Saripalli, K Subith, N Apurv Chaitanya, Varun Sharma, Israel De Leon and GK Samanta, *Frontiers in Optics*, FM7C. 3

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Chapter 1

Introduction

The invention of lasers has advanced science and technology very much ahead due to their wide variety of applications, including material processing, information technology, laser spectroscopy, photochemistry, medical diagnostics, and reaction dynamics. The majority of these applications used the laser beam in the Gaussian spatial profile. However, photons feature multiple degrees of freedom, such as wavelength/frequency, amplitude, phase, time, polarization, and spatial structure. Therefore, manipulation of different degrees of freedom of photons enhances the reach of lasers to the diversity of light-based applications. While attention has been paid with time to manipulating the standard physical parameters of light, the manipulation of the spatial structure of the laser beam has attracted increasing interest. In fact, the tailoring of the spatial structures of light beams resulted in new classes of special light beams with added features. Some of the important spatial structured beams are vortex beams, Bessel beams, Airy beams, hollow Gaussian beams, and so on. The optical vortices [1–3], having phase singularities (phase dislocations) in the wavefront, carry vanishing intensity at the singular point. Due to the screw-like (helical) phase structure around the point of

singularity, such beams carry orbital angular momentum (OAM). The OAM associated with optical vortices is important for high-resolution microscopy [4–6], quantum information [7–9], material processing [10, 11], particle micro-manipulation [12, 13] and lithography [14, 15]. Similarly, the Bessel beams [16–19] and Airy beams [20–22] form the new class of beams known as non-diffracting beams. The spatial structure of such beams remains unchanged during beam propagation. These beams also have peculiar self-healing properties. As a result, these beams retain their spatial structure after beam obstruction, making them travel deeper in biological samples [23]. In addition to the peculiar properties, like the Bessel beams, the Airy beams [20, 21] have additional self-acceleration properties. Due to such unique properties, the Bessel and Airy beams find many applications, including micro-particle manipulation [12, 24, 25], optically mediated particle clearing [25, 26], long-distance communication [27–29], and nonlinear frequency conversion [19, 30]. In addition to manipulating the spatial mode of the Gaussian beams forming the scalar structured beams, one can use both the spatial mode and the polarization degree of freedom of the light to form a new class of structured beams known as vector beams with additional features. Unlike the scalar-structured beams, which have the same polarization states, the vector beam shows different polarization states in different local positions on the transverse plane. In fact, if the vector beam is passed through a polarizer, a change in the angle of the polarizer will result in different energy density distributions. Vector beams have attracted a great deal of attention for their wide range of applications, such as microscopy imaging [5, 31], quantum information [32–35], laser manipulation [36–38], and many more.

Conventionally, structured optical beams are generated through the spatial mode conversion of a Gaussian laser beam using mode converters. Some of the very prominent mode converters are the holographic techniques based on liquid crystal spatial light modulators (SLMs) [39–41], spiral phase plates (SPPs) [42–45], q-plate [46–48],

and cylindrical mode converters [49]. However, the availability of dynamic phase modulation and control makes the SLMs the preferred choice over other mode converters to generate structured beams of any spatial features, including the Airy and Bessel beams. However, the low damage threshold of the SLMs restricts their use for the generation of structured beams with low output powers. On the other hand, many applications, including nonlinear interactions, demand structured beams with high output powers. For such applications, dielectric material-based mode converters such as SPPs, cubic phase plates, and axicon are selected based on the type of structured beams. On the other hand, the high-power vector vortex beams are conventionally generated through the superposition of the scalar-structured beams with orthogonal polarization states using a polarization-based interferometer. For example, the collinear superposition of the horizontal polarized scalar vortex (phase singular) beam of vortex order, $+l$, with the vertical polarized scalar vortex beam of vortex order, $-l$, forms a vector vortex beam of order, l . The propagation of this beam through a polarizer with a fast axis at an arbitrary angle will result in an intensity pattern in the necklace form with the number of petals twice the order of the constituent vortex. However, if one of the constituent beams of the vector vortex beam is a Gaussian beam, it forms a new class of beam known as the full Poincaré beam having both polarization and phase singularities. As the name suggests, such beams carry all possible polarization states represented on the surface of the Poincaré sphere. The increase in vortex order results in higher-order full Poincaré beams. Even though it is theoretically possible to generate full Poincaré beams to cover the entire surface of the Poincaré sphere, in practice, the polarization coverage of these beams depends on various factors such as the relative intensity, size, and divergence of the beams forming the full Poincaré beams. On the other hand, there is no established method for estimating the practical polarization coverage of such beams. Therefore, it is essential to devise a generalized method for estimating

the polarization coverage of any vector beam. On the other hand, it is essential to study the change in the polarization pattern/topology and the coverage of such a beam in the light-matter interaction. Such a study is essential to take advantage of the special features of the full Poincaré beams, such as smaller scintillation than comparable beams of uniform polarization in the presence of atmospheric turbulence [50, 51], to help in designing the optimum special polarization structured beams for such applications and also to study the physical processes manipulating the Poincaré beams on the different parts of the Poincaré sphere [52]. On the other hand, scalar Bessel beams have been extensively studied in the literature. In most cases, the spatial structure of the Bessel beams has been used for any practical application. As polarization is an important degree of freedom of light, the addition of a polarization structure to the Bessel beam will certainly enhance our fundamental understanding and also extend the reach of such beams for future applications. The inclusion of a polarization pattern in the Bessel beam can, in principle, be achieved through the generation of a full Poincaré Bessel beam. Since the full Poincaré beams generated from vortex beams of order l carry l pairs of positively and negatively charged C-point polarization singularities, and the Bessel beams have concentric dark and bright rings of infinite spatial extend, one can expect the generation of an infinite number of C-point singularity pairs. In recent years, efforts have been made to use light beams to understand mathematical concepts. One such example is the theoretical proposal on the realization of the Hilbert Hotel paradox to understand the concept of infinity [53, 54] using the phase and polarization singularities of fractional scalar and vector vortex beams. Therefore, the exploration of the polarization pattern of full Poincaré beams with the possibility of having an infinite number of C-point singularity pairs might lead to the experimental realization of the mathematical concept of infinite numbers. On the other hand, efforts have been made for the experimental realization of such theoretical proposals using scalar vor-

tex beams. However, there is no use of a fractional vector vortex beam to verify the theoretical proposal of the Hilbert hotel paradox.

To address some of the important issues discussed above, in the current thesis, we have performed various experiments to study vector vortex beams. We have devised a new generalized method for the characterization of the full Poincaré beam to measure the polarization coverage of such a beam on the surface of the Poincaré sphere. We also experimentally verified the beam parameters influencing the polarization coverage of the vector beams. We also devised a simple technique to control the coverage of full Poincaré beams, using second harmonic generation as an example, from fully scalar (no coverage) to fully vectorial (full coverage). Using mode conversion of the full Poincaré beam using an axicon, we have generated a new class of Poincaré Bessel beam and studied its polarization self-healing characteristics. We observe the transformation of a full Poincaré beam of any number of C-point polarization singularity pairs into a series of an infinite number of C-point polarization singularity pairs. In the self-healing study, we observed that in addition to the polarization self-healing, a new set of C-point polarization singularity pairs appeared at the beam-blocking point, which subsequently replaced the existing C-point singularities. After the healing distance, the beam returns to its initial C-point polarization singularity distribution, resembling the Hilbert Hotel paradox. Finally, we have generated fractional scalar and vector vortex beams from a fixed SPP and experimentally verified the theoretical proposal of the Hilbert hotel paradox with both the phase and polarization singularities of the beams.

The current thesis, organized into seven chapters, deals with three topics: polarization optics, structured optical beams, and nonlinear optics. To give a better perspective on the fields, we have discussed the basic principles and theoretical framework to understand polarization optics, structured optical beams, and nonlinear optical pro-

cesses in the second chapter. The third chapter of the thesis presents the generation and characterization of vector beams, especially the full Poincaré beam. While the characterization of such beams is done through the study of their polarization structure, one of the interesting properties of the full Poincaré beams is their polarization coverage. However, there is no standard and easy technique to evaluate polarization coverage. We have presented a new technique to estimate the polarization coverage of full Poincaré beams. The concept is presented in the third chapter. The ideal full Poincaré beams contain all possible polarization states on the surface of the Poincaré sphere. They are readily created by linear optical techniques and, more recently, by nonlinear optical processes. An inherent limitation in the latter is the inability to achieve all polarization states, coined coverage, due to modal size, polarization, and modal weighting changes during the nonlinear conversion of the constituent modes. In chapter four, we demonstrate a simple technique to control the coverage of full Poincaré beams, using second harmonic generation as an example, from fully scalar (no coverage) to fully vectorial (full coverage). We have also done a related theory that confirms the experimental results for the vectorial characteristics of the generated beams and the balancing act between mode order, modal nonlinear efficiency, and initial relative modal weights for the polarization coverage of the Full Poincaré. In the fifth chapter, we have extended the full Poincaré to generate and characterize a new class of beam called full Poincaré Bessel beams. The Bessel beams of all orders have peculiar properties in terms of non-divergence and self-healing with beam propagation. Such properties of the Bessel beams have been well studied by monitoring the intensity profile of the beams. In addition to the common properties of Bessel beams, the full Poincaré Bessel beams have a polarization structure in the transverse plane. Therefore, we have studied the non-divergence and self-healing properties of the beam in polarization degrees of freedom. We have seen that the full Poincaré Bessel beam

carries an infinite number of pairs of lemon and star polarization singularities. It is also observed that the polarization singularities of the full Poincaré Bessel beam show self-healing characteristics. We also observed that the self-healing of the polarization singularity shows a resemblance to the Hilbert hotel paradox. In 1924, David Hilbert highlighted the strangeness of infinity through a thought experiment now referred to as the Hilbert Hotel paradox, or simply Hilbert's Hotel. The investigation of wave-field singularities has uncovered the existence of a direct optical analogy to Hilbert's thought experiment. Using a multi-ramped spiral-phase plate and a supercontinuum source, we generated and controlled fractional-order vortex beams for the practical implementation of Hilbert's Hotel in scalar and vector vortex beams. Using a multi-ramped spiral-phase plate, we show the possibility for complicated transitions of the generalized Hilbert's Hotel in sixth chapter. Finally, in chapter seven, we present a future outlook on the work reported in the thesis.

Chapter 2

Fundamental principles

Polarization is one of the most important properties of light and has been known for many centuries—the first observation of polarization credited to Erasmus Bartholinus. In 1669, he noticed that when a ray of natural light goes through a calcite crystal, it splits into two new beams of the same intensity. Christian Huygens soon after figured out that the polarization of these two rays was different from each other. However, it was not until 1803, when Young showed that light vibrates in a plane perpendicular to its movement, this was linked to the fact that light is transverse. An officer in the French army named Etienne-Louis Malus was in the Palais de Luxembourg in Paris in 1808 when he made an important discovery. He looked at the sun's reflection on a window pane through a calcite crystal. Turning the calcite crystal, he saw that the two images made by double refraction went out in turns. Malus told us about this result, but he did not say why. A few years later, in 1812, Sir David Brewster also looked into how light behaves when it bounces off the glass. He found that the reflected light seen through a calcite crystal could be turned off at a certain angle of incidence, called Brewster's angle. Brewster did more research and found a simple

link between what became known as the Brewster angle and the glass's refraction index. This work was even more important because it made it possible to measure the index of refraction of optical glass by reflection instead of by refraction (transmission). While Brewster was hard at work in Great Britain, Augustin Jean Fresnel was using the Fresnel–Huygens integral to solve the problem of diffraction and give the wave theory a solid theoretical base. In 1818, the Paris Academy of Science gave him a prize for solving the diffraction problem. This was because his friend and colleague Dominique Francois Arago proved that there was a small bright spot in the shadow of a small circular disc, which Fresnel's theory had predicted would happen. The wave theory got even better when it was used to explain how polarized light moves through an optically active medium. The wave theory of light became almost universally accepted because of Fresnel's and others' work. In classical optics, the wave equation is used as a hypothesis. It was accepted because it helped people understand and describe how light moves, bends, interferes, and gets polarized. Also, the results of the calculations using the wave equations were exactly the same as the results of the experiments. Before James Clerk Maxwell's electrodynamic theory and Heinrich Hertz's experiments in the second half of the 1800s, the wave equation did not have a solid experimental basis. It is clear that the polarization state is related to the changes in the electric field over time in the plane perpendicular to the propagation plane. Also, a wave is said to be unpolarized if the electric field changes in time in a random way. On the other hand, it is said to be polarized if it changes a certain way. The best way to begin the study of the polarization of the light in the wave equation from Maxwell's equation. The wave equation for a light propagating in an isotropic medium can be written by a set of three independent wave equations as follows,

$$\nabla^2 U_i(r,t) = \frac{1}{v} \frac{\partial^2 U_i(r,t)}{\partial t^2} \quad i = x, y, z \quad (2.1)$$

The optical field vector (electric field vector $\mathbf{E}$ or magnetic field vector $\mathbf{H}$) in the cylindrical coordinate system is represented by $\mathbf{U}(\mathbf{r}, t)$, and the velocity of propagation is represented by v . When light propagates along the z direction, the transverse components are $U_x(r, t)$ and $U_y(r, t)$, and the longitudinal component is $U_z(r, t)$. Later in 1818, after a series of experiments based on Young's interference experiment using polarised light, Fresnel and Arago concluded with the absence of the longitudinal component of light. So a most simplified solution for the Equation 2.1 for an optical field propagating along the z -axis can be written as,

$$U_x(r, t) = U_{0x} \cos(A + \delta_x) \quad (2.2a)$$

$$U_y(r, t) = U_{0y} \cos(A + \delta_y) \quad (2.2b)$$

U_{0x} and U_{0y} are the maximum amplitudes, while $A = \omega t + \mathbf{k} \cdot \mathbf{r}$ is the propagation term and δ_x and δ_y are the phases for each axis component. Because each component varies with space and time, every point in space forms a resulting field as the instantaneous vector sum of each component in the equations 2.2a and 2.2b during light propagation. The locus of the electric field vector at a given point can be derived [55] by restructuring the equations 2.2a and 2.2b,

$$\frac{U_x^2}{U_{0x}^2} + \frac{U_y^2}{U_{0y}^2} - 2 \frac{U_x}{U_{0x}} \frac{U_y}{U_{0y}} \cos \delta = \sin^2 \delta \quad (2.3)$$

where,

$$\delta = \delta_y - \delta_x \quad (2.4)$$

The equation 2.3 is identical to an ellipse equation; as a result, the locus of the

electric field vector will propagate in the form of an ellipse commonly recognized as polarisation ellipse as shown in Figure 2.1. The product term $U_x U_y$ results in a tilt on the polarisation ellipse. The tilt is quantified by the angle between the x-axis and the axis along the semi-major axis of the ellipse (x') called ellipse orientation angle, ψ . The other parameter for quantifying the polarisation ellipse is the ellipticity of polarisation ellipse, χ calculated by $\tan^{-1} b/a$. The semi-major and semi-minor axes of the polarization ellipse are a and b . Six commonly used special cases of polarisation ellipses are depicted in Figure 2.1 b – g. In most situations, the light beams have a uniform polarisation throughout the transverse plane. Since this often makes the vector nature of the light insignificant. So the light beams are generally referred to as scalar beams. There is another variant of optical beams where the polarisation of the light varies all over the transverse plane. In such optical beams, the vector nature of the optical field becomes significant and such beams are often referred to as vector beams. Over the past years, many studies have been conducted to understand such beams' fundamental properties. The study revealed a list of applications of vector beams such as high numerical focusing [56], Enhanced beam stability while propagating through turbulent mediums [51], optical manipulations [57] etc. For a better understanding of vector beams, it is essential to characterize polarisation effectively. But for the optical fields oscillating 10^{15} times every second, it is a very short interval to sufficiently quantify the optical field for observing polarisation in real life. One of the best methods for characterizing polarisation is by measuring stokes parameters.

2.1 Stokes parameters

Sir Gabriel Stokes solved the problem of characterizing polarization using the magnitude and phase of the optical field during his studies on partially polarized light in 1852 [58]. Stokes figured out that light polarisation can be defined by four parameters,

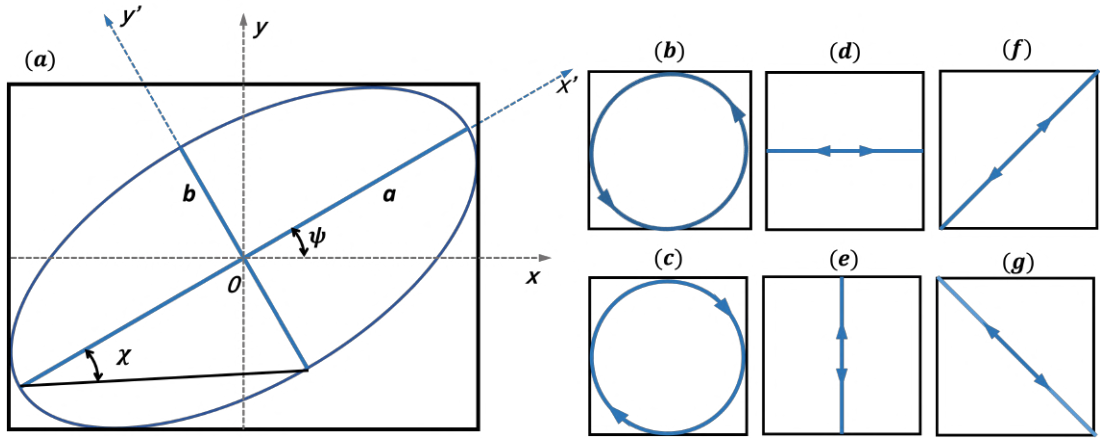


Figure 2.1: (a) polarisation ellipse, b-g: six special cases of polarisation ellipse; (b) Left circular polarisation(LCP),(c) Right circular polarisation(RCP),(d) Horizontal polarisation(H),(e) Vertical polarisation (V),(f) Diagonal polarisation (D), (g) Anti-diagonal (A).

which can be calculated from the easily measurable intensity of light. The Stokes parameters for a monochromatic plane wave in terms of electric field ($\mathbf{E}$) can be written as,

$$S_0 = E_x E_x^* + E_y E_y^* \quad (2.5a)$$

$$S_1 = E_x E_x^* - E_y E_y^* \quad (2.5b)$$

$$S_2 = E_x E_y^* + E_y E_x^* \quad (2.5c)$$

$$S_3 = i(E_x E_y^* - E_y E_x^*) \quad (2.5d)$$

The parameter S_0 is proportional to the intensity of the optical field. The horizontal/vertical, diagonal/anti-diagonal, and left/right circular contributions in a given polarization are represented by S_1 , S_2 , and S_3 . Further, the four parameters are related by the expression,

$$S_0^2 \geq S_1^2 + S_2^2 + S_3^2 \quad (2.6)$$

The equal sign is valid for completely polarised light. The sum of the squares of the Stokes parameters S_1 , S_2 , and S_3 for a partially polarized light will be less than the square S_0 . For a completely polarised light, all Stokes parameters vanish except S_0 . The ratio between the right and left-hand sides of the equation 2.6 is known as the Degree of polarisation (DoP). As the name suggests, the Degree of polarisation indicates how much the light is polarised. The degree of polarization (DoP) of completely polarized light is one; for unpolarized light, it is zero. The degree of polarization of a partially polarized light ranges between 0 and 1. Further, the Stokes parameters S_1 , S_2 , and S_3 are related to the ellipse orientation (ψ) and ellipticity(χ) by the following relation:

$$S_0 = I_0 \quad (2.7a)$$

$$S_1 = I_P \cos 2\chi \cos 2\psi \quad (2.7b)$$

$$S_2 = I_P \cos 2\chi \sin 2\psi \quad (2.7c)$$

$$S_3 = I_P \sin 2\chi \quad (2.7d)$$

Here, I_0 is the net intensity, and $I_P = \sqrt{S_1^2 + S_2^2 + S_3^2}$ denotes the polarised intensity component of the beam. In the case of totally polarized light ($I_0 = I_P$), the relations are as follows:

$$S_1 = S_0 \cos 2\chi \cos 2\psi \quad (2.8a)$$

$$S_2 = S_0 \cos 2\chi \sin 2\psi \quad (2.8b)$$

$$S_3 = S_0 \sin 2\chi \quad (2.8c)$$

From the above set of expressions, we can find the expression for the ellipse orientation and ellipticity as,

$$2\psi = \tan^{-1} \frac{S_2}{S_1} \quad (2.9a)$$

$$2\chi = \sin^{-1} \frac{S_3}{S_0} \quad (2.9b)$$

For a completely polarised light, Eq. 2.6 becomes the equation of a sphere. As a result the Stokes parameters S_1 , S_2 and S_3 can be geometrically represented as the Cartesian coordinates of a point P on a Poincaré sphere with radius S_0 as depicted in the Figure 2.2. From the equation 2.8, it can also be understood that the double of ellipse orientation (2ψ) and ellipticity (2χ) form the other spherical coordinate of the point P. In some of the special cases, e.g., when $S_1 = S_0$, all other Stokes parameters vanish, and as a result, the Cartesian coordinate of the point P becomes $(S_0, 0, 0)$, representing a horizontal polarization. Similarly, in the other special cases coordinates, $(-S_0, 0, 0)$, $(0, S_0, 0)$, $(0, -S_0, 0)$, $(0, 0, S_0)$, and $(0, 0, -S_0)$ of the point P represents the vertical, diagonal, anti-diagonal, left circular, and right circular polarisation respectively.

Poincaré sphere is a variation of the Bloch sphere in optics for visualizing polarisation. Similar to the Bloch sphere, the diametrically opposite points in the Poincaré

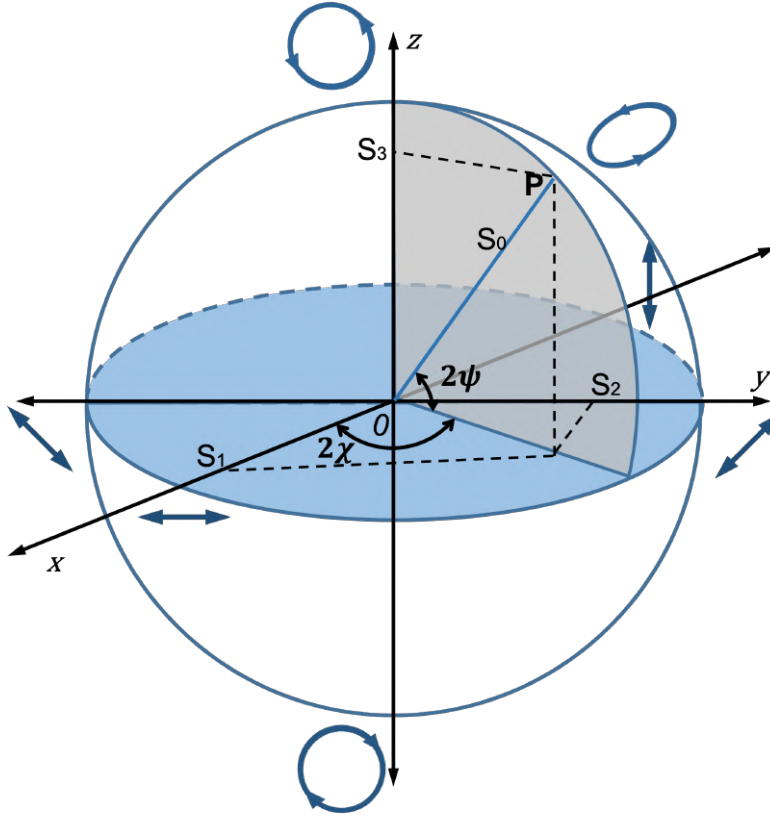


Figure 2.2: Geometrical representation of polarization as a point on the surface of the Poincaré sphere

sphere will be orthogonal. This makes any diagonally opposite polarisation can take as the basis for representing any other polarization. For example, using the complex representation $\alpha |H\rangle + \beta |V\rangle$, horizontal (H) and vertical (V) polarization (represented by the points $(S_0, 0, 0)$, $(-S_0, 0, 0)$ in a Poincaré sphere) can be used to represent all other polarizations. Here α and β are the complex coefficients for each polarisation state and can be normalized as $\alpha^2 + \beta^2 = 1$. Figure 2.3 shows the Bloch sphere representation of Poincaré sphere in terms of commonly used three pairs of polarisation states. This representation of points on the Poincaré sphere in terms of two orthogonal polarisations is handy for visualizing different polarisation states.

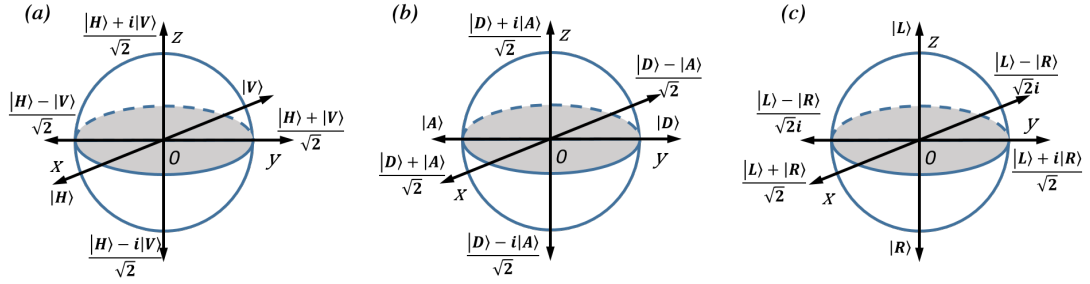


Figure 2.3: Bloch sphere representation of Poincare sphere in terms of (a) Horizontal($|H\rangle$)- Vertical($|V\rangle$), (b) Diagonal($|D\rangle$)- Anti-diagonal($|A\rangle$), (c) Left circular($|L\rangle$)- Right circular($|R\rangle$) polarisation states.

2.2 Structured beams

Structured light beams are created by varying the intensity, phase, and polarization of a light beam. Most examples of structured light fields in the past have only considered the scalar nature of the optical beams. That is, restructuring the phase and intensity profiles of the light beams by neglecting or treating polarization as a free degree of freedom in the design process. This estimation is essential for converting Eq. 2.1 into a scalar form, reducing the complexity of solving the wave equation. The solutions for the wave equation mentioned in Eq. 2.2 are the most simplified solutions for the wave equation that represents the hypothetical plane waves. In real-life situations, such infinitely extended light beams are not possible. One of the most often used approximations for solving the wave equation is the paraxial wave approximation, which reduces the wave equation into a much more easily solvable Helmholtz equation as mentioned in the equation 2.10.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik \frac{\partial}{\partial z} \right) U(x, y, z) = 0 \quad (2.10)$$

The most simple solution for the paraxial equation is the solution in the form of a

2D Gaussian function called the Gaussian beam. The Gaussian beam is the most stable solution of the paraxial beam equation in the case of a laser cavity. The solution for the paraxial equation in terms of the Gaussian equation is as follows,

$$U_G(\mathbf{r}) = U_0 e^{-i\Phi(z)} \left[\frac{1}{\sqrt{1 + z^2/z_0^2}} e^{ik(x^2+y^2)/2R(z)} \right] e^{-(x^2+y^2)/w^2(z)} \quad (2.11)$$

where, U_0 is the beam's amplitude, $z_0 = \pi w_0^2/\lambda$ is the Rayleigh range, $R(z) = z + z_0^2/z$ is the wavefront curvature, $w(z) = w_0 \sqrt{1 + z^2/z_0^2}$ is the beam width at the distance z and $\Phi(z) = \arctan(z/z_0)$ is the Gouy phase shift as the function of z . Even though the Gaussian mode is the most stable mode of the laser cavity, some of the other commonly known solutions for the paraxial equation, especially in the form of special functions, attracted enormous scientific interest in the past decades due to its vast range of applications. Some notable examples are Hermite-Gaussian beams, Laguerre-Gaussian beams, Bessel beams, and Airy beams.

2.2.1 Hermite-Gaussian beams

Because of astigmatism introduced in the laser cavity, rectangular symmetric Hermite-Gaussian beams are frequently formed in the laser cavity. The Hermite-Gaussian beams are the solutions of the paraxial wave equation in the Cartesian coordinate system. The equation for Hermite-Gaussian beams with order (m, n) can be written as,

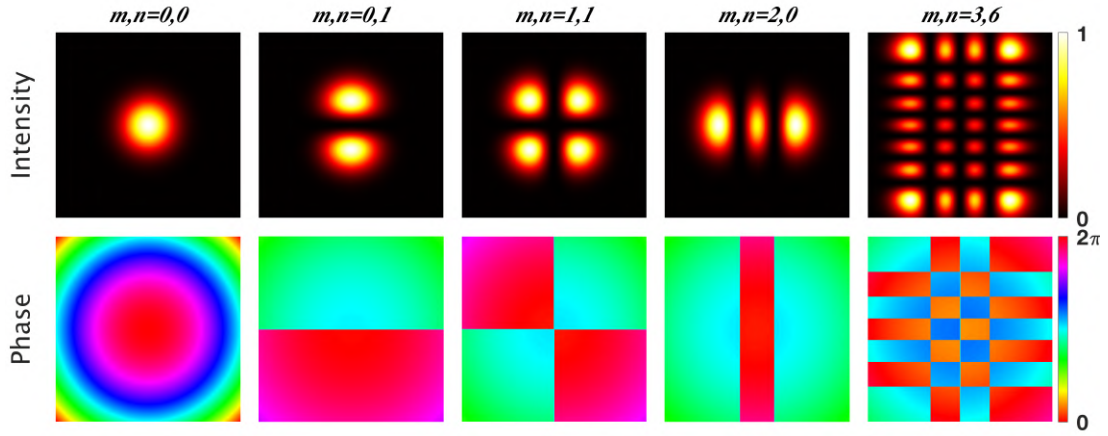


Figure 2.4: Intensity (first row) and Phase (second row) profile of the various Hermite Gaussian modes with mode order m, n .

$$U_{l,m}^{LG}(\rho, \phi, z) = U_0 \left[\frac{w_0}{w(z)} \right] \left(\frac{\rho}{w(z)} \right)^l L_m^l L_m^l \left(\frac{2\rho^2}{w^2(z)} \right) \exp \left(-\frac{\rho^2}{w^2(z)} \right) \times \exp \left[ikz + ik\frac{\rho^2}{2R(z)} + il\phi + i(l+2m+1)\Phi(z) \right] \quad (2.12)$$

here H_n and H_m are the m^{th} and n^{th} order Hermite polynomial . It needs to be noted that when $(m, n) = (0, 0)$, the Hermite-Gaussian beams will become the Gaussian beam. Figure 2.4 shows the intensity and phase profile for the different orders of Hermite-Gaussian beams. orders

2.2.2 Laguerre-Gaussian beam

The vortex beams are the solutions for the equation 2.10 in the cylindrical coordinates (ρ, ϕ, z) . Vortex beams attracted great attention after Allen, and his colleagues [1] discovered that such beams could carry orbital angular momentum. The complex amplitude profile of the vortex beams is represented by U_{lm}^{LG} and can be expressed with the equation 2.13.

$$\begin{aligned}
U_{l,m}^{LG}(\rho, \phi, z) = U_0 \left[\frac{w_0}{w(z)} \right] \left(\frac{\rho}{w(z)} \right)^{|l|} L_m^l \left(\frac{2\rho^2}{w^2(z)} \right) \exp \left(-\frac{\rho^2}{w^2(z)} \right) \\
\times \exp \left[ikz + ik\frac{\rho^2}{2R(z)} + il\phi + i(l+2m+1)\Phi(z) \right]
\end{aligned} \tag{2.13}$$

Similar to the Hermite-Gaussian beam, the above expression will form a complete set of solutions for the paraxial wave equation 2.10. The L_m^l is the Laguerre-Gaussian polynomial with azimuthal ($l = \dots, -2, -1, 0, 1, 2, \dots$) and radial ($m = 1, 2, 3, \dots$) indices respectively. Similar to the Hermite-Gaussian beams, the lowest order of the Laguerre-Gaussian beam will be the Gaussian beam. The Laguerre-Gaussian beam is circularly symmetric because the intensity, which is a function of ρ and z but not of ϕ , is proportional to the absolute square of the amplitude mentioned in the equation 2.13. The transverse intensity distribution of the LG beam takes on the form of a doughnut. In the case of Laguerre-Gaussian beam with $l \neq 0$, the maximum value of intensity will be at a radius of $\rho = \sqrt{|l|/2}w(z)$, and it increases along with the propagation distance z as similar to the divergence of Gaussian beam. This is clear in the intensity profile depicted in the first row of figure 2.5 for $l = -2$ to 2 . The most interesting properties of the LG beams are related to the azimuthally varying phase term $\exp(il\phi)$. From the second row of the figure 2.5, it was clear that this most prominent phase will produce a phase singularity (A point where the phase of the optical beam is undefined) at the center surrounded by l number of phase winding. As a result, along the z direction, the wavefront of the LG beam will form l distinct inter-winded helical (left-handed for $l > 0$ and right-handed for $l < 0$) structure resulting in a corkscrew-like motion during the beam propagation. This characteristic wavefront geometry forms a skew angle between the pointing vector and the propagation direction and results in a rotational motion of the pointing vector around the propagation direction. The skew angle of the

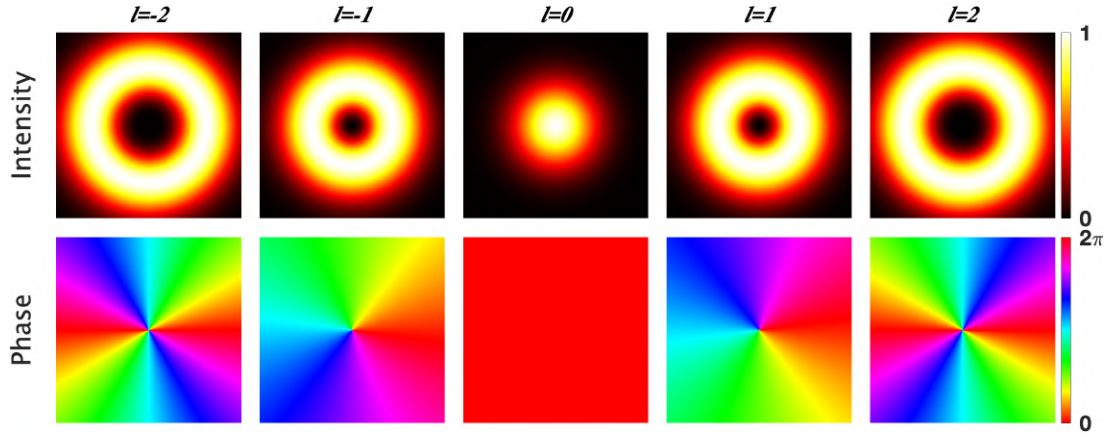


Figure 2.5: Intensity (first row) and Phase (second row) profile of the LG beam for topological order, $l = -2$ to 2 .

wave vector with respect to the optical axis can be estimated as l/kr [59]. This imparts an azimuthal component to the linear momentum flow of $l\hbar/r$, resulting in an angular momentum per photon of $l\hbar$. In contrast, to spin angular momentum, which has only two distinct states, spin up and spin down, orbital angular momentum has an infinite number of orthogonal states corresponding to the integer values of l .

Due to their unique properties, LG beams have great application potential in many fields. LG beam can be utilized for information coding, which can significantly improve optical communication's information capacity and security [60, 61]. In optical tweezers, the unique phase distribution of LG beams can be used to capture and manipulate particles to rotate [62, 63]. The distinctive "doughnut" intensity distribution can be used to improve imaging resolution in Stimulated Emission Depletion Microscopy (STED) [5, 64]. Furthermore, LG beams have found important applications in gravitational wave detection [65, 66] and quantum entanglement [67].

2.2.3 Bessel beam

Bessel light beams are immune to diffraction, making them potentially an attractive alternative to using Gaussian beams in several scenarios. Bessel beams are one of the solutions to the paraxial equation in cylindrical coordinates [18, 68] diffraction, making them. The following equation can describe the electric field of the Bessel beam:

$$E(r, \phi, z) = A_0 \exp(ik_z z) J_n(k_r r) \exp(\pm in\phi) \quad (2.14)$$

where J_n is the n^{th} order Bessel function, and k_r and k_z are the radial and longitudinal wave-vector components of the free-space wave-vector, such that $k = \sqrt{(k_z^2 + k_r^2)}$, and r , ϕ and z are the radial, azimuthal and longitudinal components respectively. Bessel beams have a set of concentric ring structures, and the zeroth order has a central maximum, whereas all the higher-order Bessel beams have zero intensity at the center. All the higher-order Bessel beams have an orbital angular momentum and phase singularity of charge l associated with the azimuthal phase term. The mathematical

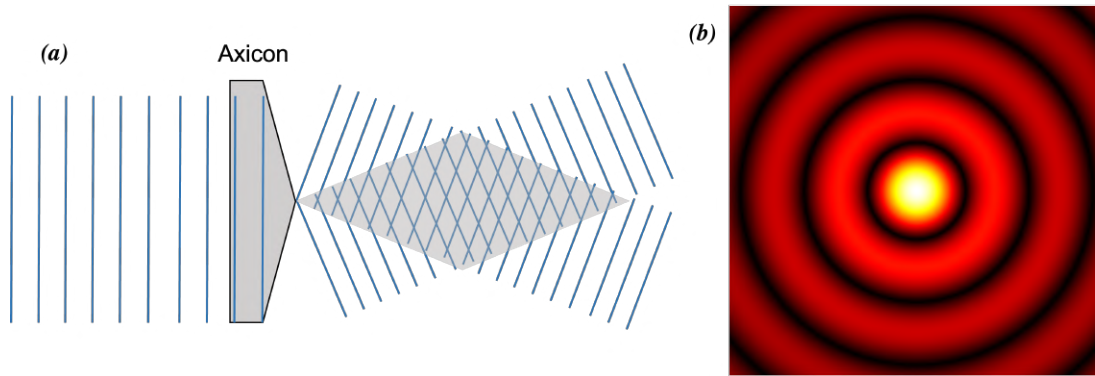


Figure 2.6: (a) The generation of Bessel beam from an Axicon (b) Intensity profile of Bessel beam

description of the Bessel beam shows an infinite number of rings; thus, an infinite area would carry an infinite amount of power. Therefore, it is evident that in the laboratory, a 'quasi-Bessel beam,' i.e., finite energy Bessel beam with finite energy, can be realized, which possesses all the properties of the Bessel beam over a finite distance.

Figure 2.6 shows the generation of the Bessel beam using Axicon and Bessel beam's intensity profiles.

2.2.4 Airy beam

The Airy beam, unlike other paraxial beams, has the unusual characteristics of propagating in a parabolic trajectory, self-healing, and non-divergence [20, 21]. An ideal Airy beam propagates up to an infinite distance without spreading in the spatial profile; however, it necessitates carrying infinite energy to exist. As a result, only finite-energy airy beams can be produced experimentally while retaining all of their peculiar properties. The 1-D and 2-D Airy beams experience an acceleration transverse to their propagation direction, resulting in the beam's parabolic trajectory.

Like other non-diffractive beams, such as Bessel beams, the Airy beam regenerates after encountering an obstruction along its propagation [22]. An airy beam is the interference of multiple waves with a cubic phase front; when an obstruction is placed in the beam path, as long as some waves are able to pass the obstruction, the waves can interfere beyond it and reconstruct the beam. These unique features of Airy beams find application in particle acceleration[69], self-imaging[70], and light-sheet microscopy[71]. The finite energy Airy beam along the propagation direction is defined as,

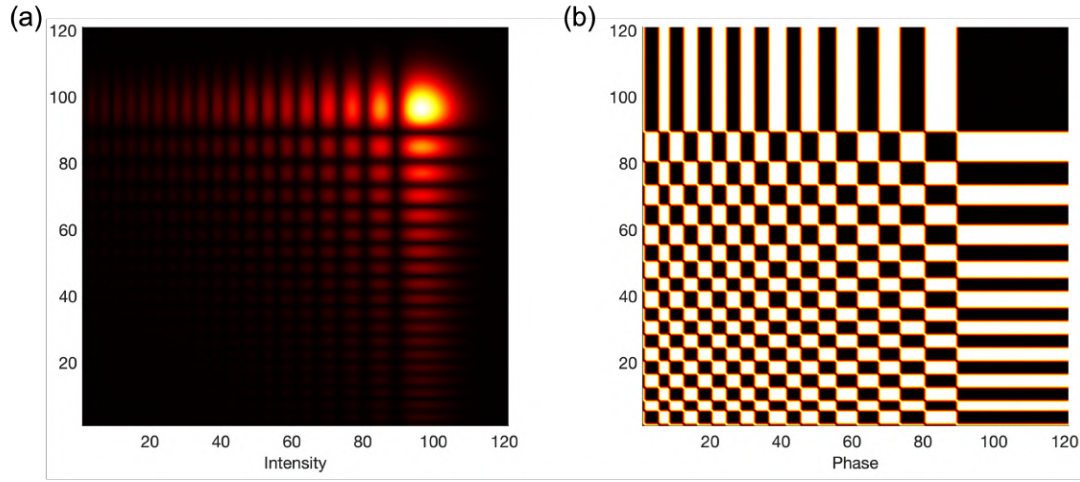


Figure 2.7: (a) 2D Airy beam at propagation distance $z = 0$ and (b) its phase profile.

$$\Phi(\xi, s) = \text{Ai}\left(s - (\xi/2)^2\right) \exp\left(i(s\xi/2) - i(\xi^3/12)\right) \quad (2.15)$$

the transverse shift x_d of the beam along the propagation direction z will be,

$$x_d = \frac{a}{4k^2x_0^3} + \theta_x z \quad (2.16)$$

where k is the propagation constant, x_0 is the arbitrary transverse scale and θ is the incident angle. Airy beams are generally generated by the Fourier transform of a cubic phase imposed Gaussian beam [72, 73]. The transverse intensity profile of two-dimensional Airy beams and its cross-sectional phase profile are shown in Figure 2.7.

2.2.5 Vector beams

In all of the structured beam examples we have talked about so far, the scalar nature of the optical beam has been taken into account. But it was well understood that the

electric and magnetic fields are vector quantities; hence, a direction is associated with this quantity. At the start of this chapter, we discussed the vector properties of electromagnetic fields in great detail. From Maxwell's equation, one can see that the electric ($\mathbf{E}$) and magnetic ($\mathbf{B}$) fields are perpendicular to the wave vector (k). In the previous sections, we discussed some of the beams that can be generated from the spatial manipulation of Electric field amplitude. Even though the electric and magnetic fields magnitudes are restricted by $E_x = -cB_y$ and $E_y = cB_x$ for an optical beam propagating along z direction. There are no restrictions involved between the x and y components of the optical field. Hence, by carefully engineering the transverse electric or magnetic field along with the previously discussed phase and amplitude, one can generate a new class of optical beams with spatially varying polarization. The optical field's vectorial property is greatly considered in such optical beams. Hence this category of beams is commonly known as "vector beams." Vector beams are also solutions of vector wave equations having spatially inhomogeneous polarization states across the beam profile [74]. Despite the fact that vector beams can be released as natural solutions to the vectorial Helmholtz equation, they are frequently generated as coaxial superpositions of orthogonal scalar fields with orthogonal polarization states as follows.

$$U(r) = U_i(r)e^{i\delta_1}\hat{e}_i + U_j(r)e^{i\delta_2}\hat{e}_j \quad (2.17)$$

here, $\hat{e}_i, \hat{e}_j$ can be any two orthogonal polarization pair such as horizontal-vertical or right-left circular. Similarly, U_i and U_j can be any orthogonal spatial modes. The $\delta_2 - \delta_1$ will be the relative phase difference between the spatial modes. Even though the polarization degree of freedom is limited to two-dimensional space, the spatial degree of freedom is infinite. In other words, since there are infinite ways to use spatial modes, the set of vector beams also makes an infinite space. Some of the well-

established examples of vector beams are given in figure 2.8.

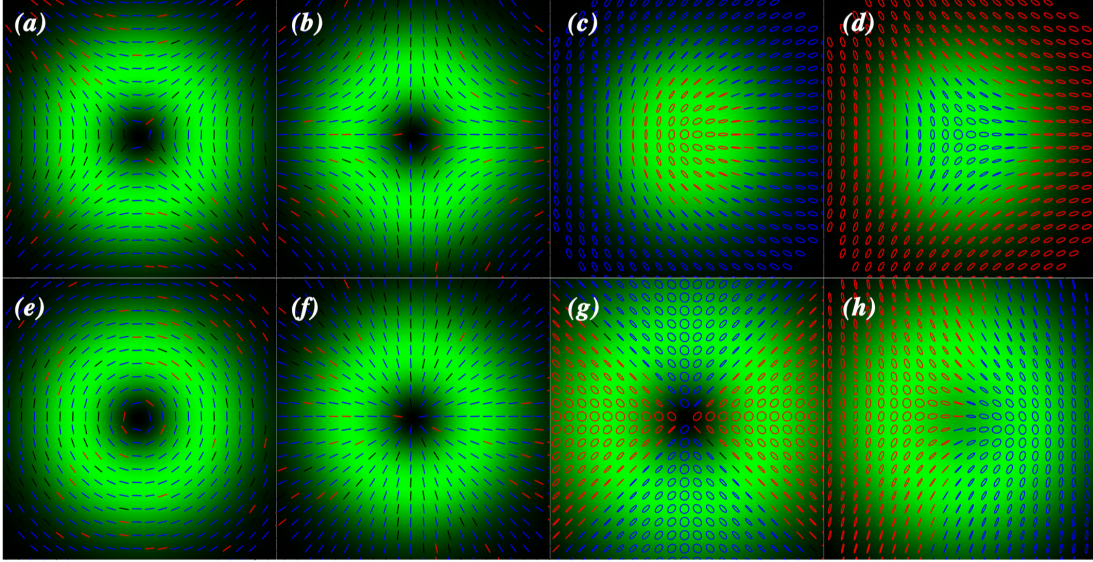


Figure 2.8: Examples of vector beams generated from various coaxial superpositions of the LG beams. (a) $LG_{-1}(r)\hat{e}_L + LG_1(r)\hat{e}_R$, (b) $LG_{-1}(r)\hat{e}_L - LG_1(r)\hat{e}_R$, (c) $LG_0(r)\hat{e}_L + LG_1(r)\hat{e}_R$, (d) $LG_1(r)\hat{e}_L + LG_0(r)\hat{e}_R$, (e) $LG_1(r)\hat{e}_L + LG_{-1}(r)\hat{e}_R$, (f) $LG_1(r)\hat{e}_L - LG_{-1}(r)\hat{e}_R$, (g) $LG_1(r)\hat{e}_H + LG_{-1}(r)\hat{e}_V$, (h) $LG_0(r)\hat{e}_H + LG_1(r)\hat{e}_V$. The left and right ellipticity are represented by red and blue ellipses.

In recent years, vector beams have shown promising applications in many fields, such as biomedical and clinical applications[75], ultra-sensitive polarimetry [76, 77], mode division multiplexing[78], and better spectroscopy[79], laser machining[10], optical trapping[80], high-resolution imaging [81]. Therefore, various methods have been proposed to generate vector vortex beams, such as conical Brewster prism, interferometry, sub-wavelength gratings, laser intracavity devices, twisted nematic liquid crystals, and metallic nanostructures.

2.3 Generation of structured beams

Spatial beam shaping techniques, encompassing active and passive methods, are employed to precisely control the intensity, polarization and phase distribution of laser

beams. Active methods, such as resonator engineering and intracavity element manipulation, directly influence the beam within the laser cavity. Passive methods, including spatial light modulators, phase plates, and diffractive optical elements, modify the beam's characteristics externally.

2.3.1 Spatial Light Modulators (SLMs)

Spatial light modulators (SLMs) have emerged as indispensable tools in modern optics, offering precise control over the amplitude, phase, and polarization of light beams. These devices have revolutionized various fields, from laser material processing to quantum optics, by enabling the generation of complex optical patterns and effects.

SLMs operate by modulating the optical properties of light as it passes through or reflects off the device. This modulation is achieved through various mechanisms, including the use of microelectromechanical systems (MEMS) or liquid crystals. By controlling the orientation of microscopic elements within the SLM, it is possible to introduce phase shifts, amplitude variations, or polarization changes into the light beam. For example, to generate optical vortices from a Gaussian beam, a forked grating is used to imprint on the SLM, which results in the generation of optical vortices of respective order, as shown in Figure.2.9

The ability to manipulate light in such a precise manner has opened up a vast array of applications for SLMs. In laser material processing, SLMs can be used to create custom beam profiles for tasks such as cutting, drilling, and engraving. In optical communication, they can modulate light signals for high-speed data transmission. In the realm of scientific research, SLMs have played a pivotal role in advancing our understanding of fundamental optical phenomena. They have been used to create optical vortices, optical tweezers, and structured light fields, enabling novel experiments in

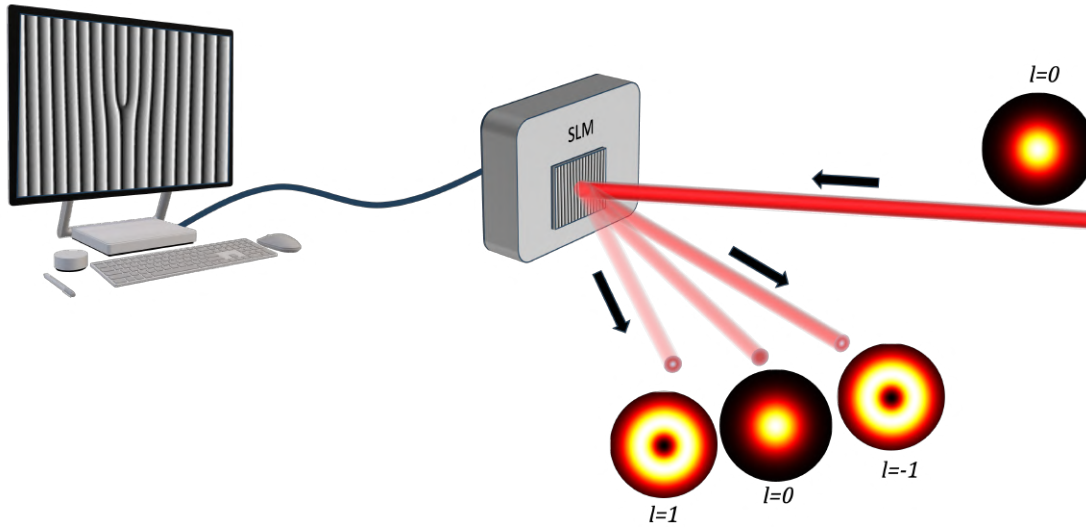


Figure 2.9: The setup used for the generation of vortex beams from Spatial Light Modulators (SLM).

fields such as quantum optics, microscopy, and biophotons.

As technology continues to evolve, SLMs are expected to play an even more significant role in shaping the future of optics. Advancements in materials science, microfabrication, and control algorithms are paving the way for the development of more efficient, versatile, and powerful SLMs.

2.3.2 Dielectric phase elements

Dielectric elements, known for their exceptional resistance to optical damage, are indispensable tools for shaping coherent light sources. These elements, such as spiral phase plates (SPPs) and cubic phase masks (CPMs), introduce specific phase variations to the input beam, resulting in the generation of structured beams with unique properties. Their high damage threshold makes them particularly suitable for applications involving high-power lasers.

While dielectric elements offer remarkable advantages, they are often limited in

their wavelength versatility. The refractive index of the dielectric material can vary with wavelength, restricting the element's effectiveness to a specific range. This limitation can be particularly challenging when working with broadband or tunable lasers.

Spiral Phase Plates (SPP) are advanced Diffractive Optical Elements (DOE) that twist a plane Gaussian beam into an optical vortex, offering unique applications. The resulting intensity distribution has a dark central area and is therefore called a doughnut-shaped beam. spiral phase plate is a unique optic, whose structure is composed entirely of spiral or helical phase steps, whose purpose is to control the phase of the transmitted beam. The topological charge, denoted in the literature as m , refers to the number of 2π cycles (i.e. "staircases") etched around 360° turn of diffractive surface. One main effect of a higher topological charge is an increase in the angular momentum of the vortex beam by a factor of l . Another effect is the dimensions magnification of the ring intensity pattern, by a factor of l .

2.4 Nonlinear optics

The study of phenomena that originate from altering a material system's optical properties by light is known as nonlinear optics. In most cases, only laser beams are powerful enough to change a material system's nonlinear optical characteristics substantially. A material system's response to an applied optical field depends nonlinearly on the intensity of the applied optical field; hence, nonlinear optical phenomena are called "nonlinear" in this sense. For instance, the aspect of the atomic response that scales quadratically with the strength of the applied optical field causes second-harmonic production. As a result, the intensity of the light produced at the second-harmonic frequency does tend to grow as the square of the laser light's applied intensity. To understand the effect of the optical field in the matter, let's consider the wave

equation for an optical beam propagating through a lossless dispersion-less medium in terms of the electric field $\mathbf{E}$ [82],

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2} \quad (2.18)$$

here P is the material polarisation quantified as the dipole moment per unit volume produced by the optical field, and the $\epsilon^{(1)}$ is the relative permittivity of the medium. To illustrate the optical nonlinearity, consider the dependence of a material system's polarisation $P(t)$ (the dipole moment per unit volume) on the applied optical field strength $E(t)$. Polarisation is crucial in describing nonlinear optical phenomena because a time-varying polarisation can be the source of new electromagnetic field components. The optical response in nonlinear optics can often be described by expressing the polarisation $P(t)$ as a power series in the field strength $E(t)$.

$$\mathbf{P}(t) = \epsilon_0 \left(\chi^{(1)} \mathbf{E}(t) + \chi^{(2)} \mathbf{E}^2(t) + \chi^{(3)} \mathbf{E}^3(t) + \dots \right) \equiv \mathbf{P}^{(1)}(t) + \mathbf{P}^{(2)}(t) + \mathbf{P}^{(3)}(t) + \dots \quad (2.19)$$

here, $\chi^{(1)}$ is known as the linear susceptibility, and the higher-order coefficients $\chi^{(2)}$ and $\chi^{(3)}$ are known as the second and third-order nonlinear optical susceptibilities, respectively. Accordingly, we can refer to the terms $P^{(1)}(t) = \epsilon_0^{(1)} \mathbf{E}(t)$, $P^{(2)}(t) = \epsilon_0^{(2)} \mathbf{E}^2(t)$, and $P^{(3)}(t) = \epsilon_0^{(3)} \mathbf{E}^3(t)$ as the first, second, and third order polarisation terms. The same approach can also be applied to the higher-order terms; however, observing the higher-order terms is challenging due to the magnitude of higher-order susceptibilities. To comprehend this, we can make a simple order-of-magnitude estimate of the size of these numbers in the typical scenario where the nonlinearity has an electronic origin. In equation 2.19, the lowest-order correction term $P^{(2)}$ would

be comparable to the linear response $P^{(1)}$ when the applied field amplitude E is of the order of the specific atomic electric field strength $E_{at} = e/(4\pi\epsilon_0 a_0^2)$, where $-e$ is the electron's charge, and $a_0 = 4\pi\epsilon_0 \hbar^2/me^2$ is the Bohr radius of the hydrogen atom (here $\hbar$ is Planck's constant divided by 2π , and m is the electron's mass). We can numerically find that $E_{at} = 5.14 \times 10^{11} \text{V/m}$. Therefore, we anticipate that under non-resonant excitation situations, the second-order susceptibility $\chi^{(2)}$ will have the form $\chi^{(1)}/E_{at}$. For condensed matter, $\chi^{(1)}$ is on the order of one; thus, we expect $\chi^{(2)}$ to be on the order of $1/E_{at}$, or that $\chi^{(2)} \approx 1.94 \times 10^{12} \text{m/V}$. Similarly, we anticipate $\chi^{(3)}$ to be on the same scale as $\chi^{(1)}/E_{at}^2$, which for condensed matter corresponds to $\chi^{(3)} \approx 3.78 \times 10^{24} \text{m}^2/\text{V}^2$. Based on these values, it was evident that higher-order terms were unfeasible in real life due to the low order of magnitude of the higher-order susceptibility, which necessitated a sizeable optical field amplitude for witnessing such phenomena. Besides that, second-order nonlinear optical interactions are only possible in non-centrosymmetric crystals, i.e., crystals that lack inversion symmetry.

While considering the first nonlinear term of polarisation P , the second-order nonlinear process will lead to a series of nonlinear optical processes such as Second harmonic generation (SHG), Sum frequency generation (SFG), Difference frequency generation (DFG), Optical rectification (OR), Optical parametric generation (OPG), Optical parametric generation (OPO). Similarly, the higher-order terms also can produce a series of optical processes. Since the third and higher-order nonlinear interactions are out of the scope of this thesis, we are restricting our discussions to the examples of second-order nonlinear interactions in the following session. Let us consider an optical field with two distinct frequency components represented by

$$E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + \text{c.c.} \quad (2.20)$$

While applying this optical field equation into the second order term of the polarisation equation mentioned in the equation 2.19, the term will result in

$$P^{(2)}(t) = \epsilon_0 \chi^{(2)} \left[E_1^2 e^{-2i\omega_1 t} + E_2^2 e^{-2i\omega_2 t} + 2E_1 E_2 e^{-i(\omega_1 + \omega_2)t} + 2E_1 E_2^* e^{-i(\omega_1 - \omega_2)t} + \text{c.c.} \right] + 2\epsilon_0 \chi^{(2)} [E_1 E_1^* + E_2 E_2^*] \quad (2.21)$$

From the above expression for the nonlinear polarisation, we can see the generation of many new frequency terms. Each of the frequency components represents various nonlinear optical interactions represented by their corresponding amplitude terms,

$$P(2\omega_1) = \epsilon_0 \chi^{(2)} E_1^2 \quad (\text{SHG}), \quad (2.22a)$$

$$P(2\omega_2) = \epsilon_0 \chi^{(2)} E_2^2 \quad (\text{SHG}), \quad (2.22b)$$

$$P(\omega_1 + \omega_2) = 2\epsilon_0 \chi^{(2)} E_1 E_2 \quad (\text{SFG}), \quad (2.22c)$$

$$P(\omega_1 - \omega_2) = 2\epsilon_0 \chi^{(2)} E_1 E_2^* \quad (\text{DFG}), \quad (2.22d)$$

$$P(0) = 2\epsilon_0 \chi^{(2)} (E_1 E_1^* + E_2 E_2^*) \quad (\text{OR}). \quad (2.22e)$$

Although four distinct nonzero frequency components are present in the nonlinear polarisation, only one of these will typically be present with considerable intensity in the radiation generated by the nonlinear optical interaction. This behavior results from the nonlinear polarisation's inability to produce an output signal efficiently unless a specific phase-matching condition is met. This criterion cannot usually be satisfied for more than one frequency component of the nonlinear polarization. By adjusting the input radiation's polarisation and the nonlinear crystal's orientation, frequency components are frequently selected for radiation. A more detailed description of the phase matching is presented in the later section.

2.4.1 Second-harmonic generation

Second-harmonic generation can be described as the exchange of photons between the different frequency components of the interacting optical field. During Second-harmonic generation, an optical field with ω frequency will be converted into the optical beam with 2ω frequency while propagating through the nonlinear medium as shown in Figure 2.10 a. As depicted in Figure 2.10 b, a single quantum-mechanical process results in the destruction of two photons with frequency ω and the creation of a photon with frequency 2ω . The solid line in the 2.10 represents the atomic ground state, whereas the dashed lines represent the virtual energy levels. The term 'virtual' was coined since these levels are not energy eigen levels of the free atom. Instant represents the combined energy of one of the atom's energy eigenstates and one or more photons of the radiation field.

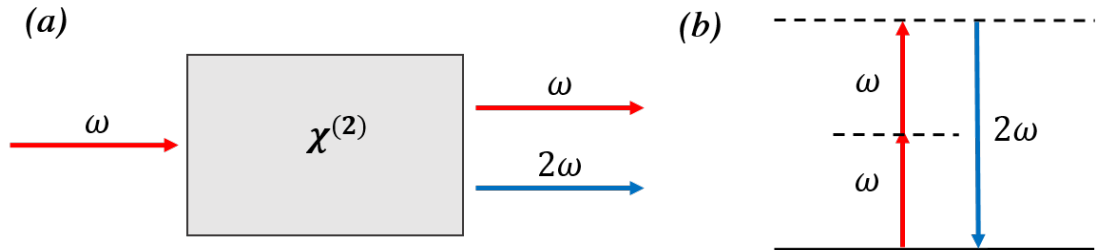


Figure 2.10: (a) The schematic representation of Second-harmonic generation (b) Virtual energy level diagram of Second harmonic generation

Second harmonic generation is the first nonlinear interaction realized by converting a 694.3 nm optical wave generated from a Ruby laser into 347.2 nm by passing through crystalline quartz in 1961 [83]. With the right experimental conditions, the second-harmonic generation process can be so effective that almost all the power in the incident beam with frequency ω is turned into radiation with frequency ω . The higher efficiency of the second harmonic generation can be used to obtain the wave-

lengths which are difficult to access with the laser gain mediums. One of the most common examples is the second harmonic generation of 532nm using Nd: YAG laser, which works at 1064 nm, or the second harmonic generation of Ti: sapphire laser, which will enable 400 nm ultra-fast optical beams from 800 nm femtosecond pulses which discussed in detail in chapter 4.

The Second-harmonic generation can be understood by considering a single frequency (ω_1) in the electric field equation 2.20 for the calculation of the material polarisation mentioned in 2.21. By doing this, we will get a polarisation term ($P(2\omega_1)$) oscillating with a frequency that double the input optical beams frequency ($2\omega_1$) and an amplitude mentioned in 2.22a. This polarization term will generate an electric field $E(2\omega_1)$ in the medium resulting in the Second harmonic generation. More analytically, If we substitute the input pump electric field $E_p = A_p e^{-i(\omega_p t + k_p z)}$ and second order polarisation term $P^{(2)} = 2\epsilon_0 d_{eff} E_p^2$ in the wave equation mentioned in the Equation 2.18. We can derive the coupled electric field equation for the second harmonic generation represented by,

$$\frac{dA_\omega}{dz} = \frac{2i\omega^2 d_{eff}}{k_\omega c^2} A_{2\omega} A_\omega^* e^{-i\Delta k z} \quad (2.23a)$$

$$\frac{dA_{2\omega}}{dz} = \frac{2i\omega^2 d_{eff}}{k_{2\omega} c^2} A_\omega^2 e^{i\Delta k z} \quad (2.23b)$$

here d_{eff} is the effective second-order nonlinearity defined by $\chi^{(2)}/2$ and the Δk is the so-called phase matching condition defined by $\Delta k = 2k_\omega - k_{2\omega}$. A detailed description of the phase-matching condition is given in the following section.

2.4.2 Phase matching

The energy transfer between the frequency components in the second harmonic generation is governed by the coupled equation mentioned in the equation 2.23. From the solutions of the equations 2.23, one can derive the intensity output of the second harmonic generation for nonlinear crystal with length L as [82],

$$I_3 = \frac{4d_{\text{eff}}^2 \omega^2 I_\omega^2 L^2}{\epsilon_0 n_\omega^2 n_{2\omega} c^3} \text{sinc}^2 \left(\frac{\Delta k L}{2} \right) \quad (2.24)$$

The parameters n_ω and $n_{2\omega}$ are the refractive indices for the crystal at ω and 2ω frequencies. One of the noticeable points in the expression 2.24 is the term $\Delta k L/2$ inside the *sinc* function. From Figure 2.11, it is clear that a small variation in the $\Delta k L/2$ term impacts a huge impact on the second harmonic efficiency. Hence it is essential to make the $\Delta k L/2$ term into zero. Since making the length of the crystal into zero is realistically impossible, the only way to make the *sinc* function is by making the term Δk into zero. This condition further makes sense when considering the conservation of momentum during the parametric process. In the case of second harmonic generation, the net momentum of the combining photons needs to be matched with the second harmonic photons such that $\hbar k_{2\omega} = \hbar k_\omega + \hbar k_\omega$. So the slight rearrangement of the momentum conservation will give the perfect condition for phase matching. Hence, by satisfying the phase-matching condition, we also satisfy the momentum conservation condition, leading to maximum efficiency in the second harmonic generation. Birefringent phase-matching and quasi-phase-matching are the two important techniques to satisfy the phase-matching condition.

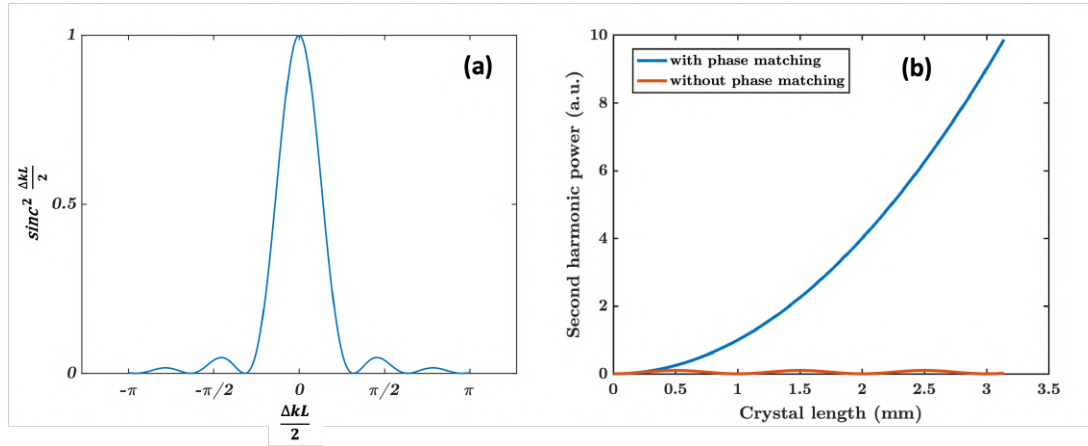


Figure 2.11: (a) The variation of second harmonic efficiency with respect to phase mismatch $\Delta k L/2$ (b) Second harmonic efficiency with and without phase matching

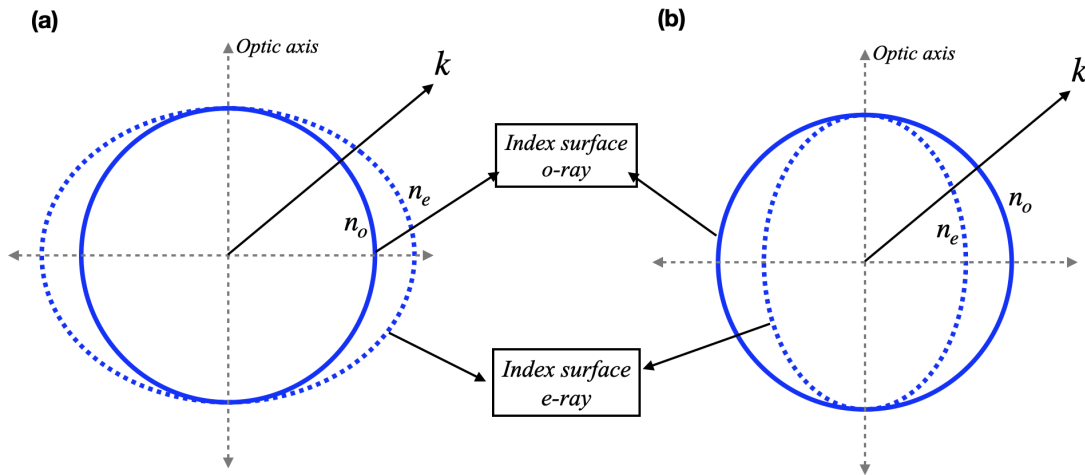


Figure 2.12: Index ellipsoids for (a) positive and (b) negative uniaxial crystals.

Birefringent phase-matching

In the case of second harmonic generation, the perfect phase matching condition ($\Delta k = 0$) can be simplified further by matching the material's refractive index ($n_\omega = n_{2\omega}$) at the pump and second harmonic frequencies [82]. Normal dispersion, in which the material's refractive index increases with frequency, makes phase matching challenging in most circumstances. On the other hand, birefringence is the dependence of the refractive index on the direction of propagation of optical radiation exhibited by

certain materials. In this material, the refractive index of one of the polarizations will change depending on the direction of propagation through the crystal. In contrast, the refractive index of the other polarization will stay the same, except along the optic axis. Figure 2.12 shows the two different types of refractive index variations (a-positive, b-negative) in birefringent crystals. In such crystals, phase-matching is achieved by choosing the propagation direction along the polarization of the second harmonic wave, which provides it with the lower of the two possible refractive indices. This option, for example, corresponds to the extraordinary polarization of a negative uni-axial crystal, as shown in Figure 2.12. When it comes to the polarization of the pump, birefringent phase-matching can be classified into two types: The first category is that both pump photons have the same polarization, which is orthogonal to the second harmonic beam. This first category is known as Type I phase-matching(2.13a). The other possibility is both pump photons have different polarization, and the second harmonic wave contains either one of the polarization known as the Type II phase matching(2.13b).

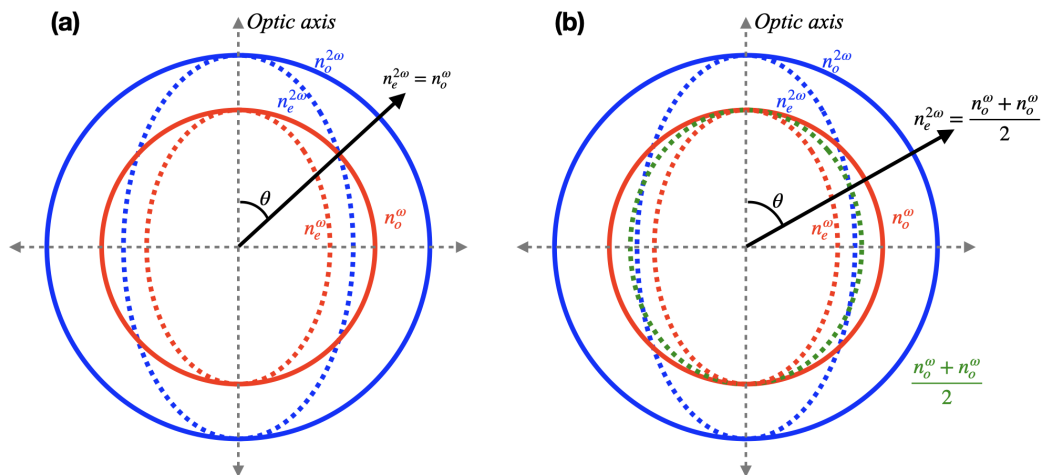


Figure 2.13: An illustration showing the ideal phase matching condition for a negative uniaxial crystal with (a) Type I and (b) Type II configurations

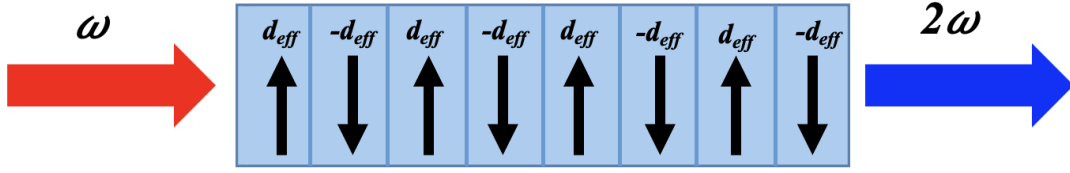


Figure 2.14: An illustration showing the periodic inversion of the sign of the nonlinear coefficient in a QPM nonlinear material by periodic poling.

Quasi phase-matching

Armstrong et al. [84] initially proposed the idea of using a method called quasi-phase-matching (QPM) to accomplish birefringent phase-matching. As can be seen in Figure 2.14, QPM involves the periodic inversion of relative phase by switching the sign of the nonlinear coefficient of the material. The QPM is often implemented by reversing the sign of a nonlinear coefficient by periodic poling of a ferro-electric medium such as lithium niobate [85, 86]. Figure 2.14 depicts a schematic of the sign reversal. In the case of SHG, one can reduce destructive interference among second-harmonic wave components produced at various points along the propagation path by switching the sign of the nonlinear polarization at every L_c , and so retain power owing from the fundamental to the second harmonic. This method is known as quasi-phase-matching because it falls short of perfect phase-velocity matching between the fundamental and second-harmonic waves. As shown in Figure 2.11b, in the case of QPM, the output intensity increases gradually rather than in the case of perfect phase-matching. In quasi-phase-matching, the nonlinear coefficient of the material is altered with the period $\Lambda = 2L_c$. For an optimal duty factor of 50 %, the effective nonlinearity for the first order grating is shown as [82],

$$d_{eff} = \frac{2}{\pi m} \quad (2.25)$$

Although, in theory, QPM reduces the nonlinear coefficient for frequency conversion processes, in practice, this is offset by the prospect of all interacting waves being polarized in the same direction. This allows access to the greatest nonlinear tensor element of the material, which is often an element in which both indices are equal, denoted by the notation $d_{ij}, (i = j)$. In the case of lithium niobate ($LiNbO_3$), lithium tantalate ($LiTaO_3$), and potassium titanyl phosphate ($KTiOPO_4$), d_{33} is the largest tensor element. The primary benefit of QPM is that it may be used in materials with low birefringence where BPM is not conceivable, and it can produce Non-critical phase matching for any non-linear interaction allowed within the transparency range of the material. Since all of the fields may be polarized parallel to one of the primary optical axes of the nonlinear crystal, the Poynting vector walk-off between the interacting fields can be avoided if the non-critical phase matching configuration is used. In addition, the periodical poling technology permits the fabrication of multiple grating crystals, which expands the phase-matching capability throughout the entire transparency range of the nonlinear material.

Chapter 3

Generation and characterization of vector beams

3.1 Vector beam generation

In the previous chapter, we discussed how light is structured in the intensity and polarization domains. This chapter will go through the creation and characterization of structured beams, particularly vector beams with both spin and orbital angular momentum. The vector vortex beam is generated through the spatial superposition of two optical beams having orthogonal polarization and distinct spatial modes, as denoted by the mathematical expression of equation [2.17](#). There are various generation techniques for the vector beams developed by different research communities over the past years. The generation methods initially utilized diffractive optics, which were created by the community focused on laser beam shaping. These optics involve sub-wave-length gratings that require the consideration of the vectorial nature of light. As a result, all solutions are inherently vector solutions [[87](#), [88](#)]. Crystal-based techniques have also

been developed for conical diffraction. The efficient generation of various vector states of light has been observed through the exploitation of the propagation of light through anisotropic crystals, as demonstrated in previous studies[89, 90]. It is evident that in the process of tight focusing, scalar light will undergo a transformation into vector light. The spin-orbit interactions are categorized into several approaches[91, 92].

3.2 Measurement of Stokes parameters

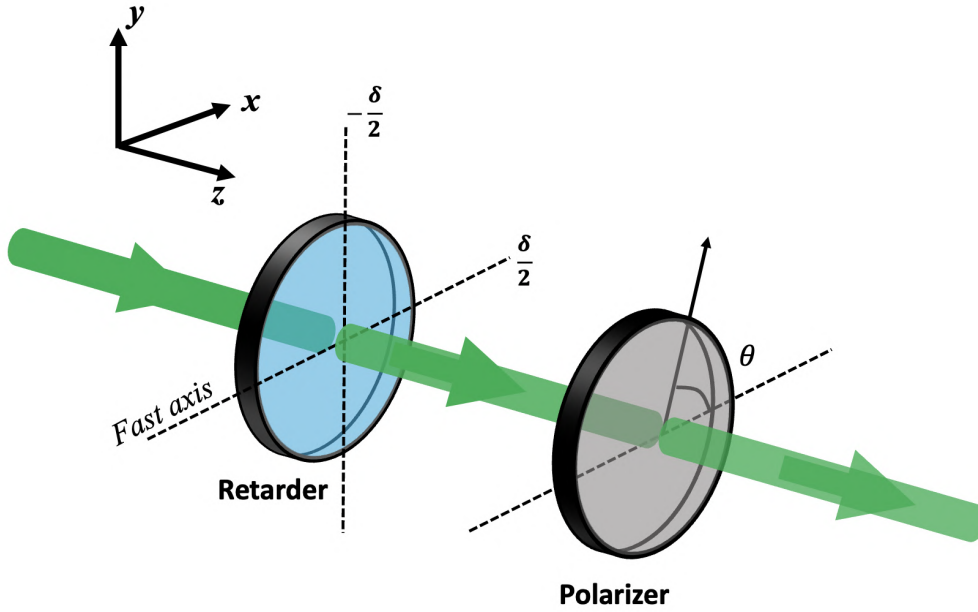


Figure 3.1: measurement setup for Stokes polarization parameters.

In this section, we will talk about how intensity profiles are used to measure the Stokes polarization parameters. The Stokes parameter measurement setup consists of an optical retarder or birefringent medium with a fast axis along the x -axis and a polarizer held at an angle θ with respect to the x -axis. A schematic diagram for the setup is shown in Fig. 3.1. The retarder will introduce a relative phase difference δ between the x and y axes. For an input optical beam with the electric field, $E_{in} = E_x e^{i(\omega t + \phi_x)} \hat{\mathbf{x}} + E_y e^{i(\omega t + \phi_y)} \hat{\mathbf{y}}$ passing through this configuration, the output electric field

can be calculated as $E_{out} = E_x \cos \theta e^{i(\omega t + \phi_x + \delta/2)} \hat{\mathbf{x}} + E_y \sin \theta e^{i(\omega t + \phi_y - \delta/2)} \hat{\mathbf{y}}$. From this electric field expression, the intensity output in terms of the optical retardance δ and the polarizer angle θ will be,

$$\begin{aligned} I(\theta, \delta) &= \frac{1}{2} [(E_x E_x^* + E_y E_y^*) + (E_x E_x^* - E_y E_y^*) \cos 2\theta \\ &\quad + (E_x E_y^* + E_y E_x^*) \cos \delta \sin 2\theta + i(E_x E_y^* - E_y E_x^*) \sin \delta \sin 2\theta] \\ &= \frac{1}{2} [S_0 + S_1 \cos 2\theta + S_2 \cos \delta \sin 2\theta + i S_3 \sin \delta \sin 2\theta] \end{aligned} \quad (3.1)$$

The output intensity is a function of four Stokes parameters: S_0 , S_1 , S_2 , and S_3 . So, we need at least four intensities to be able to figure out the four Stokes parameters. For measuring these four intensity configurations, a quarter-wave plate (QWP) can be used as the retarder with $\delta = 90^\circ$, and the polarizer can be configured at the angles, $\theta = 0^\circ, 45^\circ$, and 90° to obtain the four intensities $I(0^\circ, 0^\circ)$, $I(45^\circ, 0^\circ)$, $I(45^\circ, 90^\circ)$ and $I(90^\circ, 0^\circ)$. The four Stokes parameters can be calculated from this four intensity profile using the set of equations given in [3.2](#)

$$S_0 = I(0^\circ, 0^\circ) + I(90^\circ, 0^\circ) \quad (3.2a)$$

$$S_1 = I(0^\circ, 0^\circ) - I(90^\circ, 0^\circ) \quad (3.2b)$$

$$S_2 = 2I(45^\circ, 0^\circ) - I(0^\circ, 0^\circ) - I(90^\circ, 0^\circ) \quad (3.2c)$$

$$S_3 = 2I(45^\circ, 90^\circ) - I(0^\circ, 0^\circ) - I(90^\circ, 0^\circ) \quad (3.2d)$$

For the characterization of the vector beams, we have recorded the intensity profiles of the input beam with the above-mentioned QWP and polarizer combinations in a

CCD-based camera. Later, the intensity data for each pixel is used to measure the Stokes parameter for each pixel. These Stokes parameter distributions were later used for the calculations of the spatial distributions of polarization ellipse orientation (ψ) and ellipticity (χ) using the relation presented by the Eq. 2.9. The MATLAB code used for this calculation is given in Appendix A.

3.3 Measurement of polarization coverage

Estimation of the polarization states available in an optical vector beam or the polarization coverage is essential while utilizing such beams in real-life applications. The straightforward way of estimating polarization coverage is by estimating the area covered by all available polarization states on the Poincaré sphere. Even though the method is a good way of visualizing polarization, it does not consider the fact that an equal area near the pole can contain more polarization states than an equal area near the equatorial region. Further, in real-life situations, it is cumbersome for the generation and detection of a continuous distribution of polarization states. The polarization distribution of a practical Poincaré beam will be discrete. Since the discrete points don't have an area of their own, the only option will be to use a binning method. As such the fundamental parameters of the polarization ellipse, ellipse orientation (ψ), and ellipticity (χ) can be utilized to bin the polarization states. The first step in calculating polarization coverage with the binning technique is to define a rectangle in a 2D plane with axes, 2ψ , and 2χ , the spherical coordinates of Poincaré's sphere. In doing so, one can project each point on the curved surface of the Poincaré sphere into the 2D plane without distortion as commonly observed during the projection of 3D into 2D plane. The length ($2\psi \in [0, \pi]$) and breadth ($\chi \in [-\pi/4, +\pi/4]$) of the rectangle are then divided with n equal intervals along both axes, yielding a total of $N_0 = n \times n$ buckets. As a result, all polarization states fell into one of these buckets, and polar-

ization coverage can be calculated as $100 \times N/N_0$ if each of the N buckets contains at least one polarization state.

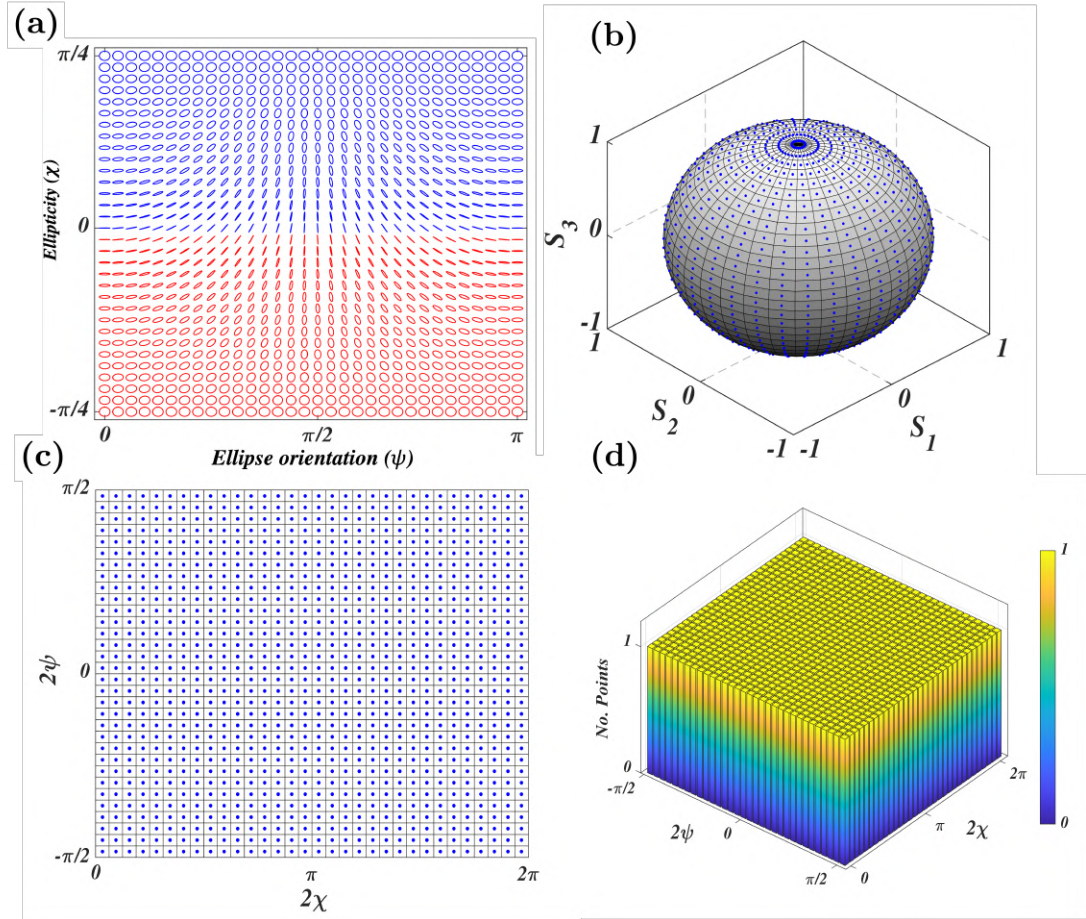


Figure 3.2: (a) Polarization distribution of a Poincaré beam with uniform polarization distribution across the transverse plane. The axis is marked as ψ and χ to indicate the uniform increments of these parameters along the x and y-axis. (b) Distribution of polarization states on Poincaré sphere. (c) Distribution of polarization states on $2\psi - 2\chi$ plane with the number of bins 32×32 . (d) Distribution of the number of points in the $2\psi - 2\chi$ plane.

The optimization of the number of buckets N_0 is a crucial part of this calculation. The maximum acceptable number of the buckets N_0 can be understood by an example with a hypothetical beam with a uniform polarization distribution of 32×32 polarization states shown in Fig 3.2(a). Here the ellipse orientation (ψ) and ellipticity (χ) is varying uniformly along x and y axes, respectively. The polarization states of the beam

is shown by the distribution on the Poincaré sphere using Fig. 3.2(b). It is evident from Fig. 3.2(b) that the polarization distribution of the beam on the surface of the Poincaré sphere is not uniform even though the beam carries a uniform distribution polarization. Further, it is important to note that the concentration of polarization states is high near the polar region in comparison to the concentration of polarization states in the equatorial region. As a result, an equal area near-equatorial region can contain less span of polarization states compared to an equal area selected near the polar region. Therefore, it is evident that simply calculating the area covered by the polarization states on the Poincaré sphere is not sufficient for the exact evaluation of the polarization coverage of any vector beams. On the other hand, these polarization states have a uniform distribution in the $2\psi - 2\chi$ plane divided into $N_0 = n \times n = 32 \times 32$ bins, as shown in Fig. 3.2(c). Figure 3.2(d) shows the histogram for the distribution of the number of points in each bin. So from the above description, it is evident that the most fundamental way of estimating the span of discrete polarisation distribution over the Poincaré sphere is by calculating the coverage of polarization distribution in the $2\psi - 2\chi$ plane.

The above-explained example is an ideal case for full coverage. In this example, a bin size less than the total number of pixels in the beam will not change the polarization coverage, on the other hand, a binning of more than the number of polarization will lead to a case of oversampling hence a reduced polarization coverage. On the hand, the selection N_0 value can be made by matching it with the total number of pixels present in the sample, the same as the maximum possible number of bins possible without over-sampling. Unfortunately, in real-life scenarios, the polarization states may not be uniformly distributed in the $2\psi - 2\chi$ plane. In such situations, the selection of the highest possible number of bins will lead to under-sampling of the polarization coverage. The best example for understanding under-sampling conditions is the case of a scalar beam. Figure 3.3(a) is the polarization distribution of a scalar beam with a

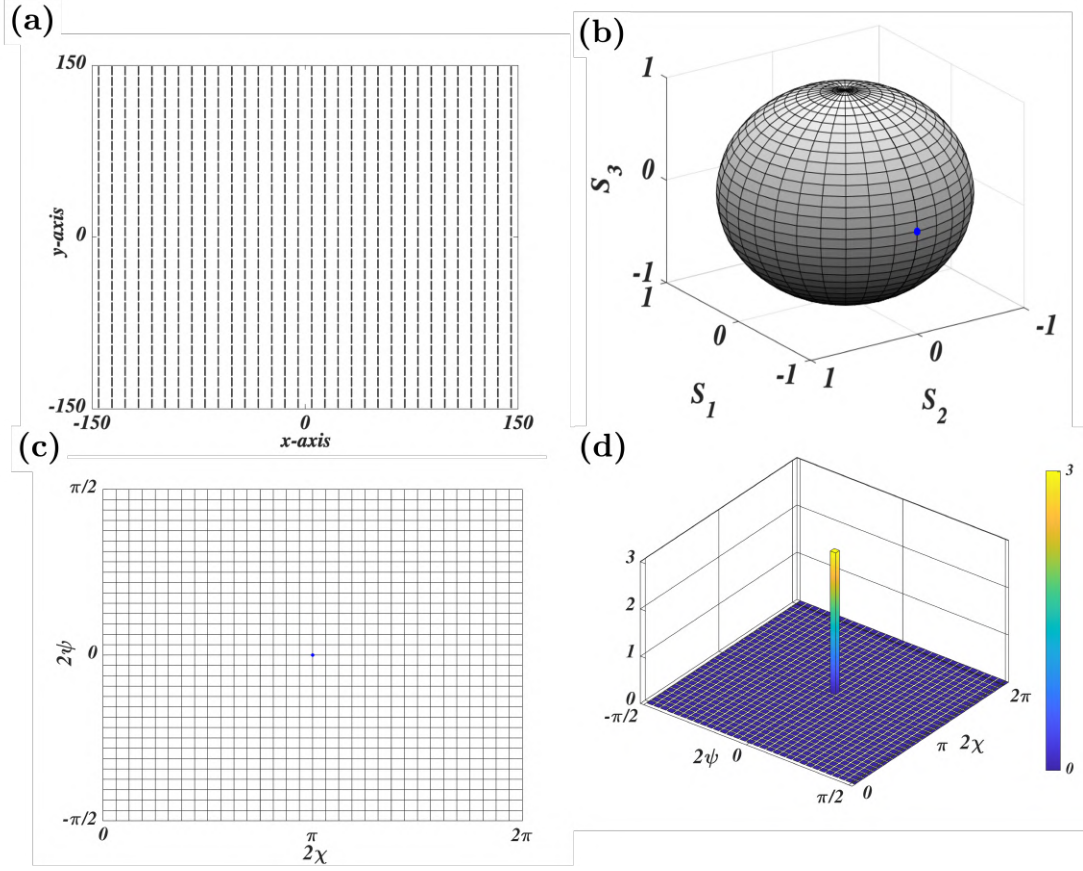


Figure 3.3: (a) Polarization distribution of a scalar beam with vertical polarization across the transverse plane. (b) Distribution of polarization states on Poincaré sphere. (c) Distribution of polarization states on $2\psi - 2\chi$ plane with the number of bins 32×32 . (d) Distribution of the number of points in the $2\psi - 2\chi$ plane in logarithmic scale.

uniform vertical polarization. In Fig. 3.3(b), we can see that all the polarization states in the beam are concentrated in a single point on the surface of the Poincaré sphere. Similarly, the single point on the $2\psi - 2\chi$ plane is shown in Fig. 3.3(c). As a result, all the polarization states will be concentrated on a single bin as shown in Fig. 3.3(d), and the coverage corresponding to the scalar beam completely depend on the choice of a number of bins. For example, the selection of N_0 as 1, 10, and 100 buckets will give a polarization coverage of 100%, 10%, and 1%, respectively. In this case, we can see that a very low value of N_0 is not an acceptable choice. Therefore, it is imperative to devise the right strategy to select the optimum value of N_0 . Throughout the thesis,

we have considered the value of N_0 to be the least count of the measured polarization. The selected N_0 value applied to measure the polarization coverage of different scalar and vector beams has provided a sensible and consistent polarization coverage.

Chapter 4

Controlling the coverage of full Poincaré beams

4.1 Introduction

Structured light, including Laguerre-Gauss (LG) modes of light that carry orbital angular momentum (OAM), has been a fast-growing field in recent years due to access to be able to use the spatially varying phase and space degree of freedom [93]. Another popular degree of freedom, the polarization of the light beam, is a consequence of the vectorial nature of the electromagnetic field. Light beams with space-varying polarization distribution, known as Poincaré beams [93], can be constructed from a coaxial superposition of orthogonally polarized fundamental Gaussian mode and an LG mode [74, 94]. With the study of beams having a nontrivial distribution of polarization, and by exploiting this vectorial nature of light, new dimensions have opened since its interaction with matter is polarization sensitive. Such vector light shows promising applications [74, 95, 96] in both classical and quantum contexts [97]. Notable vecto-

rial structures of light are the Full Poincaré (FP) beams [94], which carry all possible polarization states spanning over the entire surface of the Poincaré sphere. Since the first demonstration [94], the FP beams have attracted a great deal of interest in various science and technology applications. The inherent spatial distribution and the flattop intensity distribution [98] of polarization states over the transverse plane make the FP beam a good candidate for an on-demand polarization source. It is also well established that the spin and orbital angular momentum of the light plays an important role in material processing [10], biomedical applications [75], and optical manipulation [12]. The spatial degree of freedom for choosing the spin-orbital angular momentum in FP beams further enhances the usefulness of FP beams in similar applications. For each of these applications, it is essential to properly quantify and control the polarization content of the optical beam at different wavelengths.

On the other hand, generating structured beams with space-varying polarization can be challenging as it entails an amalgamation of beam shaping techniques, viz. liquid crystals, spatial light modulators, etc., interferometric setups, and computer resources and algorithms, that altogether make up a complex setup. This can impose certain restrictions like, for example, while the FP beam, in theory, can carry all possible polarization states on the surface of the Poincaré sphere, in practice, due to the finite beam size of the superposed beams, the generation of an FP beam with full polarization coverage is difficult. Also, the direct generation straight from lasers [99–102] has also been explored. In this regard, the modal description has been a useful tool to describe newly formed structures [40]. However, this generation type is restricted in the wavelength by the availability of gain media. Traditionally, FP beams are generated and controlled by linear optical elements. The nonlinear creation and control of FP beams is a subject very much in its infancy [103].

For the case of second harmonic generation (SHG), seminal works have revealed the interesting effects of topological doubling [101, 104], algebra [105], and analysis of the conversion of FP beams [106, 107]. Nowadays, in the forefront of this area is the nonlinear behaviour of vector light [36], with recent works accessing frequency up-conversion [37, 107–110] and down conversion [111], even in the quantum realm [112].

However, an early aspect of SHG has not been taken into consideration when using FP beams: different optical modes have different conversion efficiencies. This has been debated early in the field with seminal works using Bessel beams [19, 30, 113–115], but the principle is the same, viz., a nonlinear process efficiency is proportional to its intensity, which in turn is the average power over the area. The waist parameter of a structured beam increases proportionally to the square root of its order [116], increasing the minimum area achieved while focused and therefore decreasing conversion efficiency. A consequence is that FP beams do not remain so, with the coverage potentially increasing or decreasing depending on both beam and medium conditions.

In FP beams, the spatial and polarization degrees of freedom (DoFs) are coupled in a non-separable manner, mimicking the phenomena of entanglement in quantum mechanics. This coupling will depend on the extent of spatial overlap of the orthogonal polarization and modes. The use of FP beams for material processing [10] can be very useful to be able to create complex structures and precise features by controlling the polarization and modal distribution in the FP beams. Further, the second harmonic generation of the FP beams opens up access to the exotic wavelengths [117] where the direct structuring of the optical beams is difficult. Recent studies have shown that the nonlinear upconversion is an effective method to tune the wavelength while maintaining its quantum properties [118]. This points out the usefulness of nonlinear upconver-

sion to quantum processes that use the polarization degree of freedom in cryptography [119] or hybrid entanglement [120]. In view of the above applications, it is imperative to study the polarization coverage and modal coverage of the generated FP beams. In this work, we address this open problem and demonstrate a simple technique to control the coverage of FP beams, from fully scalar (no coverage) to fully vectorial (full coverage). Using SHG in a single-pass dual-crystal geometry with various topological charges as vectorial inputs, we show how the relative weightings and SHG effect features such as singularities and L-lines, and analyze how the coverage is affected by the change in beam sizes and different conversion efficiencies for the two constituting modes of an FP beam. We reveal the balancing act between mode order, modal non-linear efficiency, and initial relative modal weights, all in close agreement with that theoretically predicted. To the best of our knowledge, this is the first comprehensive quantitative study on the estimation and control of the accessible polarization states of an FP beam.

4.2 Background and Concept

Full Poincaré beams are generated by the coaxial superposition of orthogonally polarized Gaussian and Laguerre-Gaussian (LG) beams. The electric field expression for the FP beam can be written as, $E = \alpha |E^H, l\rangle + \beta |E^V, 0\rangle$, Where the α and β are normalized amplitude coefficients of the LG and Gaussian beams. For the theoretical generation of FP beams, we have generated the transverse electric field distribution of the horizontal ($|E^H, l\rangle$) and vertical ($|E^V, 0\rangle$) components using the experimental parameters in a two-dimensional array with the same size of the detector aperture resolution. These transverse electric field expressions are further used in calculating the theoretical intensity profile for four different combinations of $\lambda/2$ and $\lambda/4$ wave-plates sufficient for the measurement of Stokes parameters. Stokes parameters,

traditionally labeled as S_0, S_1, S_2 , and S_3 on the axis of a Poincaré sphere, play a crucial role in visualizing the unconventional polarization states of light beams. These four parameters are sufficient for calculating the ellipse orientation ($\psi = \tan^{-1} S_2/S_1$) and ellipticity ($\chi = \sin^{-1} S_3/S_0$) of the polarization ellipse in both experiment and theory.

To study polarization topology of the FP beam, we have studied the singularities on the Stokes field. The Stokes fields are hypothetical complex fields constructed from the Stokes parameters [121], described by $S_{12} = S_1 + iS_2$, $S_{23} = S_2 + iS_3$, and $S_{31} = S_3 + iS_1$. The Stokes phases, the argument of the Stokes fields, can be calculated as $\phi_{ij} = \tan^{-1} S_j/S_i$, where, $i, j = 1, 2$ and 3 . Stokes vortices are the points in the Stokes phase field with screw dislocation often quantified by charge σ_{ij} (number of $0 - 2\pi$ phase winding around the dislocation). The Stokes phase σ_{12} is used for visualizing C-point polarization singularities, which are the points in the optical fields where the orientation of the polarization ellipse is undefined. C-point singularities are characterized by circular singularity indices I_c representing the number of complete 2π rotations of polarization ellipse around the singularity point. Since the ellipse orientation is calculated by $\psi = \frac{1}{2} \tan^{-1} S_2/S_1$, the circular singularity indices I_c and Stokes vortex charge σ_{12} are further related by $\sigma_{12} = 2I_c$ [121, 122]. The vortices σ_{23} in the Stokes field, S_{23} , also known as the Poincaré field, represent the net orbital angular momentum of the beam. The number of C-point singularity pairs in the FP beam is the same as the vortex order σ_{23} of the beam.

The coverage of the FP beam on Poincaré sphere is realized by the one-to-one mapping of the polarization calculated in each of the pixels in the detector plane to a point on the surface of the Poincaré sphere. The area then measures the coverage covered by these points with respect to the total area of the Poincaré sphere. So the coverage of FP on the surface of the Poincaré sphere can be considered as the net

accessible polarization states available in an FP beam. A coverage of more than 75% of the Poincaré sphere is deemed acceptable for many applications [123]. To understand how these polarization states are generated in the detector plane, let us consider the area where two beams with orthogonal polarizations are spatially overlapping. The areas where the two orthogonally polarized beams do not overlap only contains their respective polarization state. The diverging polarization states will only exist in the space where these two beams are spatially superimposed, and their relative weights and phases will dictate the polarization state at a given position. This spatial overlap of the beams, however, decreases when considering the SHG of both beams. This can be observed in Figure 1 and can be explained as follows. In the SHG process, while a Gaussian beam generates another Gaussian with a smaller area, a Laguerre-Gaussian (LG) beam also generates an LG beam, but with its intensity distribution further away from the center and with a sharper drop in the intensity.

In Figure 4.1 (a), we illustrate the beam intensity profiles along the overlapping region in black. If this area is projected in the radial direction, as in Figure 4.1 (b), it is easier to see that the region decreases, particularly if considered a detection threshold (a minimum total intensity required for practical purposes). For the case of a Gaussian beam with right-circular polarization and an LG beam with left-circular polarization composing the fundamental/pump wavelength, we have illustrated this effect in Figure 4.1 (c). Circular polarization states are found only in the regions where only one beam has detectable intensity. All other polarization states are confined in between. A linear polarization state can be found in a radial coordinate where both have equal intensities. In this case, for the fundamental beam, equal weights were used. However, this balance is broken in SHG, where a Gaussian and an LG beam have different conversion efficiencies. The region containing all polarization states decreases, meaning that for a finite detection system, the total number of detectable states also decreases. In

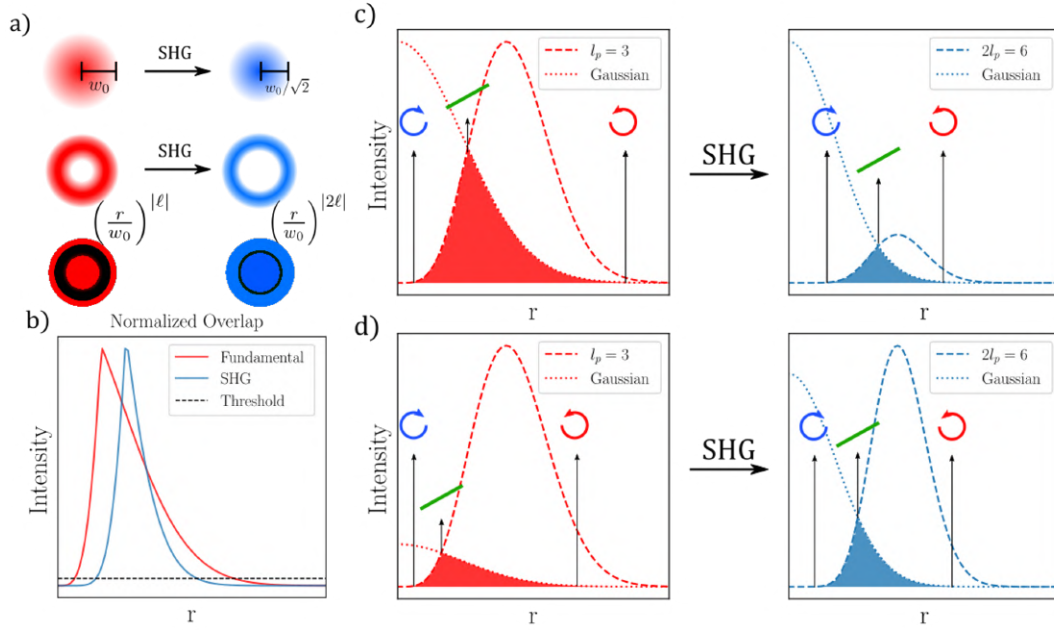


Figure 4.1: Spatial overlap and polarization. In (a), we illustrate how both constituent spatial modes of FP beam change differently in size when used in SHG, changing the total overlap area (depicted in black). A projection of both overlaps in the radial direction can be seen in (b). The region where the beams overlap can decrease even more if the different mode efficiencies are considered, as depicted in (c). In (d), the relative intensities of the fundamental wavelength beams are adjusted in order to maximize the overlap area of SHG, taking into consideration the different modal efficiencies.

Figure 4.1 (d), we try to mitigate this effect by changing the relative intensities of the fundamental wavelength in order to compensate the conversion efficiency and improve the number of states in the SHG.

4.3 Experimental set-up

The schematic of the experimental setup is shown in Figure 4.2a. A Ti: Sapphire laser delivering (pulses width 17 fs, repetition rate 80 MHz) average output power of 837 mW is used as the fundamental source. The output radiation has a wavelength of 55 nm centered at 810 nm. The combination of a $\lambda/2$ -plate (HWP1) and a polarizing

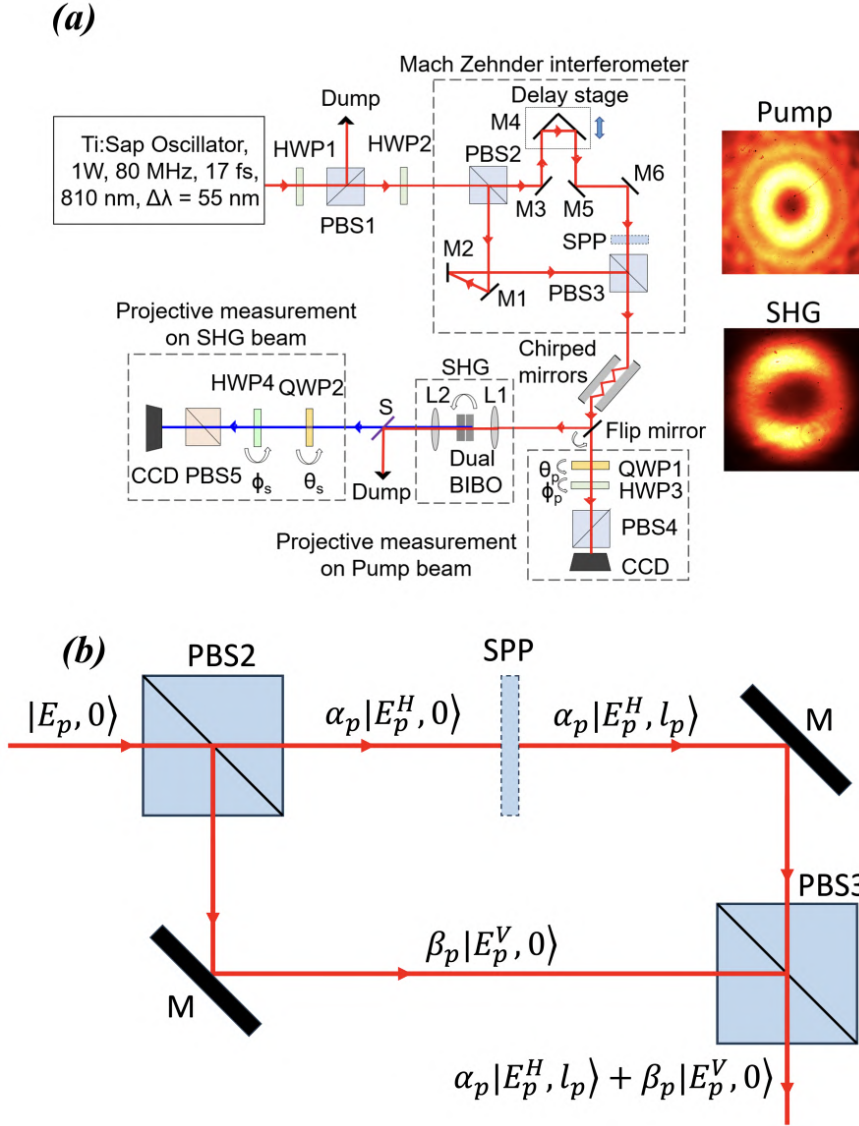


Figure 4.2: (a) Experimental setup for SHG of FP beam. HWP1-4: $\lambda/2$ plates; PBS1-5: polarizing beam splitter cubes; SPP: spiral phase plate; QWP1-2: $\lambda/4$ plates; M1-6: mirrors; L1-2: lenses; S: wavelength separator; CCD: camera; dual-BIBO: nonlinear crystal. Intensity profile of pump and SH FP beams.(b)The schematic diagram for the FB beam generation inside the Mach-Zehnder interferometer

beam splitter (PBS1) cube is used to control laser power in the experiment. The second $\lambda/2$ -plate (HWP2) controls the polarization of the input beam to the polarization-based Mach-Zehnder interferometer (MZI) comprised of PBS2, PBS3, and a set of plane mirrors, M1-6. The delay stage matches the path lengths of the two arms of the MZI. A

schismatic diagram for the generation of the FP beam is shown in Figure 4.2b. In the MZI after PBS2, the input pump beam is split into $\alpha_p |E_p^H, 0\rangle$ in the horizontal arm and $\beta_p |E_p^V, 0\rangle$ in the vertical arm. After passing through the spiral phase plate (SPP), the state will convert into $\alpha_p |E_p^H, l_p\rangle$. Here l_p is the order of the SPP. The horizontal (H) polarized vortex beam and vertical (V) polarized Gaussian beam on recombination on PBS3 produces [94] FP beam represented as $E_p = \alpha_p |E_p^H, l_p\rangle + \beta_p |E_p^V, 0\rangle$, where α_p and β_p are the amplitude coefficient of the vortex beam and Gaussian beam, respectively, and $\alpha_p^2 + \beta_p^2 = 1$. Using the SPPs of the phase winding corresponding to vortex orders, $l_p = 1 - 3$, we generate FP beams of order up to 3. A pair of chirped mirrors compensate stretching of the ultrafast FP beam resulting from the dispersion of the optical components. The flip mirror directs the pump beam either to the second harmonic (SH) setup or to the projective measurement setup comprised of $\lambda/4$ plate (QWP1), $\lambda/2$ plate (HWP3), PBS4, and CCD camera for polarization and Stokes parameters. The lens L1 of focal length, $f_1 = 150$ mm, is used to focus the pump at the center of the dual-BIBO crystal, consisting of two contiguous BIBO crystals, each having a thickness of 0.6 mm and an aperture of $1 \times 1 \text{ cm}^2$ with an orthogonal optic axis [120]. Both crystals are cut at an angle, $\theta = 151.7^\circ$ ($\phi = 90^\circ$) in the optical yz-plane for type-I (e+e $\rightarrow$ o) phase-matching for the frequency-doubling of 810 nm into 405 nm [107]. The unique geometry of the dual-BIBO crystal permits the single-pass frequency-doubling of the orthogonally polarized components of the FP beam. A lens, L2, of focal length $f_2 = 100$ mm, collimate the fundamental and SHG beams. Subsequently, the polarization and Stokes parameters of the SH beam extracted from the pump by the harmonic separator, S, are measured with the help of the projective measurement [55] setup comprised of $\lambda/4$ plate (QWP2), $\lambda/2$ plate (HWP4), PBS5, and the CCD camera. The inset of Figure 4.2 shows the typical intensity profile of the pump and SH FP beams.

4.4 Results

4.4.1 Polarization structure

To verify the pump FP beam, we have recorded the intensity distribution of the beam using different combinations of the angles of QWP1 and HWP3 and calculated the Stokes parameters, S_1, S_2 , and S_3 . Using the Stokes parameters, we have calculated the orientation (ψ) and ellipticity (χ) [55] of the polarization ellipse with the results shown in Figure. 4.3. As evident from Figure. 4.3(a), the transverse distribution of polarization ellipse of pump beam of vortex order, $l_p = 1$, contains C-point polarization singularity in the form of pair of lemon (see the black circle) and star (see the black square) and a single L-line (green line) confirming the vortex order of the FP beam to be $l_p = 1$. To verify the experimental results, we have theoretically calculated the polarization ellipse distribution of the pump FP beam. However, in the current experiment, the Ti:Sapphire laser produces the output beam in an elliptical Gaussian beam spatial profile. While spatial filtering can easily transform the elliptical beam into a high-quality Gaussian beam at the cost of overall laser power, the demand for higher laser power for the SHG process restricted us from using any mode filtering. To address such limitation and to find a close match with the experimental results, we have modified the electric field equation of the FP beam to accommodate the ellipticity of the laser beam as follows,

$$\begin{aligned}
 E &= \alpha |E^H, l\rangle + \beta |E^V, 0\rangle \\
 &= E_{LG} \left(\frac{\sqrt{2}r}{w_0} \right)^{|l|} LG_l(2r_l^2) \exp\left(\frac{-ikr^2}{2R_1} - r_l^2 + il\phi \right) \hat{\mathbf{H}}
 \end{aligned}$$

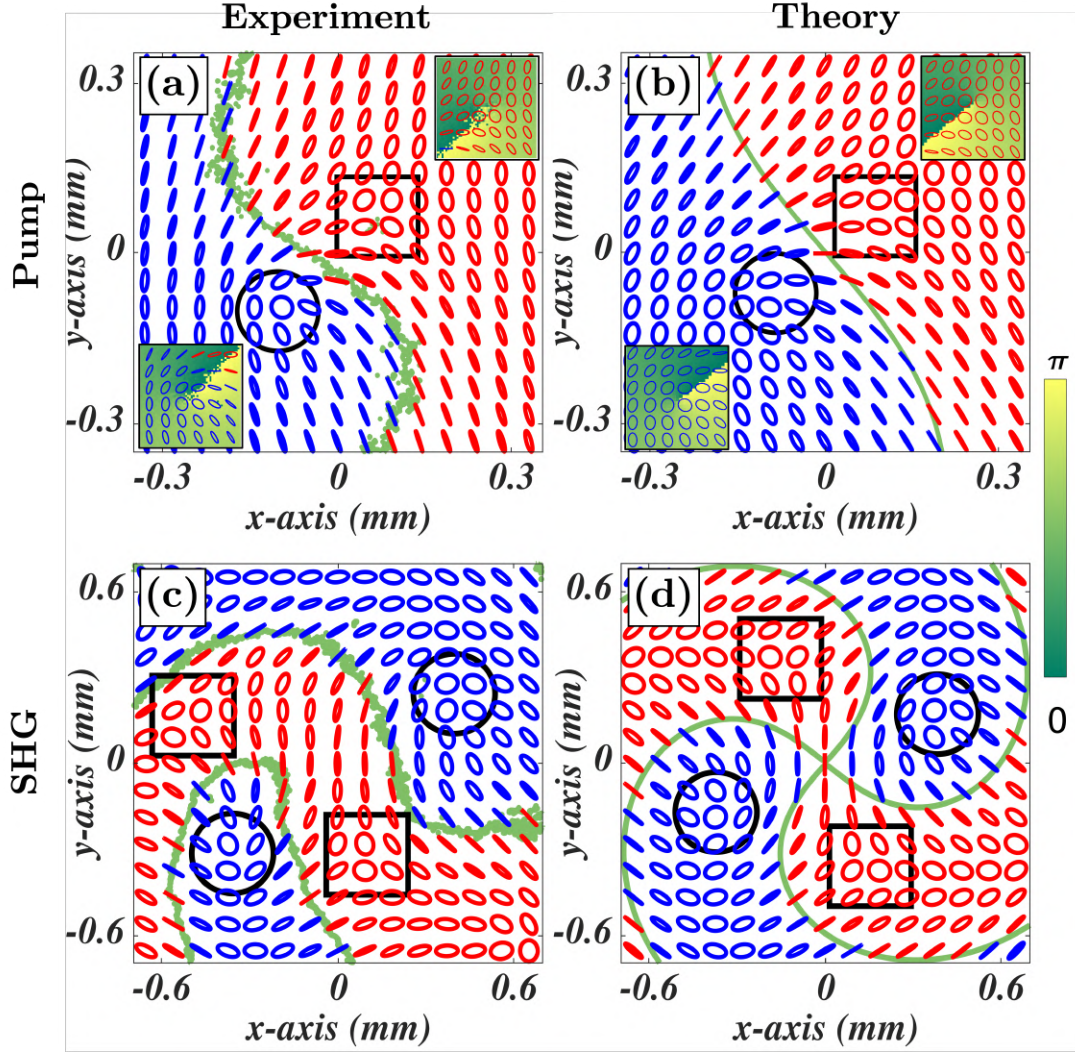


Figure 4.3: (a) Experimental and (b) theoretical transverse polarization distribution of the FP pump beam of vortex order $|l_p| = 1$. The experimental parameters used for the theoretical calculation are: beam widths, $w_{xl} = 1.2$ mm, $w_{yl} = 1.3$ mm, $w_{x0} = 2.9$ mm, $w_{y0} = 5.3$ mm, and the relative intensities, $(\alpha_p^2, \beta_p^2) = (0.75, 0.25)$. The polarization distribution of the corresponding SHG of vortex order $|l_{sh}| = 2$ is shown in (c) Experimental and (d) theoretical using the experimental parameters, beam widths $w_{xl} = 1.6$ mm, $w_{yl} = 1.9$ mm, $w_{x0} = 1.7$ mm, $w_{y0} = 4$ mm, and the relative intensities, $(\alpha_{shg}^2, \beta_{shg}^2) = (0.26, 0.74)$. Inset images are the magnified images of the lemon and star (C-point) singularities highlighted by the black circle and square, respectively. The background of the inset is the ellipse orientation and the green lines depict the L-lines.

$$+ E_G \exp\left(\frac{-ikr^2}{2R_2} - r_0^2\right) \hat{\mathbf{V}}$$

Here, α_p and β_p , the amplitude coefficient of the vortex beam and Gaussian beam, respectively, and $\alpha_p^2 + \beta_p^2 = 1$, was calculated using the intensity profile of the constituent beams and subsequently estimated the electric field amplitude terms E_{LG} and E_G . The terms, $r_l = \sqrt{(x/w_{xl})^2 + (y/w_{yl})^2}$ and $r_0 = \sqrt{(x/w_{x0})^2 + (y/w_{y0})^2}$ represents the asymmetry of the LG and Gaussian beams, having beam widths along x and y axis as w_{xl} , w_{yl} , and w_{x0} , w_{y0} , respectively. All these parameters have been calculated using the experimentally measured spatial profiles of the Gaussian and LG beams. These electric field expressions are later utilized for calculating the Stokes parameters S_1 , S_2 , and S_3 . The theoretically calculated polarization distribution of the pump (see Figure. 4.3(b)) contains a pair of C-point and single L-line singularities and is in close agreement with the experimental results. The insets of Figure. 4.3 (a) and (b) show the magnified images of the lemon and star singularities with ellipse orientation as the background color map. The parameters used for the theoretical calculations are mentioned in the caption.

Similarly, we have calculated the experimental and theoretical polarization distribution of the single-pass SHG of the pump beam of $l_p = 1$. For the theoretical simulation, the single pass efficiency of the dual crystal was calculated directly from the intensity output of the individual Gaussian and LG beams. This indirect approach opted to effectively account for various parameters that affect the efficiency of the SHG apart from the nonlinear efficiency. Some of the major parameters are the slight variation in the focal length of the Gaussian, and LG beams arise from the different divergence during propagation. This slight focal change variation is essential since the

thin crystal was used in the experiment. Further, to reduce the effect of this factor, we have optimized the focal length of the focusing lens L1 to 150 mm at the cost of SHG efficiency. The second factor that affects the SHG efficiency is the optimization that emerges from simultaneously satisfying the critical phase matching of dual crystal. Since each of the crystals in the dual crystal scheme required a pitch and yaw angle, it is cumbersome to accommodate such parameters in the theoretical model but relatively easy to optimize the experimental setup. From the relative intensity and beam size obtained from the individual Gaussian and LG beam calculations, we have theoretically calculated the polarization distribution. As evident from the second row, (c, d) of Figure. 4.3, the polarization distribution (experimental, theoretical) of the SHG beam contains two pairs of C-points and two L-lines, confirming the vortex order of the SHG beam to be $l_{sh} = 2$, twice that of the pump beam. Such observation confirms the doubling of the OAM mode or the C-point and L-line singularities of the pump beam in the SHG process [124]. The doubling of C-points (marked by black circle and square) and L-line (green line) singularities in the SHG process can be understood as follows. As reported previously [107], the dual BIBO crystal converts the pump beam of the electric field, $E_p = \alpha_p |E_p^H, l_p\rangle + \beta_p |E_p^V, 0\rangle$ into the SH beam of the electric field, $E_{sh} = \alpha_{sh} |E_{sh}^V, 2l_p\rangle + \beta_{sh} |E_{sh}^H, 0\rangle$. Here, α_{sh} and β_{sh} are the amplitude coefficients of the vortex and Gaussian SH beams, respectively, governed by the conversion efficiency of the individual beams. The relative phase due to the birefringence properties of the crystal is controlled by the time delay between the orthogonal components of the pump FP beam.

Further, using the FP beam of vortex order $l_p = 3$, we have measured the Stokes parameters, S_1, S_2 , and S_3 for pump and corresponding SH beam, we have subsequently calculated the Stokes phases, ϕ_{12}, ϕ_{23} , and ϕ_{31} . The results are shown in Figure. 4.4. As evident from the first column of Figure. 4.4, the Stokes phase, ϕ_{12} representing the

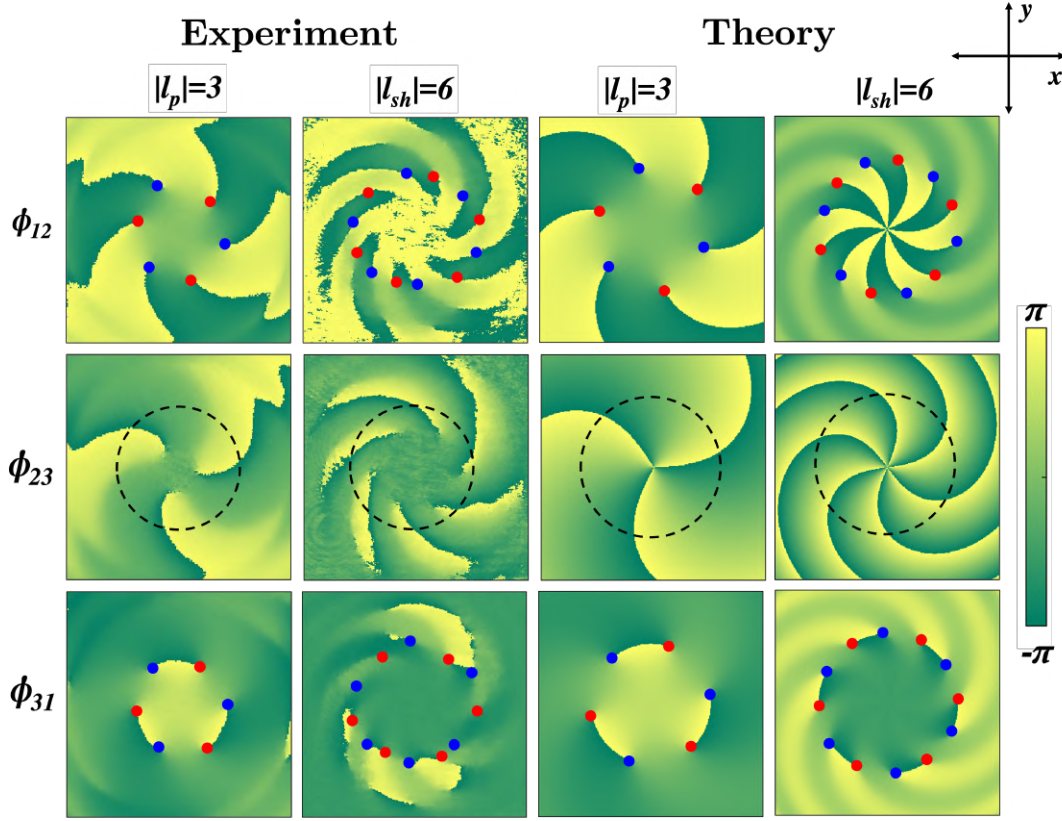


Figure 4.4: Experimental Stokes phases, ϕ_{12} , ϕ_{23} , and ϕ_{31} of the pump (first column) and SHG (second column) FP beams. Theoretical Stokes phases, ϕ_{12} , ϕ_{23} , and ϕ_{31} of the pump (third column) and SHG (fourth column) FP beams. The dotted circle guides to observe phase winding.

C-point singularity, contains three pairs of points marked by blue and red dots having phase winding corresponding to the charge $\sigma_{12} = +1$, star singularity, and $\sigma_{12} = -1$, lemon singularity, respectively. Using the formula $\sigma_{12} = 2I_c$, the singularity indices I_c of the C-points are found to be $I_c = +1/2$ and $I_c = -1/2$, respectively. The Stokes phase, ϕ_{23} , of the FP beam, shows the azimuthal phase winding corresponding to the vortex order, $l_p = 3$ of the pump beam. Therefore, the vortex order of the FP beam can be determined from the Stokes phase, ϕ_{23} . On the other hand, the Stokes phase, ϕ_{31} , represents three pairs of singularities over a ring. The second column of Figure. 4.4 shows the Stokes phases, ϕ_{12} , ϕ_{23} , and ϕ_{31} of the SHG beam of the FP beam of vortex

order, $l_p = 3$. As evident from the second column of Figure. 4.4, the Stokes phase, ϕ_{12} , has six pairs (lemon, blue, and star, red) of C-point singularities with singularity indices, $I_c = +1/2$ and $I_c = -1/2$, respectively, confirming the doubling of the C-point singularities in the SHG process. On the other hand, the Stokes phase, ϕ_{23} , shows the azimuthal phase winding corresponding to the vortex charge, $\sigma_{23} = l_{sh} = 6$, twice the order of the pump vortex charge, $l_p = 3$, of the FP beam owing to the OAM conservation in the nonlinear frequency doubling processes. Similarly, we observe six pairs of singularities over a ring in the Stokes phase, ϕ_{31} , of the FP SHG beam. For supporting the experimental results, we have performed the theoretical calculation for each of the Stokes phases for the pump by accommodating the experimental parameters $w_{xl} = 1.2$ mm, $w_{yl} = 1.7$ mm, $w_{x0} = 2.9$ mm, $w_{y0} = 2.5$ mm and $(\alpha_p^2, \beta_p^2) = (0.75, 0.25)$. Further, for SHG, we have used the parameters $w_{xl} = 1.9$ mm, $w_{yl} = 1.8$ mm, $w_{x0} = 0.9$ mm, $w_{y0} = 1.1$ mm and $(\alpha_{shg}^2, \beta_{shg}^2) = (0.35, 0.65)$ for calculating the Stokes phases of SHG. The theoretically calculated Stokes phases of the pump and corresponding SHG FP beam, as shown in the third and fourth columns of Figure. 4.4, respectively, are in close agreement with the experimental results. From the results of Figure. 4.4, it is evident that the polarization properties of the FP beams are conserved in nonlinear processes, and the Stokes phases can be explored as important tools to characterize the nonlinear optical processes.

To understand the dynamics of C-point singularities for different relative intensity of the constituent modes. We have measured the Stokes parameters for both pump and SHG beams while changing the relative intensities, α_p^2 and $\beta_p^2 = 1 - \alpha_p^2$ of the vortex and Gaussian beams, respectively, of the FP pump beam and calculate the Stokes phase, ϕ_{12} , with the results shown in Figure. 4.5. As evident from the first row, (a-e), of Figure. 4.5, the Stokes phase, ϕ_{12} , of the pump beam, maintains a uniform phase distribution for $(\alpha_p^2, \beta_p^2) = (0, 1)$ and $(1, 0)$ due to the absence of orthogonal

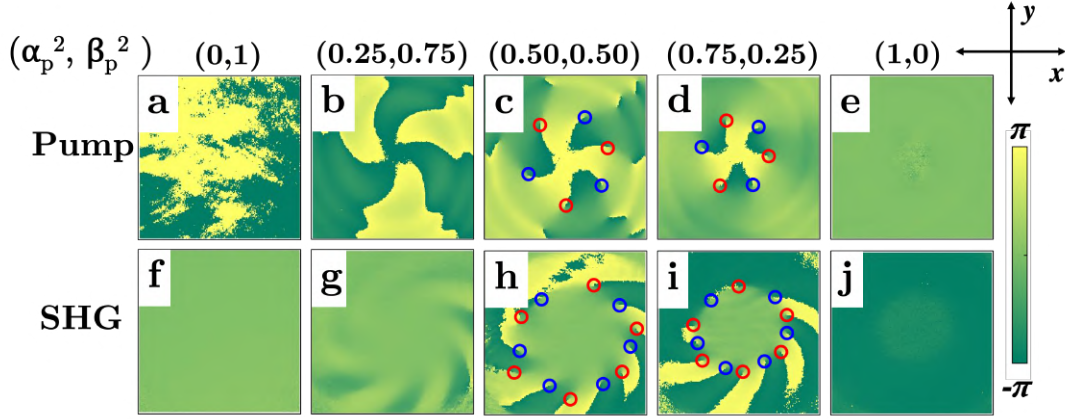


Figure 4.5: Variation of Stokes phase, ϕ_{12} , of (a-e) pump and (f-j) SH FP beam for different combinations of (α_p^2, β_p^2) .

polarization states. However, the distribution of the Stokes phase, ϕ_{12} , changes with the combination of (α_p^2, β_p^2) values showing the presence of three pairs of lemon and star singularities for the $(\alpha_p^2, \beta_p^2) = (0.5, 0.5)$ and $(0.75, 0.25)$. The distribution of the Stokes phase, ϕ_{12} , of the SHG beam, as shown in the second row, (f-j), of Figure. 4.5, follows the Stokes phase distribution of the pump beam with six pairs of lemon and star singularities for the $(\alpha_p^2, \beta_p^2) = (0.5, 0.5)$ and $(0.75, 0.25)$. It is evident from the first and second rows of Figure. 4.5 that the increase of α_p^2 (vortex beam intensity) brings the singularity points (lemon and star) towards the center and finally annihilates each other to form the uniform Stokes phase, ϕ_{12} distribution.

4.4.2 Coverage

We have further studied the polarization coverage of the pump and corresponding SH FP beams. Using the pump FP beam of order, $l_p = 3$, and $(\alpha_p^2, \beta_p^2) = (0.5, 0.5)$, we have calculated the Stokes parameters of the pump and SHG beam and projected them on the surface of the Poincaré sphere with the results shown in Figure. 4.6(a) and Figure. 4.6(b), respectively. As evident from Figure. 4.6(a, b), the polarization states

on the Poincaré sphere has a complicated distribution with void regions. Therefore, it is not easy to estimate the polarization coverage of the FP beam using the conventional area integration method [123]. As such, one can transform the 3D surface onto the 2D plane and estimate the polarization coverage. However, in doing so, one has to make the necessary correction to encounter the distortion effect commonly observed while transforming the surface of a sphere into a flat plane, especially for the sections in the pole region of the surface; a rectangle near the poles corresponds to a smaller area on the sphere than a rectangle near the equator. On the other hand, each discrete point on the surface of the Poincaré sphere represented by the Stokes parameters S_1 , S_2 , S_3 in Cartesian coordinates can also be represented by the ellipse orientation (ψ), ellipticity (χ), and Stokes parameter S_0 in the spherical coordinates. Therefore, the polarization states represented on the surface of the Poincaré sphere can also be represented in the rectangular Cartesian coordinate by considering x, y axis as 2ψ and 2χ with the limits $\psi \in [0, \pi]$ and $\chi \in [-\pi/4, +\pi/4]$ respectively. In doing so, one can transform the polarization states commonly depicted on the curved surface into a 2D plain surface.

However, the number of data points in a bucket does not contribute to the polarization coverage. Therefore, we have considered the bucket as full if the corresponding polarization state is detected in one or more data points. Now, if the number of buckets containing at least one point is N , then the polarization coverage of the beam can be calculated as $100 \times N/N_0$. For a beam with 100% polarization coverage, N should be equal to N_0 . However, proper selection of the number of buckets is very important. For example, suppose we consider N to be small. In that case, the bucket size will be large enough to cover the area not having any polarization state of the beam and overestimate the polarization coverage of the beam. On the other hand, if N is very large, then the bucket size will be small and fewer buckets will carry a polarization state with respect to the total number of buckets, thus underestimating the result. Therefore, to

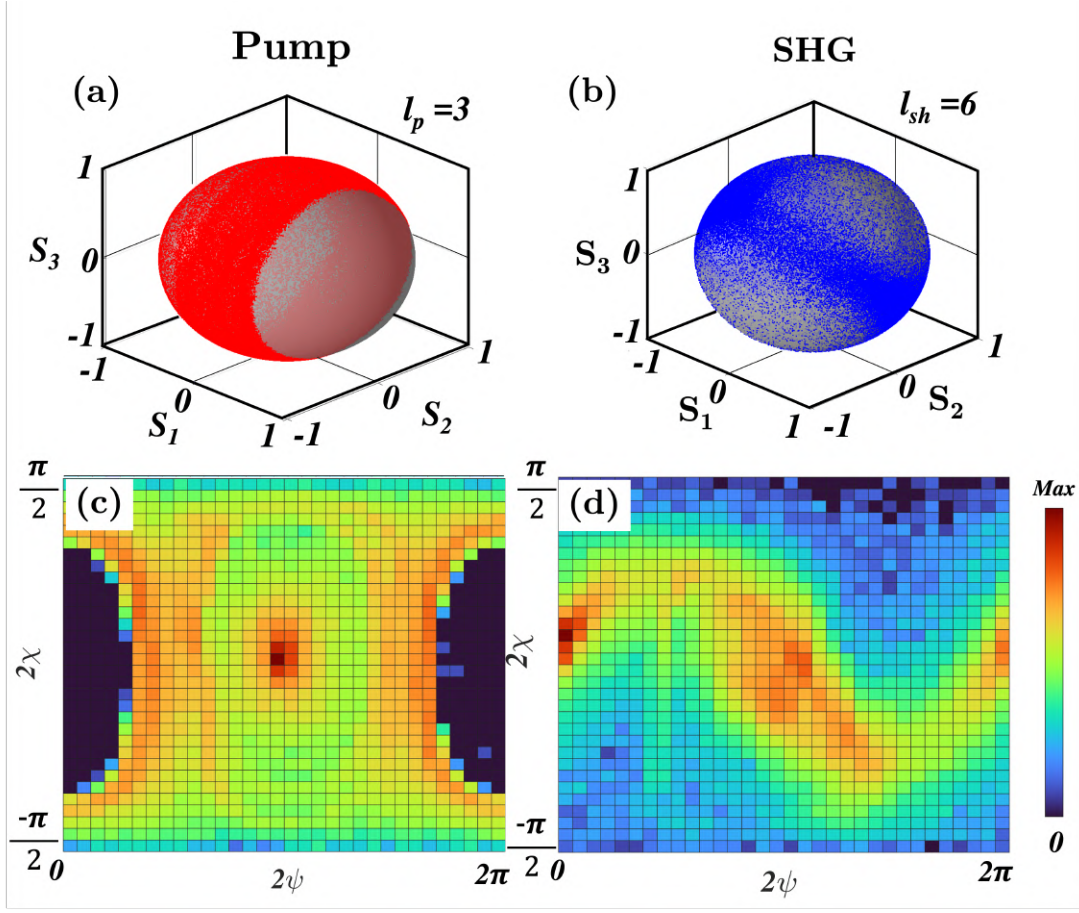


Figure 4.6: The Stokes parameters of (a) pump and (b) SH FP beams of pump vortex order, $l_p = 3$, are projected on the surface of the Poincaré sphere for $(\alpha_p^2, \beta_p^2) = (0.5, 0.5)$. Distribution of polarization states in $2\psi - 2\chi$ plane for (c) pump (d) SH FP beams.

optimize the number of buckets, N , we locked the least count of the area measurement to be 0.1%. Such a small value of the least count can be obtained by dividing the areas of the square-shaped $2\psi - 2\chi$ plane by $N = n \times n \sim 1000$ buckets. Therefore, we have considered $n \sim 32$ and measured the polarization coverage of the pump FP beam of order, $l_p = 1, 2, 3$, and 6 while varying the value of (α_p^2, β_p^2) . The results are shown in Figure. 4.7(a). As evident from Figure. 4.7(a), the polarization coverage of the FP beam of all orders lies in the range of 75 – 95% for $(\alpha_p^2, \beta_p^2) = (0.5, 0.5)$ and $(0.75, 0.25)$ and around 1% for $(\alpha_p^2, \beta_p^2) = (0, 1)$ and $(1, 0)$. The low coverage

($\sim 1\%$) can be attributed to the absence of both polarization states in the beam. On the other hand, for $(\alpha_p^2, \beta_p^2) = (0.25, 0.75)$, the polarization coverage varies in the range of 20 – 40% for all vortex orders indicating that in the presence of the azimuthal phase, the relative intensity of the vortex beam plays the crucial role to the polarization coverage of the FP beams. Using the experimental parameters, we have also calculated the polarization coverage of the FP beam of different orders.

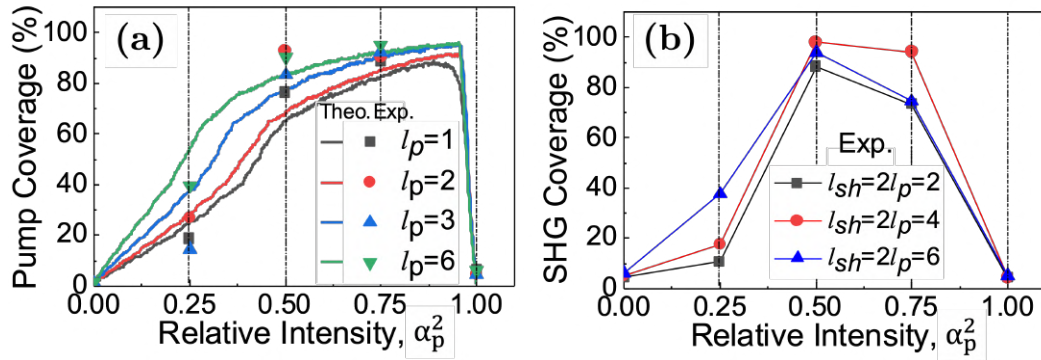


Figure 4.7: Variation of the polarization coverage (a) of the pump and (b) corresponding SH Poincaré beams of different pump vortex orders for a different combination of (α_p^2, β_p^2) values. Solid lines are theoretical results.

As evident from Figure. 4.7(a), the experimental polarization coverage represented as dots on the plot lie very close to the theory, which is the lines in the plots for FP beams of all orders. However, the small deviation in the experimental and theoretical results can be attributed to the asymmetry in the spatial profiles of the experimental beams. The maximum error for the polarization coverage measurement is the same as the least count area, 0.1%, used in this study. We have also measured the polarization coverage of the SHG FP beam for pump orders, $l_p = 1, 2$, and 3, for different values of (α_p^2, β_p^2) with results shown in Figure. 4.7(b). Interestingly, from Figure. 4.7(b), we see that the best weightings for the fundamental field do not give optimal coverage for all topological charges. For example, the relative weight of 0.25 has the best coverage for $l_{sh} = 6$, but this changes for relative weights of 0.75 where the best coverage is

given for $l_{sh} = 4$. When the relative weight is increased above 0.5, then we see a decrease in SHG coverage. This indicates that the size mismatch between the SHG vortex component and the Gaussian component bears a bigger influence. This size mismatch comes from the fact that the SHG of the Gaussian component has a waist resizing effect of a factor of the square root of 2, and this can be seen as a circle getting shrunk, while the phase vortex component can be seen as a ring getting thinner. Ultimately, this decreases the overlap between the modes and, thus, the accessible states of polarization and coverage. A striking consequence is that, for topological charge 6 and relative weightings 0.75, the fundamental beam is a FP beam as it is above the threshold of 75% coverage, but if the same beam is generated through the SHG of a FP beam of $l_p=3$ with the same weights, it falls below the threshold of 75%, no longer being considered a FP beam. When comparing the coverage of the pump and SHG for the same alpha values, we note that while pump coverage increases for values higher than 0.5, the SHG coverage decreases. This means that, counterintuitively, by decreasing the coverage of the pump beam, it is possible to increase the coverage for the SHG. Since the beam size of the Gaussian beam reduces and the LG beam size increases during the SHG process, it is difficult to maintain the spatial overlap between the two beams and generate the FP beam for the higher order. So, we note that for any lower order modal combination of the FP beam, we are able to experimentally control the coverage on the Poincaré sphere from 0 to 100% continuously by adjusting the relative intensity between the two modes.

To observe the effect of the size of the constituting beams of the FP beam on the polarization coverage, we varied the pump beam waist of the Gaussian beam (w_0) from 2.1 mm to 3.6 mm and measured the pump polarization coverage while keeping the annular width (FWHM) (w_1) of the vortex beam of order $l = 1$ constant (3 mm) and the $\alpha^2 = 0.9$. The results are shown in Figure. 4.8 (a). As expected, the polarization

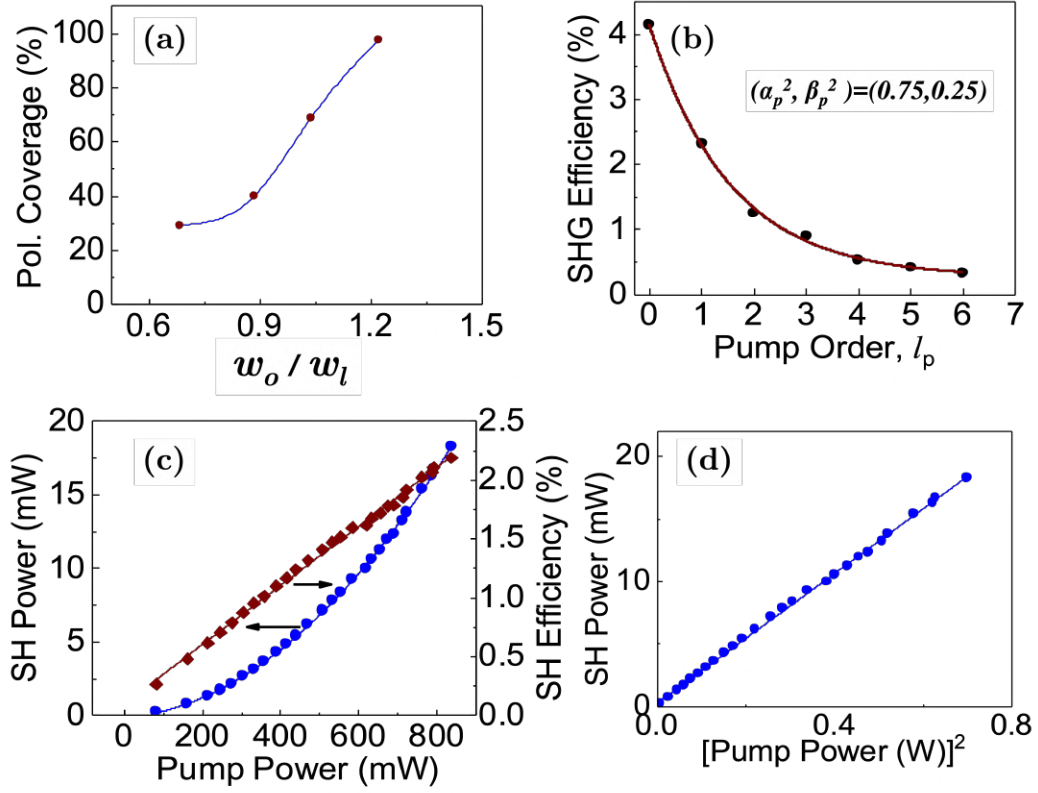


Figure 4.8: (a) Pump polarization coverage variation with relative FWHM of Gaussian and vortex beams. (b) Variation of SHG efficiency with the vortex order of the FP beams. (c) Variation of SH power (blue) and SH efficiency (brown) with the input power of the FP beams of vortex order, $l_p = 1$. (d) Dependence of SH power with the square of the pump power. Lines are guides to the eyes.

coverage increases from 29.3% for $w_o/w_l=0.7$ to a maximum of 97.75% at $w_o/w_l=1.2$. However, our numerical simulations (see Figure. 4.7(a)) show that the range of polarization coverage as a function of beam width ratio decreases with the decrease in the intensity ratio α^2/β^2 . Therefore, in our study, we have used the $(\alpha^2 = 0.9, \beta^2 = 0.1)$ to observe the higher control of the polarization coverage using the beam width ratio. Unfortunately, under such intensity ratio α^2/β^2 , the conversion efficiency of the Gaussian beam is so low that the FP beam becomes a scalar vortex beam. On the other hand, optimization in the intensity ratio results in almost constant polarization coverage in SHG with varying beam waist ratios. Therefore, a systematic study while consider-

ing the conversion efficiency of the constituent beams is essential to observe the large variation of the polarization coverage in the SHG process. We have also measured the SHG efficiency of the FP pump beam of different orders with the results shown in Figure. 4.8(b). As observed in Figure.8 (b), the single-pass SHG efficiency of the FP beam decreases with the vortex order from 4.19% for $l_p = 0$ (Gaussian beam with diagonal polarization) to 0.32% for $l_p = 6$ similar to the single-pass SHG of the vortex beam [124] due to the increase of the dark core size with the vortex order. keeping $(\alpha_p^2, \beta_p^2) = (0.5, 0.5)$, we have measured the power scaling of the FP beam of order, $l_p = 1$, with the results shown in Figure. 4.8(c). As evident from Figure. 4.8(c), the output power and SH efficiency increase quadratic and linear, respectively, with the pump power producing a maximum average output power of 18.3 mW at a single-pass conversion efficiency as high as 2.19% without any sign of saturation. The linear variation of the SH power with the square of the pump power, as shown in Figure.8 (d), further confirms the possibility of generating an SH FP beam of higher average power with the increase of pump power.

4.5 Conclusion

In conclusion, we have studied the nonlinear generation of ultra-fast FP beams while preserving the polarization characteristics of the pump FP beam and validated our experimental findings with theory. The Stokes parameters and Stokes phases analysis reveal the conservation of C-points and L-lines singularities in nonlinear processes. Even though beams having spatially varying polarization are being utilized in a wide variety of applications, the effective measurement and control of the accessible polarization state is not explored. In this work, we have devised a new method to estimate the polarization coverage of the FP beam, which can be extended to any optical beam with spatially varying polarization. Further, we show that the variation of the

polarization coverage for different intensity weightage of the constituent beams controls the available polarization state in an optical beam. We also showed that through the second harmonic generation of the FP beam, we could further control the accessible polarization while converting the beam into a new wavelength. The control of accessible polarization states via nonlinear interaction can be effectively utilized in applications that require short wavelengths without the requirement of any new optical elements in the new wavelength. It is also interesting to note that the SHG FP beam has the highest polarization coverage for equal weightings between the beams in the fundamental frequency. We have also devised a method to control the effective polarization of FP beam through second harmonic generation, which will be a very useful tool for applications demanding short wavelengths, which are widely accessed through second harmonic generation. Additionally, it is observed in the literature that the FP beams have smaller scintillation than comparable beams of uniform polarization in the presence of atmospheric turbulence [50, 51]. The robustness of the FP beams against atmospheric turbulence can further be understood by studying the change in the overall polarization coverage of such beams during free space propagation and helping to design the optimum special polarization structured beams for such applications. The current polarization coverage measurement technique can also be useful to study the manipulation of Poincaré beams on the different parts of the Poincaré sphere [52].

Chapter 5

Generation and characterization of self-healing properties of full Poincaré Bessel beam

5.1 Introduction

Spatial structured optical beams, tailoring or shaping light beams in the spatial degree of freedom, have attracted a great deal of attention recently due to their wide range of applications, including optical manipulation [12, 57], micromachining [10, 11, 36], imaging [5, 31], and optical communications [32, 125]. Typically the structured beams are generated through the mode conversion of the Gaussian beams. Depending upon the polarization modulation in the transverse plane of the beam, the structured beams are termed as the scalar, polarization states are same, and vector, polarization states are different, beams. One such widely used structured beams are optical vortices, which manifest as a line of zero intensity around which the phase has a circulating or helical

structure; the study of these and related singularities now form a field known as singular optics [126]. In a transverse plane, an optical vortex manifests as a point, and the phase always changes by an integer multiple of 2π around the vortex; this multiple is called the topological charge. The topological charge is generally a conserved quantity and is usually only created or annihilated in pairs of equal and opposite charges. On the other hand, the superposition of vortex beams of different orders in orthogonal polarization states results in vector vortex beams. The full Poincaré beam is a special class of vector vortex beams, ideally containing all polarization states that can be present on the surface of the Poincaré sphere. As a result, such beams have smaller scintillation than comparable beams of uniform polarization in the presence of atmospheric turbulence [50, 51]. Recently, efforts have been made to find a general method to estimate the polarization coverage and the parameters influencing the polarization coverage of the Poincaré beam [44] to broaden the scope of such beams for different applications [50, 51].

On the other hand, Bessel beams [127, 128], propagation invariant optical fields, have found numerous applications in various areas of optics [129–132], owing to their high intensity extended focus, finite beam width, nondiffractive propagation over considerable distances, and self-healing properties behind obstacles. In addition to such intrinsic properties, interest has grown to widen the scope of the Bessel beams by incorporating the polarization singularities through the generation of vector Bessel beams [133, 134] to study quantum effects [135, 136], microscopy, and imaging [137, 138], and turbulent media [50, 51, 139]. On the other hand, efforts have been made to include the ideally 100% polarization coverage of the Poincaré beam to the Bessel beam through the generation of Poincaré Bessel beam [140, 141], and study their structure and propagation characteristics. However, many exciting properties, such as the dynamics in the polarization pattern, changes in the degree of polarization, and polariza-

tion coverage of the beam, especially in the self-healing process, are left unanswered.

In this paper, we report on the theoretical and experimental study of the polarization characteristics of the Poincaré Bessel beam. Using an axicon, we have transformed the Poincaré beam in the rectangular basis having polarization coverage $>98\%$ into Poincaré Bessel beam. The study of polarization characteristics of the Poincaré Bessel beam reveals exciting features, such as each ring of the Poincaré Bessel beam behaving like the Poincaré beam with polarization coverage $> 75\%$, self-healing of the polarization structure, and degree-of-polarization is independent of the beam obstruction. Using the polarization ellipse orientation map having an infinite number of C-point singularity pairs in the self-healing process, we study the dynamics of the C-point singularities in the reconstruction process and the connections to the mathematics of infinite sets.

5.2 Experimental details

The schematic experimental scheme for the generation and study of full Poincaré Bessel beam is shown in Fig. 5.1. A continuous wave (cw), single-frequency, green laser (Coherent, Verdi V10) providing maximum output power of 10 W in TEM_{00} spatial profile with $M^2 < 1.1$ at 532 nm is used as the primary laser source. Operating the laser at its maximum output power for reliable system performance, we have used a power attenuator comprised of the combination of $\lambda/2$ (HWP1) and a polarizing beam splitter (PBS1) cube to control the laser power to the experiment. A pair of plano-convex lenses, L1 and L2, of focal length, $f_1 = 50$ mm and $f_2 = 200$ mm, respectively, placed in $2f_1-2f_2$ configuration is used to expand the laser beam. The $\lambda/2$ (HWP2) plate is used to control the relative intensity between the two arms of the Mach-Zehnder interferometer (MZI) configured with two plane mirrors, M1 and M2, and two PBSs,

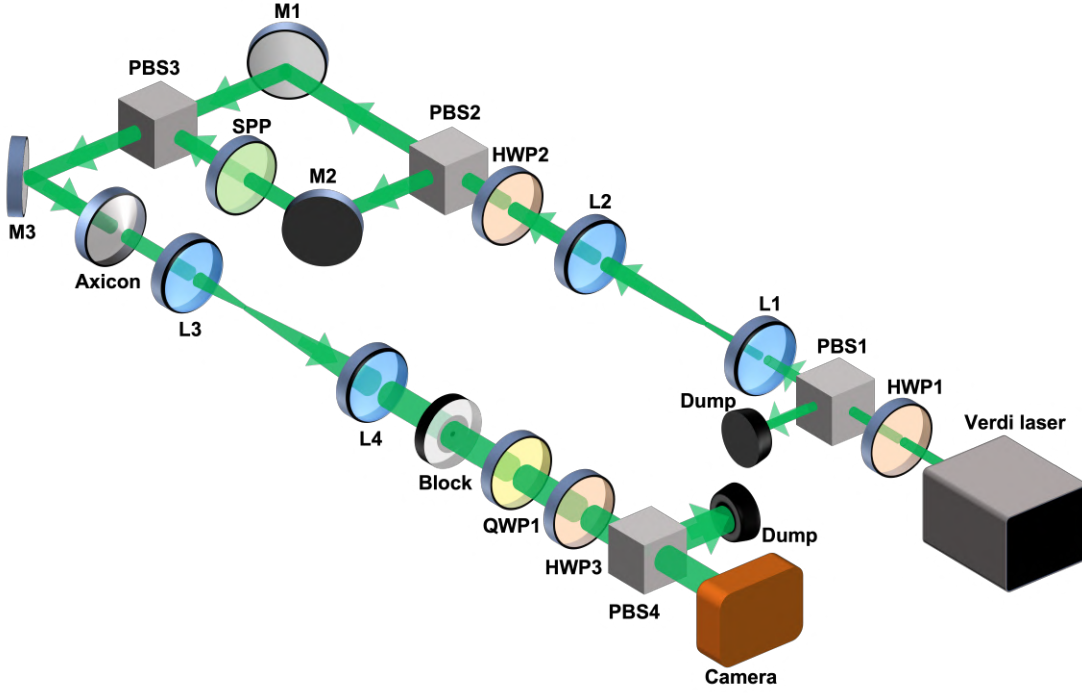


Figure 5.1: Experimental setup for the generation of Poincaré Bessel beam. HWP1-3: $\lambda/2$ plates; PBS1-4: polarizing beam splitter cubes; SPP: spiral phase plate; QWP1: $\lambda/4$ plate; M1-3: mirrors; L1-4: lenses

PBS2, and PBS3. The full Poincaré beam is generated [94] by placing the spiral phase plate (SPP) in one of the arms (here between mirror, M2, and PBS3) of MZI. The SPP has the transverse thickness variation corresponding to the phase variation of the vortex order of $l = 1$. As a result, the vertical polarized Gaussian beam of the reflected arm of MZI on propagation through the SPP and subsequent coaxial superposition with the horizontal polarized Gaussian beam of the transmitted arm on the PBS3 produces the full Poincaré beam with the electric field, $\alpha |H, 0\rangle + \beta |V, l\rangle$. Here α and β satisfying $\alpha^2 + \beta^2 = 1$, are the relative amplitudes of the orthogonal polarization modes of the FP beam. The H , and V are the horizontal and vertical polarization states of the constituent beams, and l is the order of the vortex beam. The values of α and β , can be controlled by varying the angle, θ , of the HWP2 angle by $\alpha = \cos \theta/2$ and $\beta = \sin \theta/2$. The full Poincaré beam on propagation through the Axicon with an apex angle of 196° is

transformed into a Poincaré Bessel beam. The Poincaré Bessel beam then expanded with the second pair of plano-convex lenses, L3 and L4, of focal lengths of $f_3 = 50$ mm and $f_4 = 300$ mm, respectively in $2f_3$ - $2f_4$ imaging configuration. The polarization state of the beam is characterized using the standard Stokes measurement technique [55] with the help of a quarter-wave plate (QWP), HWP3, PBS4, and the CCD camera. For the self-healing study, an obstacle (Block) made of a microscope cover slip with a black dot of diameter 0.2 mm at the center is used in the experiment.

5.3 Results and discussion

5.3.1 Characteristics of Poincaré Bessel beam

First, we have characterized the polarization characteristics of the full Poincaré Bessel beam generated through the mode transformation of the full Poincaré beam by the axicon. The electric field of the full Poincaré beam at the output of the MZI having SPP corresponding to vortex order, l , can be written as [94]

$$\begin{aligned} \mathbf{E} &= \alpha U_G^0(r, \phi) \hat{\mathbf{x}} + \beta U_{LG}^l(r, \phi) \hat{\mathbf{y}} \\ &= U_0 \left[\alpha \hat{\mathbf{x}} + \beta \left(\frac{\sqrt{2}r}{w} \right)^{|l|} LG_l \left(\frac{2r^2}{w^2} \right) e^{\pm il\phi} \hat{\mathbf{y}} \right] \end{aligned} \quad (5.1)$$

with $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ representing horizontal and vertical polarization, respectively. α and β are the relative weights of the two orthogonal polarized beams, and $\alpha^2 + \beta^2 = 1$. Here, U_{LG}^l , U_G^0 are the electric field amplitude of the vortex and Gaussian beams, respectively. Further, the Gaussian electric field amplitude at the origin is represented by U_0 . The parameters w , r , and ϕ correspond to the beam waist, radial, and azimuthal components of the beam. On propagation through the axicon, the input full Poincaré

beam represented by Eq. (6.2) transformed into a Poincaré-Bessel beam; the superposition of orthogonal polarized Bessel beams of order same as the topological charge or order of the respective orthogonal polarized beams of the input full Poincaré beam. Therefore, the electric field of the Poincaré-Bessel beam can be represented as,

$$\begin{aligned}\mathbf{E} &= \alpha U_J^0(r, \phi) \hat{\mathbf{x}} + \beta U_J^l(r, \phi) \hat{\mathbf{y}} \\ &= U_0 \left[\alpha J_0(k_r r) \hat{\mathbf{x}} + \beta J_l(k_r r) e^{\pm i l \phi} \hat{\mathbf{y}} \right]\end{aligned}\quad (5.2)$$

Here, $J_0(k_r r)$ and $J_l(k_r r)$ are 0^{th} and l^{th} order Bessel beams, respectively. The maximum electric field amplitude at the origin is represented by the constant U_0 . The radial wave vector k_r can be written in terms of the wave vector(k) and axicon apex angle(γ) and refractive index (n) as $k_r = k(n - 1)\cos(\gamma)$.

Using the experimental parameters (diameter (FWHM) of Gaussian and vortex beams of ~ 3.4 mm and ~ 6.8 mm, respectively, and their relative intensity weightage, $\alpha/\beta = 1$) in Eq. (6.2) and Eq. (6.3) and the theoretically calculated Stokes parameters, we have calculated the orientation (ψ) and ellipticity (χ) [55] of the polarization ellipse of the input full Poincaré beam and corresponding Poincaré Bessel beams. The results are shown in Fig. 5.2. As evident from Fig. 5.2(a), the transverse distribution of the polarization ellipse of the input beam of vortex order, $l = 1$, contains C-point polarization singularity in the form of a pair of lemon (see the white circle) and star (see the yellow circle) and a single L-line confirming the vortex order of the full Poincaré beam to be $l = 1$, same as our recent report [44]. For clear observation, we have magnified the section of polarization singularities region as shown in the inset of Fig. 5.2. Throughout the manuscript, we identify the white and yellow color circles as the lemon and star polarization singularities, respectively, if otherwise presented. However, the polarization distribution of the Poincaré Bessel beam, as shown in Fig. 5.2(b), shows

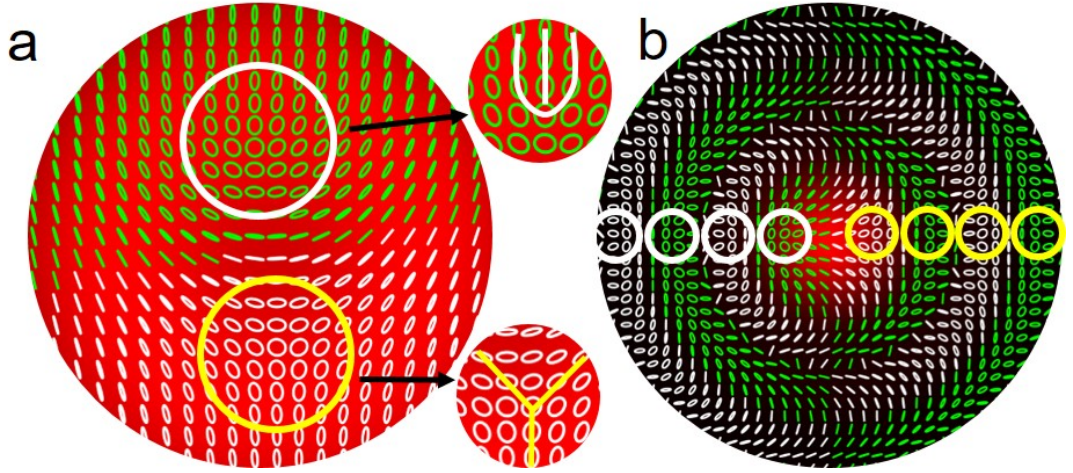


Figure 5.2: Polarization distributions of (a) full Poincaré and (b) Poincaré Bessel beams calculated using the experimental parameters in Eq. 6.2 and Eq. 6.3. The insets are the magnified images of lemon and star polarization singularities marked by white and yellow circles.

a very interesting pattern where each concentric circle contains a pair of lemons (see the white circle) and stars (see the yellow circle) singularities. As the Bessel beam has characteristic intensity distribution of concentric rings and infinite spatial extend, we observe the Poincaré Bessel beam to carry infinite pairs of lemon and star singularities. A careful observation of the polarization distribution indicates that each of the rings of the Poincaré Bessel beam contains a large number of polarization states, and the same polarization states are repeating in all the rings as if the Poincaré Bessel beam consists of an infinite number of full Poincaré beams.

To confirm such interesting polarization characteristics of the experimentally generated Poincaré Bessel beam, we recorded the intensity distribution for different polarization projections (using the combination of $\lambda/4$ and $\lambda/2$ plates at different combinations of the angles, the PBS). Using these intensity distributions, we have calculated the Stokes parameters, S_1 , S_2 , and S_3 , and measure the orientation (ψ) and ellipticity (χ) of the polarization ellipse. The results are shown in Fig. 5.3. As evident from

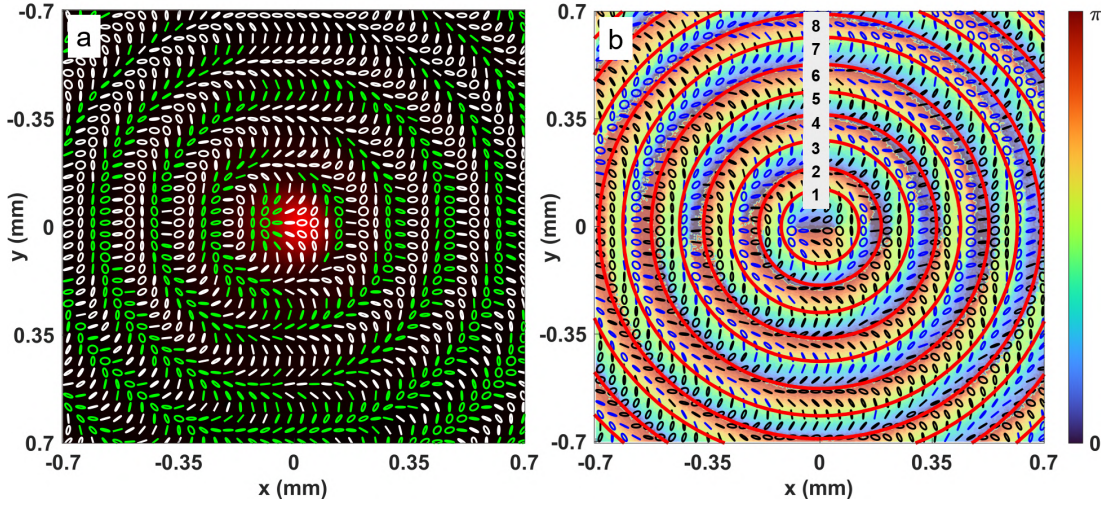


Figure 5.3: Experimentally measured polarization distribution of Poincaré Bessel beam with (a) beam intensity and (b) ellipse orientation map at the background. The red colour rings identify the characteristic rings of the Poincaré Bessel beams. The rings are also marked integer numbers for further studies.

Fig. 5.3(a), the generated Poincaré Bessel beam has polarization distribution in close agreement with the theoretical results (see Fig. 5.2(b)) with an infinite number of pairs lemon and star singularities (one pair in each ring of the Bessel beam containing an infinite number of concentric rings). As expected, the Poincaré Bessel beam generated by the full Poincaré beam has central intensity maxima, and each ring contains a large number of polarization states. To get further perspective, we have recorded the polarization ellipse orientation ($0 - \pi$) of the Poincaré Bessel beam along with the polarization distribution. The results are shown in Fig. 5.3(b). As evident from the Fig. 5.3(b), the ellipse orientation pattern of each ring (marked by red colour and identified by the numbers 1, 2, 3,....) has two singular points with opposite polarization ellipse orientation (counterclockwise, star and clockwise, lemon directions) confirming the presence of pair of polarization singularities in each ring and an infinite number of polarization singularity pairs in the transverse spatial distribution of the Poincaré Bessel beam.

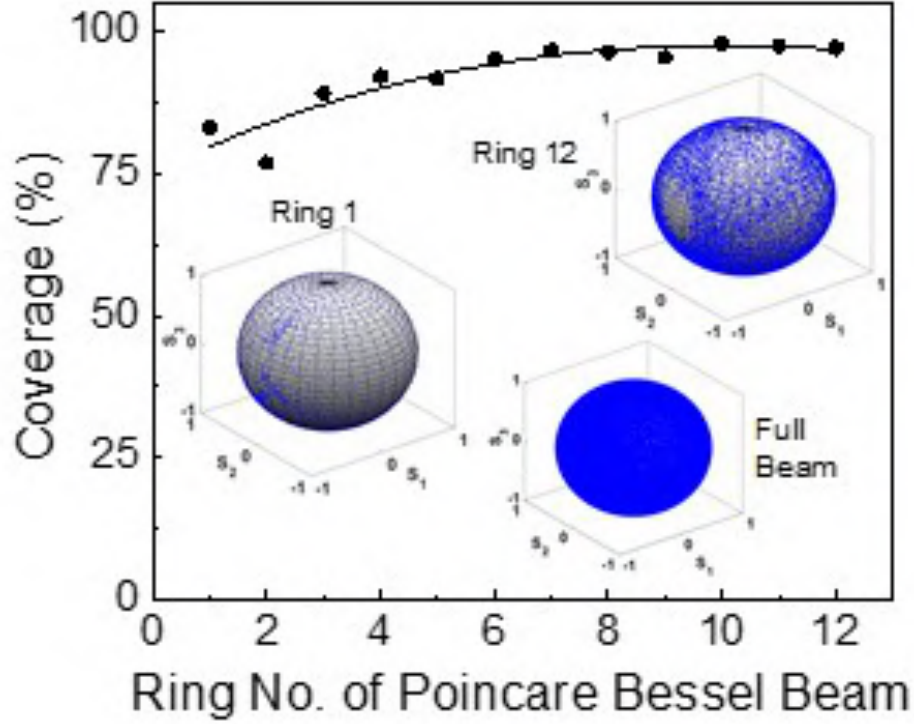


Figure 5.4: The variation of polarization coverage of each ring of Poincaré Bessel beam. Inset images show the polarization distribution of ring 1, ring 12, and the entire Poincaré Bessel beam on the Poincaré sphere.

Using the method discussed in section 3.3, we have calculated the polarization coverage of each ring of the Poincaré Bessel beam as marked in Fig. 5.3(b). The results are shown in Fig. 5.4. As evident from Fig. 5.4, the polarization coverage of the first ring of the Poincaré Bessel beam is $>75\%$. However, there is an increase in polarization coverage with ring number, and finally, all rings have polarization coverage $>97\%$. Although we expect to have the same polarization coverage in all rings, however, relatively lower polarization coverage for the central ring of the Bessel beam arises from the experimental imitations; the restriction in the number of useful camera pixels in the measurement process arising from the lower spatial extent of the central ring. The slightly lower polarization coverage for the second ring is due to the asym-

metry in the central lobe of the first-order Bessel beam generated from the vortex. The artifacts gradually die out with the increase in camera pixel numbers to accommodate the increase in the spatial extent of the Bessel rings. Therefore, we observe the initial increase of polarization coverage, which finally saturates, resulting in the same value for all rings, as expected. For further support, we have calculated the area (number of pixels) of vary from $4.8 \times 10^{-2} \text{mm}^2$ (2019) to $53.7 \times 10^{-2} \text{mm}^2$ (23633) for Ring number 1 to Ring no. 12 of the Poincaré Bessel beam. However, as the coverage of more than 75% of the Poincaré sphere is deemed acceptable as a full Poincaré beam for many applications [123], we can safely say that each ring of the Poincaré Bessel beam is a full Poincaré beam. We have also presented the polarization Poincaré sphere of each ring of Poincaré Bessel beam as the inset of Fig. 5.4. As expected, the Poincaré sphere, as evident from the inset of Fig. 5.4, gets populated with the number of points for the Bessel beam rings away from the center without increasing the polarization coverage substantially. Finally, the polarization coverage of the entire Poincaré Bessel beam, as also seen from the inset of Fig. 5.4, is around 100% and contains a large number of data points on the corresponding Poincaré sphere. Since the polarization states present in each ring of the Poincaré Bessel beam covers the entire surface of the Poincaré sphere, one can imagine the polarization coverage of the whole Poincaré Bessel beam as the superposition of an infinite number of same polarization states resulting the net polarization coverage same as the single ring. Such interesting property of Poincaré Bessel beam supports the self-healing characteristics of polarization coverage, the same as the intensity self-healing of the Bessel beam.

5.3.2 Self healing of Poincaré Bessel beam

Knowing the polarization characteristics, we have studied the intensity and polarization self-healing properties of the Poincaré Bessel beam. For the intensity self-healing

study, we have recorded the intensity profile of the Bessel beam generated through the Gaussian, scalar vortex of order, $l = 1$, and the full Poincaré beams as input to the axicon. On the other hand, we have estimated the degree of polarization, ellipse orientation, and ellipticity of the Poincaré Bessel beam using the Stokes parameters calculated from the intensity profile of the Bessel beam recorded for different polarization projections. The results are shown in Fig. 5.5. As expected, the zero-order Bessel beam, first-order Bessel beams, and Poincaré Bessel beam, shown by the first, second, and third columns of Fig. 5.5, respectively, have disturbed intensity distribution at the beam center resulting from the beam abstraction. However, all the beams start regaining and maintaining their initial spatial intensity distribution after a propagation distance of $d = 10$ cm with complete healing at $d = 59$ cm. To show the variation of the degree of polarization of the Poincaré Bessel beam during the self-healing process, we have used the colour map with blue and yellow colours representing unpolarized and perfectly polarized beams, respectively. The fourth column of Fig. 5.5 shows the polarization distribution in combination with the degree of polarization in the background. As evident from the fourth column of Fig. 5.5, the abstraction, although it disturbs the polarization distribution of the beam, has negligible or no impact on the degree of polarization. The Poincaré Bessel beam maintains a high degree of polarisation (≈ 1) throughout its cross-section. On the other hand, the concentric ring profile of the polarization distribution follows the same self-healing characteristics as the beam's intensity profile with propagation. To understand further the polarization self-healing characteristics of the Poincaré Bessel beam, we have estimated the polarization coverage of each ring along propagation distance. The results are shown by colour chart in Fig. 5.6. The rows and columns of Fig. 5.6 represent the ring number of propagation distance, respectively. It is evident from Fig. 5.6 all the rings before the beam obstruction carry polarization coverage $> 84\%$, same as Fig. 5.4. As expected,

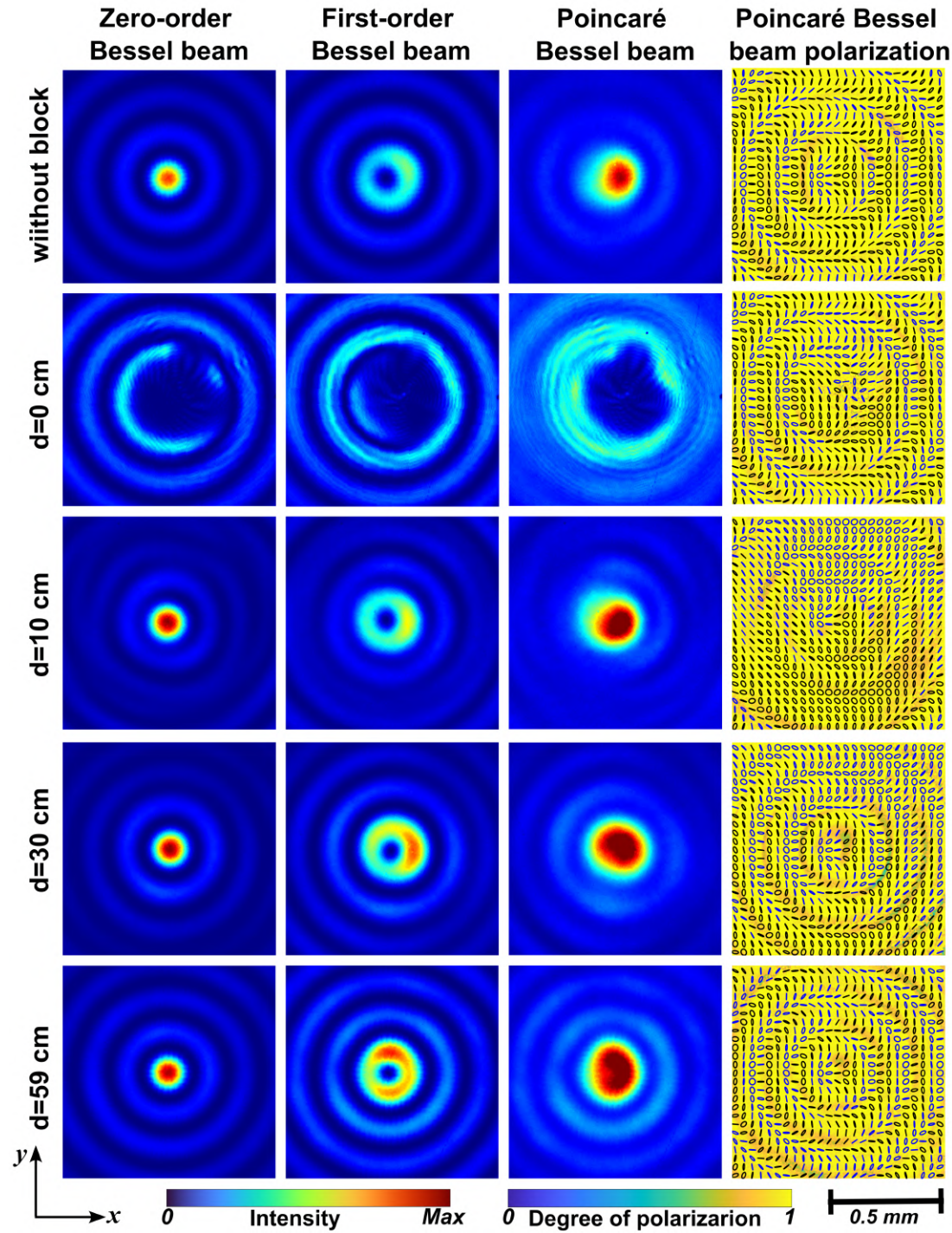


Figure 5.5: Verification of self-healing characteristics of (first column) zero-order, (second column) first-order, and (third column) Poincaré Bessel beam from the intensity distribution of the beam along propagation. Observation of (fourth column) polarization self-healing characteristics of the Poincaré Bessel beam.

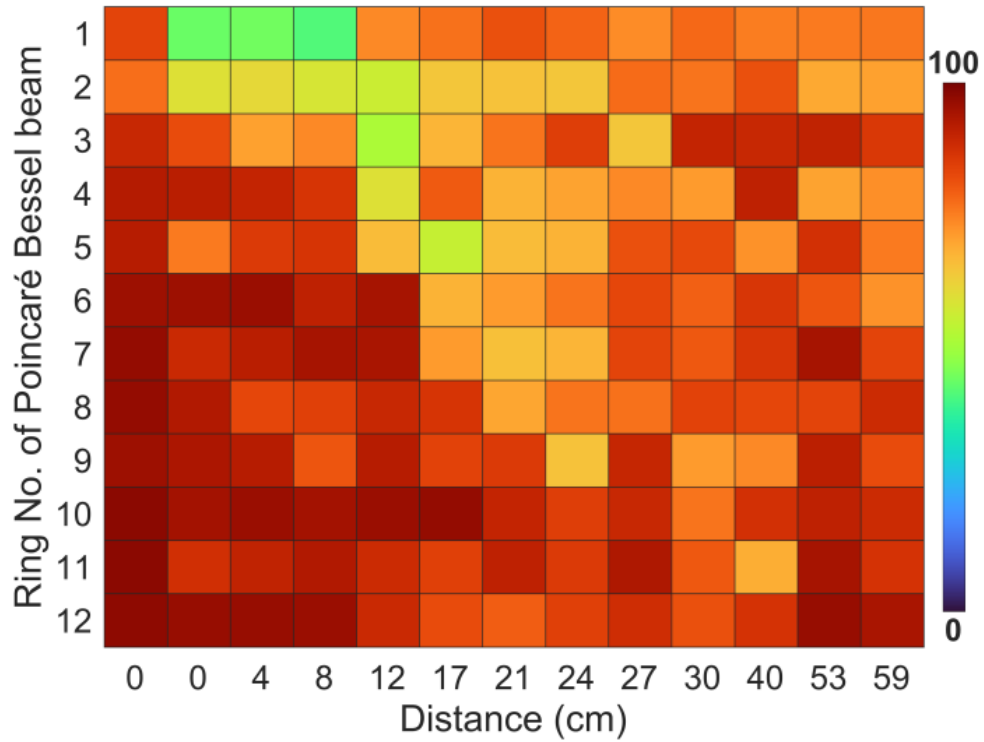


Figure 5.6: Variation of polarization coverage of different rings of the Poincaré Bessel beam along propagation during the self-healing process. The lower polarization coverage due to the beam obstruction is gradually moving in the outward direction during the self-healing process.

the polarization coverage of the central ring is low due to beam obstruction, while the polarization coverage of other rings remains unaffected. However, it is interesting to observe that the disturbance in the polarization coverage of the beam gradually travels away from the inner ring to the outer rings along the beam propagation and finally regains high polarization coverage for all rings. This observation confirms that the self-healing of the polarization coverage of the Poincaré Bessel beam occurs due to the energy flow of the Bessel beam from the outward rings to the inside rings. We also blocked the beam at various positions on the transverse plane and calculated the degree of polarization and polarization distribution to understand the impact of beam block sites on the self-healing process. We have selected obstructions of two sizes (circles of diameter 0.7 mm and 0.5 mm) and positions. The results are shown in Fig. 5.7.

As expected, in both cases, the degree of polarization, as shown by the background color map to the polarization distribution of the Poincaré Bessel beam in Fig. 5.7, is unperturbed to the beam obstruction.

On the other hand, it is observed from the first column of Fig. 5.7 that the beam block, as identified by the black circle, disrupts the polarization distribution initially at the locations it is placed. However, with beam propagation, it is observed from the second row of Fig. 5.7 that the disturbance gradually spreads away from the disturbed site. With further beam propagation, it is observed from the third row of Fig. 5.7 that the disturbance appears on the diametrically opposite side of the beam before resetting the effect beam block and returning to the initial polarization distribution. Like the intensity self-healing of the Bessel beams, the current observation clearly indicates that beam block size (much smaller than the beam size) and position does not have a detrimental effect on the intensity and polarization self-healing characteristics of the Poincaré Bessel beam.

5.3.3 Polarization singularities

Knowing the complete characteristics of the Poincaré Bessel beam in different degrees of freedom, including the intensity, polarization distribution, and degree of polarization, we have studied the effect of the C-point singularity of the beam in the self-healing process. As presented in Fig. 5.3(a), the Poincaré Bessel beam carries an infinite number of pairs of lemon and star singularities. To appreciate the observation and identify the C-points singularities, we have calculated the orientation of the polarisation ellipse varying from $0 - \pi$ with the results shown in Fig. 5.8. For easy comprehension, we have identified the direction of ellipse orientation about the singularity point by $0 - \pi$ in the counterclockwise direction with the white circle rep-

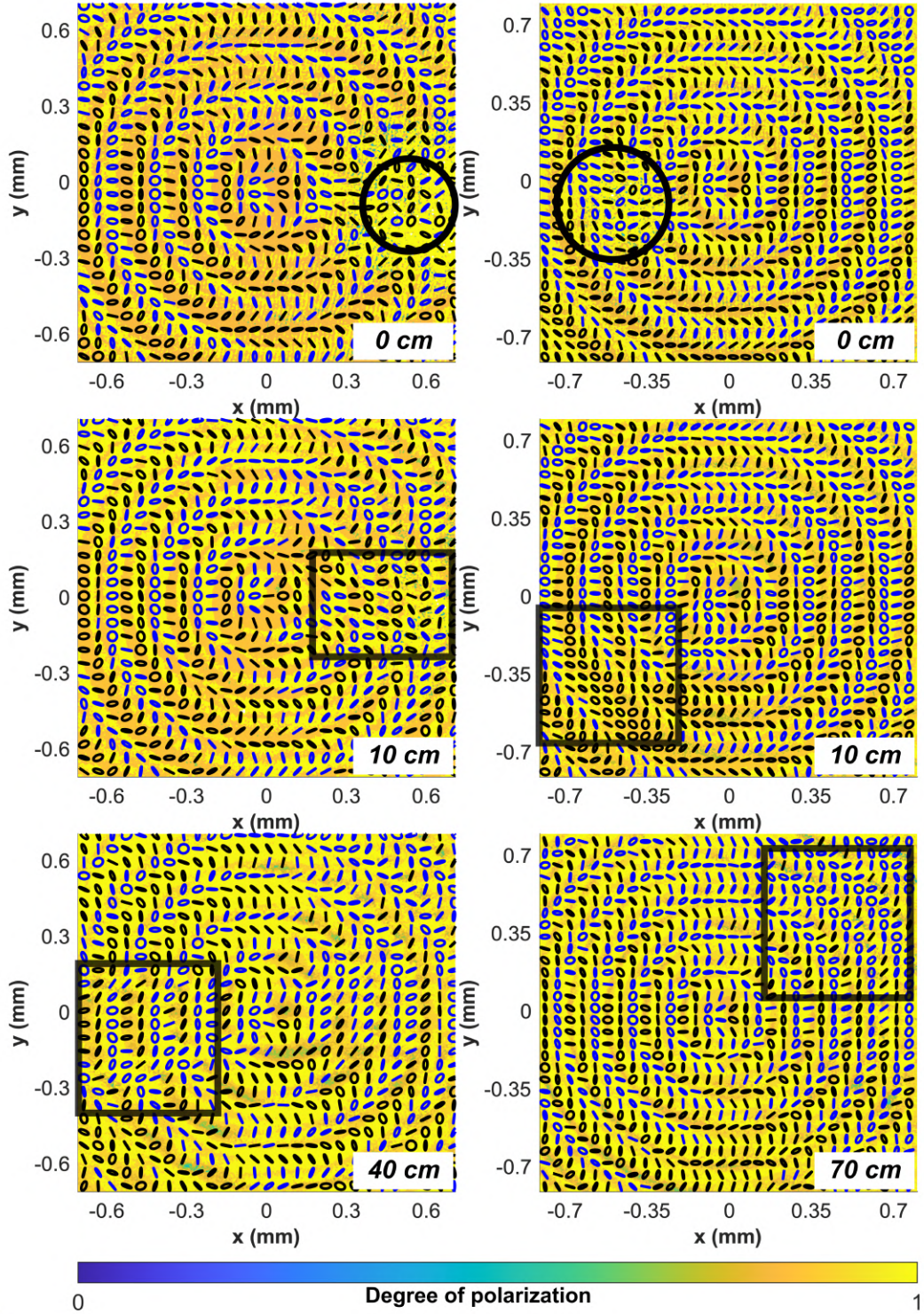


Figure 5.7: Dynamics of polarization distribution and degree of polarization of Poincaré Bessel beam along propagation after the beam obstruction by two blocks of different sizes and positions. Black circles and squares mark the position of polarization disturbance for easy identification.

representing star singularity and clockwise direction with the black circle representing lemon singularity. As expected, we observe the polarization ellipse orientation map of

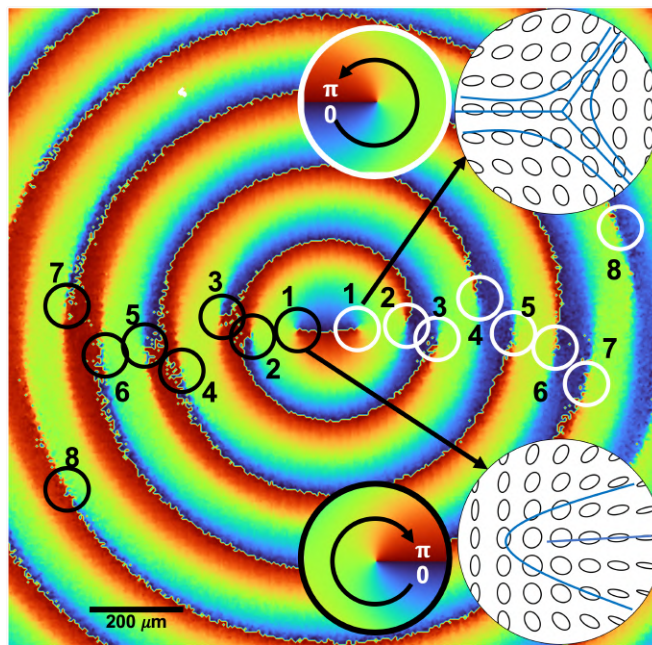


Figure 5.8: Polarization ellipse orientation map of the Poincaré Bessel beam showing the infinite series of C-point singularity pairs. The white and black circles identify the star (polarisation ellipse varying from $0 - \pi$ in the counterclockwise direction) and lemon (polarisation ellipse varying from $0 - \pi$ in the clockwise direction) singularities, respectively. The insets show the polarization distribution and corresponding polarisation ellipse orientation at C-point singularities.

the Poincaré Bessel beam in the rectangular basis to contain a series of lemon and star singularity pairs in each ring. As the ideal Bessel beams have infinite spatial extend and thus an infinite number of rings, we can clearly confirm the generation of infinite series of lemon and star polarization singularity pairs by transforming the full Poincaré beam into Poincaré Bessel beam. The number of series is decided by the number of polarization singularity pairs of the input full Poincaré beam or simply the order of the vortex. Using higher order full Poincaré beam of vortex orders, $l = 2, 3$ and 4 , we have observed the resultant Poincaré Bessel beam to carry 2, 3 and 4 infinite series of lemon and star polarization singularity pairs.

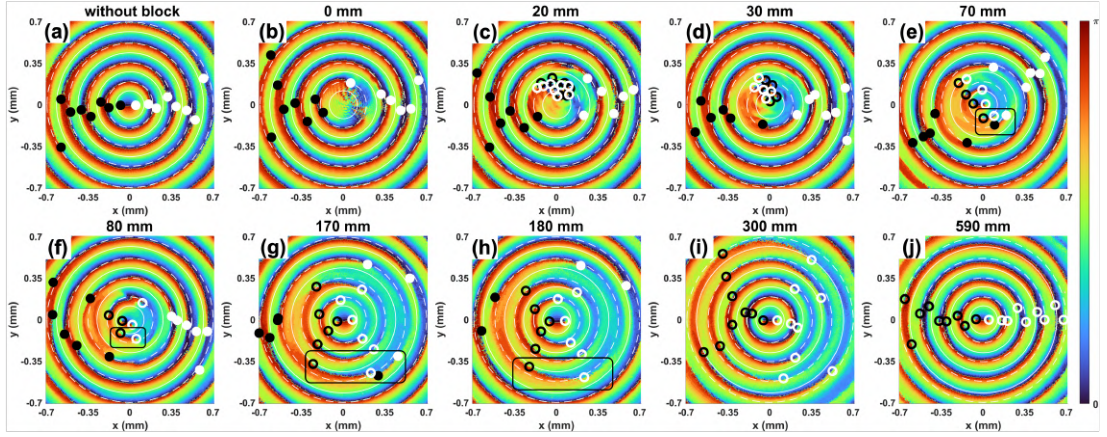


Figure 5.9: Observation of dynamics of the infinite number of C-point singularity pairs of the Poincaré Bessel beam along propagation after the beam obstruction. The star and lemon C-point singularities of the initial beam are marked by white and black dots, respectively. The newly formed star and lemon singularity pairs due to beam obstruction are identified by white and black circles. The black rectangles mark the annihilation of the old singularity with the new singularity.

Further, we have studied the dynamics of the c-point singularity of the Poincaré Bessel beam in the self-healing process. Using the intensity distribution of the beam for different projections along beam propagation, we have derived the polarization ellipse orientation map with the results shown in Fig. 5.9. As expected, the Poincaré Bessel beam contained an infinite series of star and lemon polarization singularities identified by the black and white dots in Fig. 5.9(a). As the star and lemon polarization singularities have singularity indices, $I_c = +1/2$ and $I_c = -1/2$, respectively, the net topological charge of the polarization singularities in the Poincaré Bessel beam can be considered to be zero. In the self-healing study, we purposefully adjusted the block position so that it created an asymmetry in the total number of star and lemon singularities. As evident from Fig. 5.9(b), the block has removed two stars and one lemon singularity from the polarization ellipse orientation map at the center, confirming the presence of asymmetry in the number of singularities right after the beam block. However, along beam propagation, as shown by Fig. 5.9(c-j), we observe exciting dynamics of C-point singularities. The beam block-induced perturbation results in the produc-

tion of an infinite number of new C-point singularity pairs, lemon, and star, marked by black and white open circles (see Fig. 5.9(c-d)). Due to the limited spatial resolution arising from the beam size and the pixel size of the CCD, we have marked a limited number of new C-point singularity pairs and observed their propagation dynamics. It is interesting to observe that the newly generated singularity pairs (black and white open circles) annihilate with each other and also with the intrinsic singularity pairs (black and white dots) with beam propagation (see Fig. 5.9 (e-i) and eventually take over the original singularity pairs in the center (Fig. 5.9(j)). After the self-healing process, it is interesting to observe that the Poincaré Bessel beam has an infinite number of C-point singularity pairs resulting in the net topological charge of zero, the same as the initial beam. However, after further beam propagation, the self-healed Poincaré Bessel beam, as shown by Fig. 5.9(j), regains the position of the C-point singularity and reproduces the polarization ellipse orientation map, the same as the initial beam (see Fig. 5.9(a)). A close look at the dynamics of the Poincaré Bessel beam in the self-healing process, especially the polarization ellipse orientation map of Fig. 5.9(a), (b) and (j), we see the existence of an optical analogy to the mathematics of transfinite numbers through the Hilbert's Hotel like setting. To elaborate further, let's consider the polarization ellipse orientation map as Hilbert's hotel, where the star singularity (black dots) and lemon singularity (white dots) represent the rooms and guests, respectively. From Fig. 5.9(a), it is evident that Hilbert's hotel, having an infinite number of rooms, is fully occupied by the guest, as the Poincaré Bessel beam carries an infinite number of C-point singularity pairs. Using the beam block, we have created a situation (see Fig. 5.9(b)) where the number of lemon singularities (white dot) is more than one than the number of star singularities (black dots). This situation can be considered as if the hotel is fully occupied, but an extra guest (white dot) has appeared for accommodation. Although we don't see the exact transition as described in the famous lecture of

the mathematician David Hilbert and demonstrated in optics [53, 54, 142, 143] during the self-healing process of the Poincaré Bessel beam, however, at the end of the self-healing process we do observe that the additional guest has been accommodated in Hilbert's hotel, making the hotel again fully occupied with a number of rooms (star singularity) is equal to the number of guests (lemon singularity). However, during the self-healing process, we see exciting features that can be used as optical analogies to understand the rich mathematics of transfinite numbers.

5.4 Conclusion

In conclusion, we have experimentally studied the polarization characteristics of the Poincaré Bessel beam in close agreement with the theoretical results. The use of a Poincaré beam in a rectangular basis containing all polarization states covered by the surface of the Poincaré sphere produces Poincaré Bessel beam with each ring having polarization coverage $>75\%$. Again the polarization coverage of the Poincaré Bessel beam is the same or slightly higher than the polarization coverage of any of the rings. Therefore, one can consider the Poincaré Bessel beam as the superposition of an infinite number of Poincaré beams. Further, it is observed that the polarization structure of the Poincaré Bessel beam shows self-healing characteristics like intensity self-healing after being abstracted. We also observed the degree of polarization of the beam has no impact on the beam obstruction. Using the polarization ellipse orientation map, we observe the beam to carry an infinite number of C-point singularity (lemon and star) pairs. The number of such infinite series is decided by the number of C-point singularity (lemon and star) pairs present in the input full Poincaré beam. As the number of C-point singularity pairs of the full Poincaré beam is equal to the vortex order of the constituent superposed orthogonal polarized beams, one can generate any number infinite series of C-point singularity (lemon and star) pairs by simply adjusting

the vortex order. Using the single series of infinite numbers of C-point singularity (lemon and star) pairs, we transition the dynamics of the C-point singularities in the self-healing process and observe Hilbert's hotel-like process addressing the connection to the mathematics of infinite sets. The current study can, in principle, be used for imaging objects even in the presence of depolarizing surroundings, studying turbulent atmospheric channels for communication and rich mathematical concepts of transfinite numbers.

Chapter 6

Experimental realization of Hilbert Hotel using scalar and vector fractional vortex beams

6.1 Introduction

The study of wavefield singularities in light, now referred to as singular optics [144], has revealed many interesting phenomena and new applications. In scalar waves, these singularities typically manifest as lines of zero intensity in three-dimensional space around which the phase of the field has a circulating or helical structure, as first demonstrated by Nye and Berry [145]; such structures are now referred to as optical vortices. In a closed path around an optical vortex, the phase always changes by an integer multiple of 2π ; this multiple is referred to as the topological charge. In vector waves, singularities manifest as lines of circular polarization in three-dimensional space, upon which the orientation of the major axis of the polarization ellipse is unde-

finer; the typical form of these singularities are usually referred to as C-lines [60, 146]. In a closed path around a C-line, the orientation changes by a half-integer multiple of 2π , and this multiple is called the topological index. Phase and polarization singularities typically intersect a transverse plane of an optical beam at a point; for polarization, we then refer to C-points.

Optical singularities have been considered for a number of applications due to their unusual properties. Beams carrying a pure optical vortex on their central axis, such as Laguerre-Gauss beams, possess a well-defined orbital angular momentum (OAM), and this OAM has been used for the trapping and rotation of particles and creation of light-driven micromachines [147]. Pure OAM states can be multiplexed and demultiplexed, and there are numerous investigations in using an OAM basis to increase the data transmission rate in optical communications [60, 148]. Both the topological charge of vortices and the topological index of C-lines are stable under weak perturbations of the wavefield, and typically only created or destroyed in equal and opposite pairs; because of this, these structures have been studied as alternative information carriers in optical communications [149].

One particularly surprising discovery to come from investigations of wavefield singularities is the existence of an optical analogy to the mathematics of transfinite numbers. In a famous lecture, mathematician David Hilbert introduced what is now known as Hilbert's Hotel to highlight how strange the mathematics of infinity would be in a real-world setting; his description was later popularized by Gamow [150]. Hilbert imagined a Hotel with a countably infinite number of rooms, numbered $1, 2, 3, \dots$, all occupied, so the Hotel has no vacancies. However, each guest can be asked to move to the next highest room, making room 1 available, and this process can be repeated indefinitely. Hilbert's Hotel, therefore, simultaneously has no vacancies and an infinite

number of vacancies.

Despite the paradoxical nature of Hilbert’s Hotel, it has now been recognized to manifest in systems that possess wavefield singularities, with singularities of positive and negative topological charge (or index) representing the “guests” and “rooms” of the Hotel. The first hint of this behaviour was shown by Berry [151] in 2004, who noticed that the creation of vortices by a fractional order spiral phase plate goes through a state with an infinite number of vortex pairs when the effective order of the plate is half-integer, with the topological charge of the field discontinuously changing at that moment; his results were confirmed experimentally the same year [152]. In 2016, Gbur [53] argued that this system violates the conservation of topological charge through a mechanism directly analogous to Hilbert’s Hotel and shows that multi-ramp spiral phase plates use the Hotel effect to create multiple vortices simultaneously. Furthermore, in 2017, Wang and Gbur [54] showed theoretically that novel Hotel effects could be created with polarization singularities in vector beams. The creation of infinite pairs of singularities in space is not the only way to realize Hilbert’s Hotel with OAM states; in 2015, researchers demonstrated a version of Hilbert’s Hotel through multiplicative mapping of OAM modes [153].

With such non-intuitive phenomena predicted, and the possibility of discovering more unusual vortex phenomena related to transfinite mathematics, it is worthwhile to have robust and versatile experimental techniques for testing these effects. Recently, the optical vortex version of Hilbert’s Hotel for a single “room” was demonstrated experimentally using a spatial light modulator (SLM) to produce the fractional vortex states [143]. The SLM was encoded with the desired fractional order and illuminated by a Gaussian beam to produce a fractional vortex beam. However, detailed investigations of Hilbert’s Hotel require nearly continuous changes in the fractional vortex

order, and despite the flexibility of SLMs in terms of dynamic phase modulation and wide wavelength coverage, it would be advantageous to have methods of studying the phenomenon that do not rely on their discrete nature. Furthermore, high-power vortex beams used in many applications are typically generated using spiral phase plates (SPPs), refractive elements that use a helical ramp structure to generate a vortex. If the unusual effects of the optical Hilbert's Hotel find practical application, having methods of generating them using SPPs would be beneficial.

SPPs were originally designed to produce a beam with a specific topological charge at a desired wavelength. Based on the design wavelength, λ , and the target vortex order, l , the maximum step height s is engineered in such a way that the incident Gaussian beam acquires an azimuthal shift of $2\pi l$. The vortex order of the beam is related to the optical parameters by the formula,

$$l = [n(\lambda) - 1]s/\lambda, \quad (6.1)$$

where $n(\lambda)$ is the wavelength-dependent refractive index of the material. As a result, SPPs are very wavelength specific and will not generate an integer vortex beam of order l for a laser wavelength away from the designed wavelength, λ . This limitation turns out to be an advantage for demonstrating Hilbert's Hotel: from Eq. (6.1) it is evident that one can continuously vary the effective vortex order of the SPP from l to $2l$ by varying the beam wavelength from λ to $\lambda/2$, with a small modification due to the variation of refractive index with wavelength. Therefore, by simply changing the wavelength of a Gaussian beam incident upon an SPP, one can easily generate vortex beams that effectively have a continuously variable fractional charge.

In this work we introduce, a simple experimental technique for studying fractional vortex effects, including not only the original vortex Hilbert's Hotel but also its vector

beam generalization. Using a double-ramp SPP having a step height corresponding to the vortex order $l = 2$ at the designed wavelength of 1064 nm in a modified Mach-Zehnder interferometer, and a supercontinuum laser tunable across 400-800 nm, we have generated scalar and vector vortex beams with tunable fractional topological order and verified several varieties of the optical Hilbert's Hotel.

6.2 Theoretical framework of Hilbert's Hotel Evolution in fractional singularities

Before describing the experiment, we briefly review the mathematics related to fractional spiral phase plates for both scalar phase singularities and vector polarization singularities. The results of these calculations are shown in Section 6.3 to compare with experimental measurements.

6.2.1 Phase Singularities

A common method of generating a vortex beam is to illuminate a spiral phase plate with a normally incident plane wave. As noted in the Introduction, a standard SPP has a helical ramp structure that, for a given wavelength, produces a continuous phase change $2\pi\alpha$ in the azimuthal direction; we refer to α as the order of the SPP. Beams produced by a SPP will always have an integer topological charge, regardless of whether α is integer or fractional. For brevity, however, we will refer to α as the order of the transmitted beam.

We first consider a unit amplitude scalar plane wave passing through a SPP with integer order n and transmission function $\exp(in\phi)$. At a distance z from the phase plate, the transmitted field can be determined by Fresnel diffraction, and is of the form

[151],

$$U_n(r, \phi, z) = \frac{\pi}{2} \sqrt{\frac{(-i)^{|n|}}{\lambda z}} r \exp(in\phi) \exp(ikz) \exp\left(\frac{ikr^2}{4z}\right) \times \left[J_{\frac{|n|-1}{2}}\left(\frac{kr^2}{4z}\right) - iJ_{\frac{|n|+1}{2}}\left(\frac{kr^2}{4z}\right) \right], \quad (6.2)$$

where $\mathbf{r} = (r, \phi)$ denotes the position vector in the transverse plane, λ and k denote the wavelength and wave number of the beam, respectively, and J_n represents the n^{th} order Bessel function of the first kind.

Traditional SPPs have a single ramp along the azimuthal direction and a single step discontinuity; we consider the more general case given by Gbur [53] of a SPP with m ramps of equal azimuthal width $\phi_m = 2\pi/m$, and overall fractional order α . The transmission function of the SPP is given by

$$T(\phi) = \exp[i\alpha(\phi - p\phi_m)], \quad p\phi_m \leq \phi \leq (p+1)\phi_m, \quad (6.3)$$

where $p = 0, 1, 2, \dots, m-1$ and α can have any integer or fractional value.

To evaluate the propagation of a plane wave through this fractional SPP, we may expand the transmission function of the multi-ramp fractional SPP in a Fourier series,

$$T(\phi) = \sum_{n=-\infty}^{\infty} C_n e^{in\phi}, \quad (6.4)$$

where the expansion coefficients are readily found to be of the form,

$$C_n = \frac{i}{2\pi} \sum_{p=0}^{m-1} e^{-ipn\phi_m} \left[\frac{1 - e^{i\phi_m(\alpha-n)}}{\alpha - n} \right]. \quad (6.5)$$

Equation (6.4) expresses the transmission function of the fractional SPP as a super-

position of integer SPP transmission functions. The fractional scalar vortex beam $U_\alpha(\mathbf{r}, \phi, z)$ generated by the fractional SPP can then also be written as the superposition of integer order vortex beams with different amplitudes (expansion coefficients) depending on the order α of the SPP,

$$U_\alpha(r, \phi, z) = \sum_{n=-\infty}^{\infty} C_n U_n(r, \phi, z). \quad (6.6)$$

It was shown by Gbur that the net topological charge of the transmitted field increases by integer multiples of m , and the transition happens when the fractional order of the SPP is an odd multiple of $m/2$.

In our experiment, we have used a double ramp SPP ($m = 2$) with the fractional order

$$\alpha = \frac{s}{\lambda} \left[n(\lambda) - \frac{m}{2} \right], \quad (6.7)$$

where $n(\lambda)$ is the wavelength dependent refractive index of the BK7 glass, determined by the empirical Sellmeier formula [154]. The maximum height difference in the SPP is given by the relation

$$s = \left[\frac{l_d \lambda_d}{n(\lambda_d) - \frac{m}{2}} \right], \quad (6.8)$$

where l_d and $n(\lambda_d)$ are the vortex order and refractive index of the SPP at the designed wavelength, $\lambda_d = 1064$ nm. By continuously modulating the wavelength of the source, the optical path length difference the light experiences on transmission will change and hence its output phase can change by fractional values of 2π , implying the generation of fractional vortex beams of varying order.

6.2.2 Polarization Singularities

According to Wang and Gbur [54], a similar mathematical method can be applied to generating vector beams with an effective fractional topological index. In the experiment described in Section 6.3, we superimpose a right-hand circularly (RHC) polarized Gaussian beam with a left-hand circularly (LHC) polarized beam that has passed through the fractional SPP. The output field may therefore be written in the form,

$$\mathbf{E}_\alpha(r, \phi) = U_\alpha(r, \phi)\hat{\mathbf{e}}_+ + U_0(r, \phi)\hat{\mathbf{e}}_-, \quad (6.9)$$

where $\hat{\mathbf{e}}_+$ and $\hat{\mathbf{e}}_-$ are the left- and right-handed circular polarization unit vectors, $\hat{\mathbf{e}}_\pm = \hat{\mathbf{x}} \pm i\hat{\mathbf{y}}$. In Eq. (6.9), C-points can be readily identified as points where $U_\alpha(r, \phi) = 0$, resulting in a point of pure circular polarization. In terms of linear polarization, we may write

$$\mathbf{E}_\alpha(r, \phi) = [U_\alpha(r, \phi) + U_0(r, \phi)]\hat{\mathbf{x}} + i[U_\alpha(r, \phi) - U_0(r, \phi)]\hat{\mathbf{y}}. \quad (6.10)$$

We may then use Eq. (6.6) to express the fractional order field $U_\alpha(r, \phi)$ in terms of integer order beams, as in the scalar case. The topological index of polarization singularities is determined by the change of the phase Ψ of the Stokes vector [126] $S_1 + iS_2$ in a closed loop around the singularity, where Ψ is given by

$$\begin{aligned} \Psi &= \frac{1}{2} \tan^{-1} \left(\frac{S_2}{S_1} \right), \\ S_1 &= E_x^2 - E_y^2, \\ S_2 &= 2\text{Re} [E_x E_y^*]. \end{aligned} \quad (6.11)$$

From Eq. (6.9) and the discussion of scalar phase singularities, we expect that the total topological index will increase by 1 when the fractional order $\alpha = 1$, with two $n = 1/2$ polarization singularities created in the transition.

6.2.3 Topological charge and index

The topological charge t or index n of a singularity can be formally defined by the expression,

$$t, n = \frac{1}{2\pi} \oint_C \nabla \Psi(\mathbf{r}) \cdot d\mathbf{r} \quad (6.12)$$

where C is a closed path of integration around the singularity line and $\Psi(\mathbf{r})$ is the phase of the scalar field or the Stokes vector (orientation angle) for phase and polarization singularities, respectively. For a scalar field, the topological charge t of an output field corresponding to a fractional singularity α can be written in the simple form,

$$t = m \times \text{floor} \left[\frac{\alpha}{m} + \frac{1}{2} \right], \quad (6.13)$$

where “floor(x)” refers to the largest integer not exceeding x . Equation (6.13) implies that jumps in topological charge of a scalar field can be achieved by modulating the fractional order α of the SPP exhibiting Hilbert’s Hotel evolution of phase and polarization singularities. From Eq. (6.9), we may conclude that every new scalar vortex in U_α of unit topological charge produces a C-point of $n = 1/2$ topological index.

6.3 Experimental Setup

The schematic of the experimental setup for the realization of the Hilbert hotel is shown in Fig. 6.1. A supercontinuum laser (NKT Photonics) producing unpolarized radiation with an average power of 2 W tunable across 400 nm to 2200 nm is used

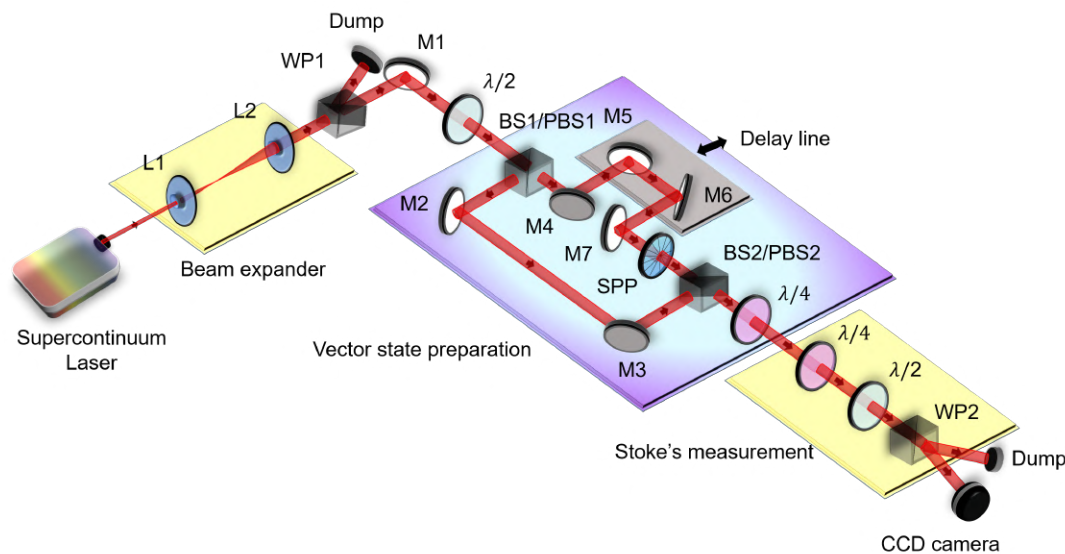


Figure 6.1: Experimental setup for the observation of optical Hilbert Hotel in phase and polarization. L1-2, Plano-convex lenses; WP1-2, Wollaston prisms; M1-7, dielectric mirrors; $\lambda/2$, half-wave plate; BS1-2, Beam splitter cubes; PBS1-2, Polarising beam splitter cubes; SPP, Spiral phase plate of order 2; $\lambda/4$, quarter-wave plate; CCD, charge-coupled device camera.

as the primary laser for the experiment. Using a variable tunable filter, we tuned the laser wavelength in the visible range across 400 nm to 800 nm with a minimum laser bandwidth of 10 nm. The coherence length of the laser is calculated to be $\sim 50 \mu\text{m}$ at 700 nm. Using a beam expander comprised of pair of plano-convex lenses L1 and L2 of focal lengths $f_1 = 50 \text{ mm}$ and $f_2 = 300 \text{ mm}$, respectively, we have collimated the Gaussian beam (TEM_{00} mode) of a diameter (full width at half maximum) of $\sim 6 \text{ mm}$. The Wollaston prism (WP1) with a polarization extinction ratio of 100000:1 is used to extract the linear polarized beam corresponding to an average power of 10 mW for a bandwidth of 10 nm across the tuning range. A $\lambda/2$ -plate is used to control the laser power in the modified polarization Mach-Zehnder interferometer (MZI), comprised of PBS1, PBS2, and a set of plane mirrors, M1-6. We used a delay line in one of the arms in order to temporally overlap the two beams at the output of the MZI. A double-ramp SPP made of BK7 glass having a topological charge of $l = 2$ at the designed wavelength

of 1064 nm was kept in one arm of the MZI to generate a vector vortex beam.

The electric field at the output of the MZI can be written as

$$\mathbf{E}_1(r, \phi) = aU_\alpha(r, \phi)\hat{\mathbf{x}} + bU_0(r, \phi)\hat{\mathbf{y}}, \quad (6.14)$$

with $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ representing horizontal and vertical polarization, respectively. However, after propagation through the quarter wave-plate ($\lambda/4$), the electric field of the vector beam transformed into

$$\mathbf{E}_2(r, \phi) = aU_\alpha(r, \phi)\hat{\mathbf{e}}_+ + bU_0(r, \phi)\hat{\mathbf{e}}_-, \quad (6.15)$$

where again $\hat{\mathbf{e}}_+$ and $\hat{\mathbf{e}}_-$ are again the unit vectors of left- and right circular polarization of light, and a and b are the relative weights of the two orthogonally-polarized beams. The set of elements $\lambda/4$, $\lambda/2$, and WP2 are used to measure the Stokes parameters of the vector beam. On the other hand, to study the scalar fractional vortex beam, we redesigned the MZI by replacing the PBS1 and PBS2 with 50:50 beam splitters, BS1 and BS2, respectively. In such a configuration, the arm of the MZI having the SPP generates a fractional vortex beam, while the second arm acts as the reference Gaussian beam or planar wavefront for interference study.

6.4 RESULTS AND DISCUSSION

We discuss the scalar case first. Using the dispersion relations of N-BK7 glass [154] in Eq. (6.7) for the double-ramped SPP ($m = 2$), we have calculated the effective fractional order α of the SPP as a function of laser wavelength; the results are shown in Fig. 6.2(a). As evident from the figure, the fractional order of the output beam continuously changes from the integer order, $\alpha = 2$ to $\alpha = 4$ as the laser wavelength

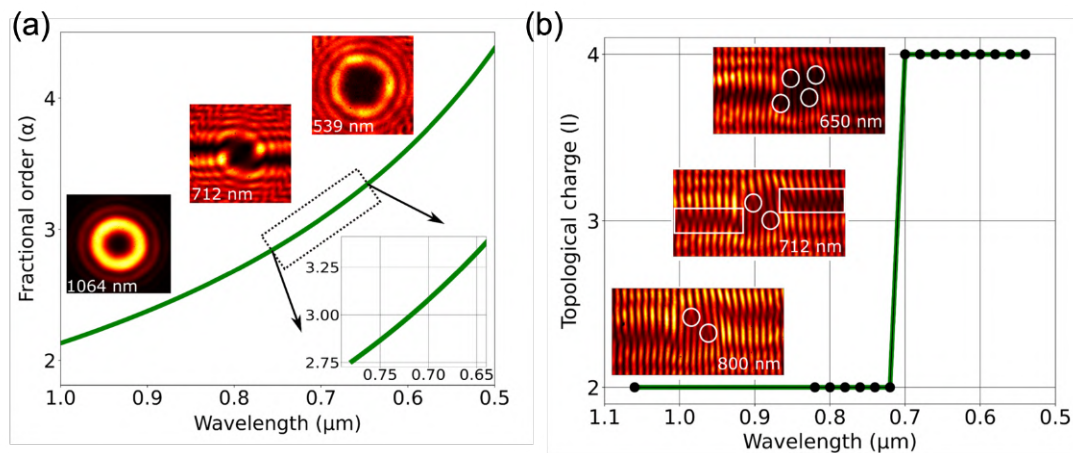


Figure 6.2: (a) Theoretical variation of the fractional order, α of the SPP as a function of the laser wavelength. (Inset) Magnified section of the topological charge variation near 712 nm and the intensity pattern of the beam after SPP at 1064, 712, and 539 nm. (b) Experimental variation of topological charge, l with wavelength showing step jump near 712 nm. (Inset) Interference pattern showing the characteristic fork pattern of the vortex beam at 800, 712, and 650 nm.

changes from 1064 nm to 539 nm. We have recorded the intensity profile of the output beam at 1064 nm and 539 nm, as shown by the insets of Fig. 6.2(a), which have the clear doughnut intensity profiles of integer vortices. By measuring the characteristic fork intensity interference pattern of the doughnut beam with the reference plane wavefront, we confirm the order of the generated vortex beam to be $\alpha = 2$ and 4 at 1064 nm and 539 nm, respectively. However, according to Eq. (6.13), a transition of the field topological charge should occur when the plate fractional order $\alpha = 3$. Therefore, we have magnified the wavelength range from 650 nm to 750 nm as shown by the inset of Fig. 6.2(a), and find the required laser wavelength for $\alpha = 3$ to be $\lambda = 712$ nm. Subsequently, we have observed the intensity profile of the beam, which is expected and confirmed to carry two expected singular lines for a $m = 2$ -step SPP.

To evaluate how the topological charge of the field changes with the variation of wavelength, we recorded the intensity interference pattern of the output beam with the reference beam and counted the net number of forks while varying the laser wave-

length. The results are shown in Fig. 6.2(b). As expected from the theory and Ref. [53], we observe the net number of forks (marked by white circles) of the interferogram of the vortex beam for the wavelength range from 1064 nm to 720 nm to be two even as the intensity pattern of the vortex beam changes from doughnut shape to carrying two low-intensity lines. Similarly, the net number of forks of the interference pattern in the wavelength range 700 nm to 539 nm is four despite the change of intensity pattern from carrying low-intensity lines to a doughnut shape. Therefore, we can easily conclude the topological charge of the vortex beam to be $l = 2$ for the wavelength range of 1064 – 720 nm and $l = 4$ for the wavelength range of 700 – 530 nm.

At the wavelength 712 nm, corresponding to $\alpha = 3$, the interference pattern contains an in principle infinite number of fork pairs representing vortex dipoles lying along the two singular lines of the SPP, as predicted by Berry [151] and Gbur [53]. When an infinite number of dipoles are present, the topological charge of the field is indeterminate. However, from Fig. 6.2(b), it is evident that the topological charge of the vortex beam generated by two-step SPP has a two-step jump from $l = 2$ to $l + m = 4$ while the laser wavelength drops below 712 nm in agreement with Eq. (6.13).

These observations suggest the experimental realization of Hilbert’s Hotel. They also demonstrate the simplicity and elegance of the overall experiment; it is very easy to select any fractional order between $l = 2$ to $l = 4$ by adjusting the laser wavelength to the SPP without moving any optical elements. However, due to the material dispersion and the ultrafast nature of the laser, one needs to ensure the temporal overlap of the beams of MZI using the delay stage. Such delay adjustment can be avoided using a continuous-wave laser or laser of a broader pulse width.

6.4.1 Scalar fractional vortex beam

To truly demonstrate Hilbert's Hotel, however, we need to show the rearrangement of positive and negative charge vortices in the manner that "rooms" and "guests" are rearranged in Hilbert's thought experiment. Therefore, we have focused our investigation on the wavelength range 760 - 660 nm in the immediate neighbourhood of the wavelength where the fractional order of the field makes the transition from order $\alpha = 2$ to 4. The results are shown in Fig. 6.3. We have recorded the interference pattern of the vortex beam for five different wavelengths, $\lambda = 760, 740, 712, 690$, and 660 nm, corresponding to the topological orders calculated from Fig. 6.2, of $\alpha = 2.8, 2.88, 3, 3.1$, and 3.25 respectively, with the results shown first column of Fig. 6.3.

We observe two forks for $\lambda = 760$ nm; however, the decrease in laser wavelength to 740 nm results in the creation of fork pairs (upward and downward-opening fork-shaped fringes) corresponding to vortex dipoles (marked by the green ellipses) in the singular lines of low intensity. The series of fork pairs (marked by the green boxes) extends in principle to infinity (here, restricted to the spatial extent of the beams) for a laser wavelength of 712 nm. A further decrease in laser wavelength results in the annihilation of the vortex dipoles, as evident from the reduction in the number of fork pairs at 690 nm. Finally, it attains a net of four forks, corresponding to the topological charge of 4 at 660 nm.

To understand and appreciate the creation and annihilation of the vortex dipole while changing the laser wavelength to the SPP, we have extracted the phase of the beam from the interference pattern by using Fourier spectrum analysis [155] of a non-contour type fringe pattern. The results are shown in the second column of Fig. 6.3.

It is evident that the phase distribution of the vortex beam at 760 nm has two phase

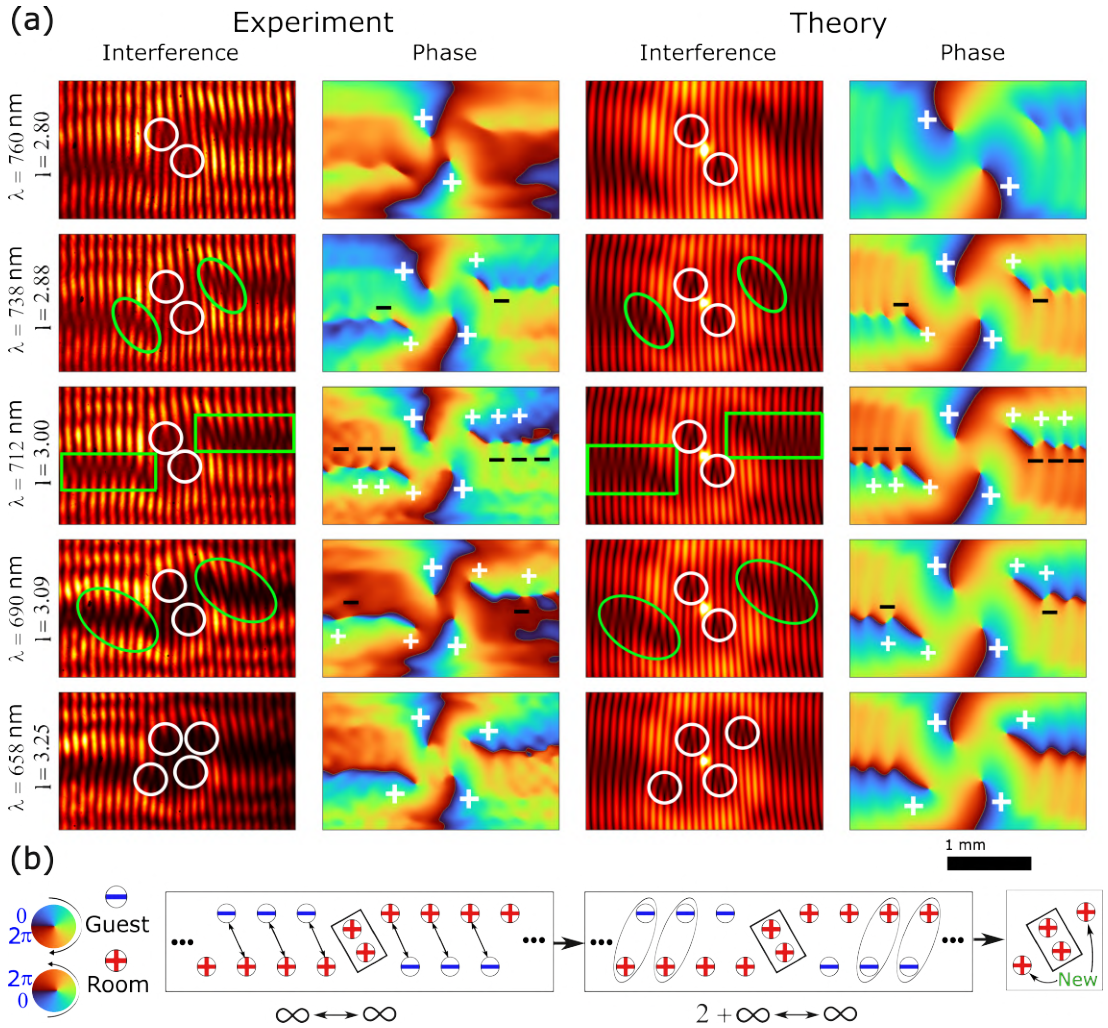


Figure 6.3: (a) Experimental interference pattern (first column) and corresponding phase distribution extracted using Fourier spectrum analysis (second column), theoretical interference pattern (third column), and corresponding phase distribution (fourth column) of the fractional vortex generated for different laser wavelength. The unit fork patterns are marked with white circles, and the dipoles are marked using green ovals or rectangles. The clockwise and counterclockwise phase variations are marked by “-” and “+” signs. (b) Illustration of vortex dynamics with the laser wavelength mimicking Hilbert’s Hotel. The phase variation “+” and “-” are labelled by “room” and “guest”, respectively.

singularities marked by “+” at the center of the beam with phase varying from 0 to 2π in the anticlockwise direction. The decrease in wavelength to 740 nm results in the creation of a pair of vortices in the singular lines marked by “+” and “-” (phase varying from 0 to 2π in the clockwise direction) on either side of the existing two vortices at

the beam center. However, at 712 nm, corresponding to the topological order of $\alpha = 3$, we see an increase in the creation of vortex pair extending away from the center of the beam. Due to the finite size of the beam and the apertures of the CCD camera, we have restricted our study to three pairs on either side of the beam center. In principle, however, the number of pairs extends indefinitely, eventually becoming unmeasurable in the low-intensity outskirts of the beam.

As predicted in Ref. [53], we experimentally observe each vortex to annihilate with its opposite neighbour instead of the neighbour it was created with. We see this occur starting at the laser wavelength of 690 nm, corresponding to fractional order $\alpha = 3.10$, and progressing towards the origin from the most distant points with further decrease of laser wavelength (an increase of topological order away from $\alpha = 3$), leaving four “+” vortices at the center of the beam at a wavelength of 660 nm. Using the experimental parameters in Eq. 6.6, we have simulated the interference fringes and extracted the phase distribution with the results shown in the third and fourth columns of Fig. 6.3, finding them in excellent agreement with the experimental results.

To make a clear relationship between the experimental results and Hilbert’s Hotel, we have labelled the phase singularities marked by “+” and “-” as the “room” and “guest” of the Hotel and illustrated the vortex dynamics using the phase distribution of the beam with fractional order $\alpha = 3, 3.10$ and 3.25 . The results are shown in Fig. 6.3(b). As evident from Fig. 6.3(b), the phase distribution has two individual charges at the center and a series of vortex pairs in the singular lines on either side of the beam center for $\alpha = 3$. Those vortex pairs created together are connected by arrows. Let us ignore the unit charges (contained in the black box) present at the center of the beam and focus on the vortex dipoles that mimic the situation of a fully occupied Hilbert Hotel, i.e., $\infty \leftrightarrow \infty$. Due to the experimental constraint, we have restricted our study

to three pairs of dipoles. However, for $\alpha = 3.10$, we observe the annihilation of the vortex dipole pairs of the series away from the beam center, leaving the first vortex dipole and a unit charge “+” from the second pair. The existence of unit charge “+” of the second pair and the increase of the separation of vortex charge “+” and “-” of the first dipole confirm that the vortex charge “-” of each dipole has annihilated with the vortex charge “+” of the next neighbouring dipole, i.e., the vortex charge “-” of N^{th} dipole annihilates with vortex charge “+” of the $(N+1)^{th}$ dipole, as if the “guest” of one “room” has moved to the adjacent “room”. Finally, for $\alpha = 3.25$, we see that the vortex charge “-” (“guest”) of the first dipole has annihilated with vortex charge “+” (“room”) of the second dipole, leaving an extra vortex charge of “+” (“room”) on either side of the beam mimicking the creation of two vacant “rooms” for the new “guests”, $\infty \leftrightarrow 2 + \infty$, in the fully occupied Hilbert Hotel. This is, to the best of our knowledge, the first demonstration of an optical Hilbert’s Hotel for a scalar field using a multi-ramp SPP. It demonstrates that a multi-ramp SPP can in fact produce multiple new vortices simultaneously, just as it is possible to open up multiple rooms in Hilbert’s Hotel by asking each guest to move more than one room over.

6.4.2 Vector fractional vortex beam

We now turn to the case of Hilbert’s Hotel with polarization singularities in a vector beam. It is known that the coaxial superposition of a vortex and a Gaussian beam, with opposite circular polarizations, transforms the phase singularity of a scalar vortex beam into a polarization singularity [94] of a vector vortex beam. Here the polarization singularities, or C-points, are the points on the vector beam where the polarization ellipse is circular, and the orientation of the polarization ellipse is undefined. The “star” and “lemon” singularities are popular C-point singularities where the polarization ellipse around the singularity point orientations from $0 - \pi$ in the clockwise and

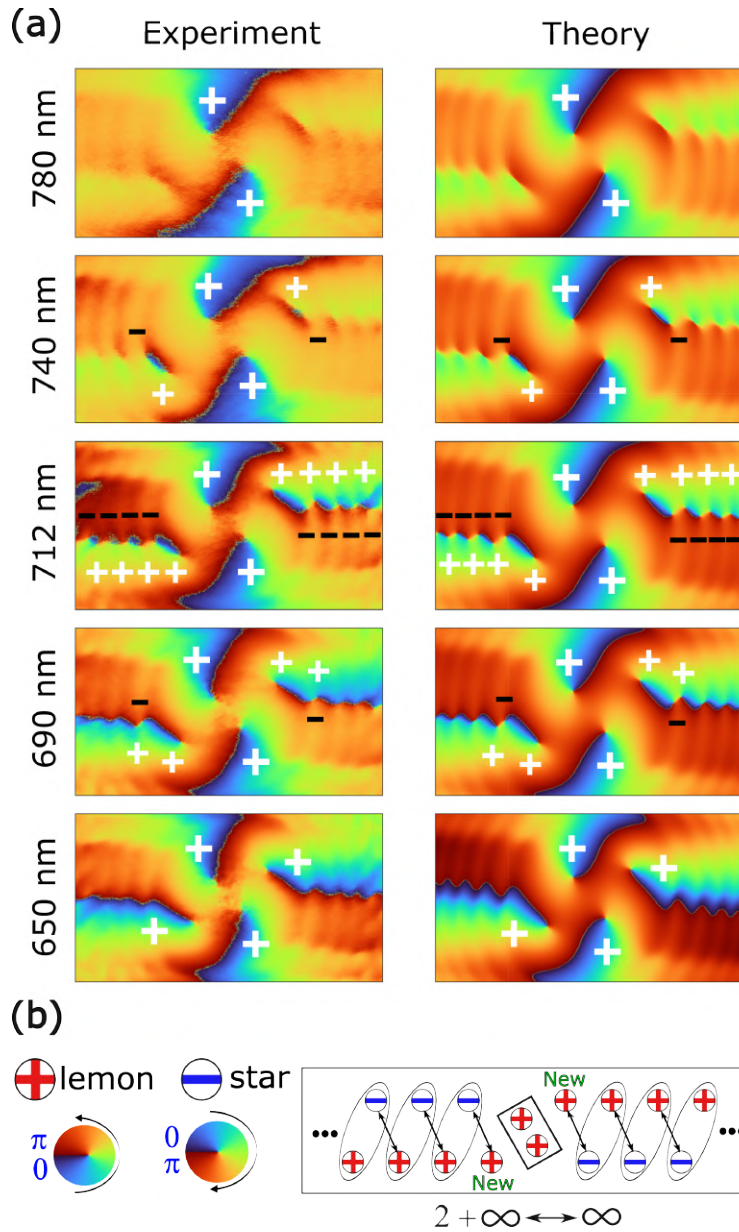


Figure 6.4: (a) Experimental (first column) and theoretical (second column) dynamics of polarization singularity of fractional vector beam with laser wavelength. The polarization ellipse orientation ($0 - \pi$) about the singular point in the counterclockwise and clockwise directions are identified by “+” and “-” respectively. (b) Illustration of dynamics of polarization singularity, “+” (lemon) and “-” (star) with the laser wavelength mimicking $\infty \leftrightarrow \infty$, to $\infty \leftrightarrow 2 + \infty$, transition of Hilbert Hotel.

counterclockwise directions, respectively. We have observed the vector beam to carry the “star” and “lemon” singularities in pairs and to carry topological charges of $+1/2$ and $-1/2$, respectively [142]. As a result, in 2017, Wang and Gbur [54] theoretically demonstrated the Hilbert Hotel using polarization singularities in vector beams.

Building upon the simple experimental realization of Hilbert Hotel using fractional scalar vortex beams, we have explored the experimental realization of Hilbert Hotel using polarization singularities. In the current experiment (see Fig. 6.1), the replacement of BS1 and BS2 with PBS1 and PBS2 of the MZI results in the vector beam with an electric field given by Eq. 6.14. Keeping a $\lambda/4$ plate at $+45^\circ$ with respect to the horizontal polarization, we have converted the vector vortex beam from a linear basis to a circular basis with a generalized form represented by Eq. (6.15). Similar to the scalar vortex study (see Fig. 6.3, we have adjusted the laser wavelength at 770, 740, 712, 690, and 650 nm and recorded the beam intensity profile for four distinct configurations of $\lambda/4$ and $\lambda/2$ plates, adequate for estimating the Stokes parameters of the polarisation distribution [156].

Using these intensity distributions, we have calculated the ellipse orientation distribution with the help of Stoke’s parameters, S1 and S2, given by Eq. (6.11). The results are shown in Fig. 6.4. As evident from the first column of Fig. 6.4(a), the polarization ellipse orientation map of fractional vector vortex in circular basis has two lemon singularities denoted by “+” at the center of the beam for laser wavelength $\lambda = 770$ nm. However, for laser wavelength of $\lambda = 740$ nm, we observe, in addition to the initial lemons, the appearance of pairs of lemon and star singularities identified by “+” and “-”, respectively, on either side of the beam center. The number of lemons and stars keeps on increasing with the decrease of laser wavelength, in principle giving a countably infinite number of pairs extended away from the beam center for the

laser wavelength of $\lambda = 712$ nm corresponding to the topological order $\alpha = 3$ of the fractional vector beam of the two-step SPP.

However, as we further reduce the laser wavelength to 690 nm, we observe, similar to the vortex pair annihilation of scalar fractional case as shown in Fig. 6.3, the annihilation of star and lemon singularities; star of N^{th} pair annihilates with the lemon of $(N+1)^{th}$ pair, leaving one pair of lemon and star and a single lemon singularity of the adjacent pair on either side of the beam center. Finally, the star of the first pair annihilates with the lemon of the second pair creating an extra lemon singularity on either side of the beam center with a result of a total of four lemons, including the initial lemons for the laser wavelength of $\lambda = 650$ nm corresponding to $\alpha = 3.25$.

Using the mathematical Eqs. (6.9), (6.11) and (6.15) and the experimental parameters, we have calculated the evolution of polarization singularity of the fractional vector beam as shown in the second column of Fig. 6.4(a) in close agreement with the experimental results. The evaluation of polarization singularities as summarized in Fig. 6.4(b) show the transition of $\infty \leftrightarrow \infty$, to $\infty \leftrightarrow 2 + \infty$, the transition of Hilbert Hotel. Therefore, we have successfully observed the first experimental realization of more general examples of Hilbert's Hotel with polarization singularities in vector beams, as predicted theoretically [54].

We have further studied the propagation characteristics of the polarization singularity of the fractional vector vortex. In doing so, we measured the beam intensity for different projections at different distances along the beam propagation for the laser wavelength of $\lambda = 712$ nm ($\alpha = 3$) and extracted the ellipse orientation distribution. The results are shown in Fig. 6.5. As evident from the first column of Fig. 6.5, the ellipse orientation map contains the infinitely extended chain of polarization singularities at the propagation of 35 cm measured from the MZI. On further propagation to a

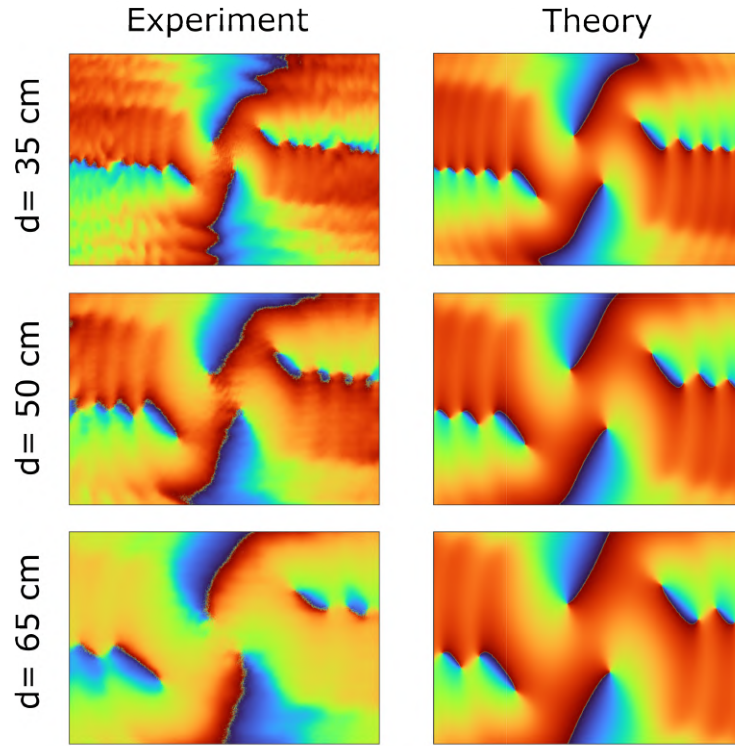


Figure 6.5: Experimental (first column) and theoretical (second column) evolution of the polarization singularities chain of the fractional vector beam corresponding to a topological order of $\alpha = 3$ at 712 nm at three different propagation distances in free space.

distance of 50 cm and 65 cm, we observe the positions and size of singularity change significantly due to the beam divergence. However, the infinite polarization singularity chains remain unchanged and preserve the signature of Hilbert's Hotel. It is to be noted that, due to the divergence of the finite beam, the infinite line of phase singularities at a long propagation distance is lost in the low-intensity region of the beam tail. Using the same experimental parameters, we have also simulated the propagation characteristics of the infinite lines of polarization singularity chains as shown in the second column of Fig. 6.5 in close agreement with the experimental results.

6.5 Conclusions

We have demonstrated, to the best of our knowledge, the first experimental realization of Hilbert's Hotel using both fractional order scalar and vector beams with a multi-ramp SPP. The generation of fractional scalar and vector vortex beams from a fixed SPP by changing the laser wavelength through a filter makes the overall experiment very simple, alignment-free, and easy to implement. While we have demonstrated the proof of concept using a SPP of order $l = 2$, the current demonstration reveals that the use of multi-ramp SPP can produce multiple new vortices simultaneously. The generation of such multiple vortices can open the possibility of verifying the complicated transitions to prove the generalized Hilbert's Hotel paradox. The generic experimental scheme can also be useful for understanding the behaviour of complex polarization-sensitive optical elements as required in many fields, including designing novel devices, quantum communication, and sensing.

Chapter 7

Summary, and Outlook

In summary, we have devised a new method to estimate the polarization coverage of any arbitrary vector vortex beams. Using the same method we estimated the polarization coverage of the vector vortex beam and compared the experimental results in close agreement with the theory.

In addition to the generation of full Poincaré (FP) beams using linear optical elements such as spiral phase plate, polarization beam splitter, and waveplate, we also explored the nonlinear generation of ultra-fast FP beams into a new wavelength while preserving the polarization characteristics of the pump FP beam generated through the linear optics. We have validated our experimental findings with theory. Using the Stokes parameters and Stokes phases analysis, we observed the conservation of C-points and L-lines singularities in nonlinear processes. We also explored the beam parameters influencing the polarization coverage and found that different intensity weightings of the constituent beams control the available polarization state in an optical beam. We also observed that in the second harmonic generation of the FP beam,

one could control the accessible polarization and achieve the highest polarization coverage for equal intensity weightings of the constituent beams of the fundamental frequency.

Having full characterization of the FP beam, we generated the Poincaré-Bessel beam and studied its polarization characteristics. Typically the peculiar characteristics of Bessel beams in terms of non-divergence and self-healing are studied while observing the intensity pattern of the beam. We have studied the polarization characteristics of the Poincaré-Bessel beam. While the intensity profile of the new beam maintains all characteristics of the Bessel beam, we observe the polarization structure of the Poincaré-Bessel beam to carry an infinite series of C-point singularity pairs even though the input FP beam has a particular pair of C-point singularities based on its vortex order. In the self-healing study, we observe the degree of polarization of the Poincaré-Bessel beam remains unchanged with propagation despite beam obstruction. The Stokes phase study of those beams during the self-healing process reveals the appearance of a new set of C-point singularity pairs at the disturbed section of the beam. Further, with beam propagation, the new set of singularities replaces the existing C-point singularity pairs. It retains the polarization distribution of the beam as the initial after the healing length. Such healing processes of the C-point singularities of the Poincaré-Bessel beam resemble with the mathematical concept of infinity as presented by the Hilbert Hotel paradox.

Finally, using the fractional scalar and vector vortex beams, we have demonstrated the experimental realization of Hilbert's Hotel. We used a very simple experimental architecture to generate the fractional scalar and vector vortex beams using a fixed SPP and continuous wavelength tunable supercontinuum laser. While we have demonstrated the proof of concept using a SPP of order $l = 2$, the current demonstration

reveals that the use of multi-ramp SPP can produce multiple new vortices simultaneously. The generation of such multiple vortices can open the possibility of verifying the complicated transitions to prove the generalized Hilbert's Hotel paradox. The present experimental scheme can also be useful for understanding the behaviour of complex polarization-sensitive optical elements as required in many fields, including designing novel devices, quantum communication, and sensing.

As a direction of future work, we would like to extend our new approach to experimentally measure the polarization coverage of any arbitrary vector beams. It is observed in the literature that the FP beams have smaller scintillation than comparable beams of uniform polarization in the presence of atmospheric turbulence [50, 51]. Therefore, one can study the robustness of the FP beams against atmospheric turbulence by monitoring the polarization distribution and the change in the overall polarization coverage of such beams during free space propagation. Such a study can be helpful in designing the optimum special polarization structured beams for long-distance free-space classical and quantum communication. We have a plan to extend our study for in-field deployment of the FP beam. In the study of self-healing characteristics of the Poincaré Bessel beam, we have blocked the beam and studied the change in the polarization distribution with propagation. As a future study, we would like to use a depolarizer and see the effect of the polarization pattern of the beam with propagation. Since the Poincaré-Bessel beams have self-healing characteristics, we can expect the ordering of the beam polarization pattern from the polarization scrambling due to the depolarizer. Although we have used a fractional vector vortex beam to verify the Hilbert hotel paradox, we would like to use the higher-order vector vortex beam for sensing applications. For example, the measurement of the optical activity of sugar solution. The polarization projection of the vector vortex beam results in a ring-shaped petal pattern with the number of petals twice the order of the vector vortex

beam. However, the introduction of any relative phase between the constituent beams of the vector vortex beam results in a shift in the petal pattern. One can estimate the relative phase between the beams by measuring the shift of the intensity pattern. We would like to use this beam to measure the concentration of sugar solution with high accuracy.

Appendix A

MATLAB code for the measurement of polarization distribution

Matlab code for estimating the polarization distribution and polarization coverage.

```
1  clc
2  clear
3  close all
4
5
6  cd data
7  n=0;
8  % Input parameters
9      crop=300;
10     x=499;
```

```
11     y=789;
12     threshold=0;
13     map='y';
14     %% Reading images
15     Ia = importdata('p0_0001.ascii.csv');
16     Ib = importdata('p45_0001.ascii.csv');
17     Ic = importdata('p90_0001.ascii.csv');
18     Id = importdata('p45q90_0001.ascii.csv');
19
20     % Center selection
21     % figure, imshow(Ia,[]), colormap hot
22     % figure, imshow(Ic,[]), colormap hot
23     % prompt = 'What is the center X value? ';
24     % y=input(prompt);
25     % prompt = 'What is the center Y value? ';
26     % x=input(prompt);
27
28     %% Calculating parameters
29     % Cropping image
30     I1a = Ia(x-crop:x+crop,y-crop:y+crop);
31     I2b = Ib(x-crop:x+crop,y-crop:y+crop);
32     I3c = Ic(x-crop:x+crop,y-crop:y+crop);
33     I4d = Id(x-crop:x+crop,y-crop:y+crop);
34
35     I1=zeros(2*crop+1,2*crop+1);
36     I2=zeros(2*crop+1,2*crop+1);
```

```
37     I3=zeros(2*crop+1,2*crop+1);
38     I4=zeros(2*crop+1,2*crop+1);
39
40     for j = 1:size(I1,1)
41         for i = 1:size(I1,2)
42             if 2*crop ≥ sqrt((crop-i)^2+(crop-j)^2)
43                 I1(i,j) = I1a(i,j);
44                 I2(i,j) = I2b(i,j);
45                 I3(i,j) = I3c(i,j);
46                 I4(i,j) = I4d(i,j);
47             end
48         end
49     end
50     figure('NumberTitle','off','Name', ...
51           'Experiment Intensity');
52     subplot(2,2,1)
53     imshow(I1/max(max(I1+I3)),[]),colormap hot
54     colorbar
55     subplot(2,2,2)
56     imshow(I2/max(max(I1+I3)),[]),colormap hot
57     colorbar
58     subplot(2,2,3)
59     imshow(I3/max(max(I1+I3)),[]),colormap hot
60     colorbar
61     subplot(2,2,4)
62     imshow(I4/max(max(I1+I3)),[]),colormap hot
```

```

62     colorbar
63     %% calculating the Stokes parameters
64     S0 = I1 + I3;
65     S1 = I1 - I3;
66     S2 = I2.*2-(I1+I3);
67     S3 = I4.*2-(I1+I3) ;
68     s0=sqrt((S1.^2)+(S2.^2)+(S3.^2)) ;% normalized ...
        Stokes factor
69
70
71     % Calculation of the normalized stokes parameters
72     NORMs1=zeros(size(s0,1),size(s0,2));
73     NORMs2=zeros(size(s0,1),size(s0,2));
74     NORMs3=zeros(size(s0,1),size(s0,2));
75     P=zeros(size(s0,1),size(s0,2));
76     thr=0.01*threshold*max(max(s0));
77     for j = 1:size(s0,1)
78         for k = 1:size(s0,2)
79             if s0(j,k)> thr
80                 NORMs3(j,k) = ...
                        NORMs3(j,k)+(S3(j,k)/s0(j,k));
81                 NORMs1(j,k) = ...
                        NORMs1(j,k)+(S1(j,k)/s0(j,k));
82                 NORMs2(j,k) = ...
                        NORMs2(j,k)+(S2(j,k)/s0(j,k));
83                 P(j,k) = P(j,k)+(s0(j,k)/S0(j,k));

```

```
84         end
85     end
86 end
87
88
89     %     figure('NumberTitle', 'off', 'Name', ' ...
        Stokes parameters Experiment');
90     %     subplot(2,2,1)
91     %     imshow(NORMs1,[]),colormap winter
92     %     colorbar
93     %     subplot(2,2,2)
94     %     imshow(NORMs2,[]),colormap winter
95     %     colorbar
96     %     subplot(2,2,3)
97     %     imshow(NORMs3,[]),colormap winter
98     %     colorbar
99     %     subplot(2,2,4)
100    %     imshow(s0,[]),colormap winter
101    %     colorbar
102    % CALCULATING STOCKS PHASES
103
104    phi12=zeros(size(s0,1),size(s0,2));
105    phi23=zeros(size(s0,1),size(s0,2));
106    phi31=zeros(size(s0,1),size(s0,2));
107
108    for i=1:size(s0,1)
```

```
109         for j=1:size(s0,2)
110             p1=angle(NORMs1(i,j)+1.i*NORMs2(i,j));
111             p2=angle(NORMs2(i,j)+1.i*NORMs3(i,j));
112             p3=angle(NORMs3(i,j)+1.i*NORMs1(i,j));
113             phi12(i,j)=phi12(i,j)+p1;
114             phi23(i,j)=phi23(i,j)+p2;
115             phi31(i,j)=phi31(i,j)+p3;
116         end
117     end
118
119
120
121     %% Calculationg Polarization ellipse parameter
122     % calculating 'chi' ratio of semi major and ...
123     % minor axis and the sense in which eliipse is ...
124     % rotating
125
126     chi=zeros(size(s0,1),size(s0,2));
127
128     for i=1:size(s0,1)
129         for j=1:size(s0,2)
130             if s0(i,j)>0
131                 q1=0.5.*asin(NORMs3(i,j));
132                 chi(i,j)=chi(i,j)+q1;
133             end
134         end
135     end
```

```
133     end
134
135     % calculating 'phi' the orientation of the ...
136         ellipse i.e. the angle made by major axis ...
137         with the X-axis
138
139     phi=zeros(size(s0,1),size(s0,2));
140
141     for i=1:size(s0,1)
142         for j=1:size(s0,2)
143             q2=.5*angle(NORMs1(i,j)+1.i*NORMs2(i,j));
144             if q2<0
145                 q21=q2+pi;
146             else
147                 q21=q2;
148             end
149             phi(i,j)=phi(i,j)+q21;
150         end
151     end
152
153     % Polarization map
154     if map == 'y'
155         back='i';
156         if back == 'h'
157             figure,imshow(I1,[]), colormap hot
158         elseif back == 'v'
```

```
157         figure,imshow(I3,[], colormap hot
158     elseif back == 'i'
159         colorMap1 = [linspace(0,1,256)', ...
160                     zeros(256,1),zeros(256,1)];
161         figure,imshow(S0,[], colormap(colorMap1)
162     elseif back == 'n'
163         figure,imshow(S0,[], colormap white
164     elseif back == 'p'
165         figure,imshow(P,[], colormap hot
166     elseif back == 'e'
167         figure,imshow(chi,[], colormap summer
168     elseif back == 'o'
169         figure,imshow(phi,[], colormap summer
170     elseif back == 's'
171         figure,imshow(phi12,[], colormap summer
172     end
173
174     % averaging chi & phi
175     nom=33;
176     f=round(log2(size(chi,1)/nom));
177     avgfact = 15;
178     avchi=imresize(chi,1/avgfact,'nearest'); ...
179         %figure,imshow(rszI1,[0 max(max(rszI1))]);
180     avphi=imresize(phi,1/avgfact,'nearest'); ...
181         %figure,imshow(rszI2,[0 max(max(rszI2))]);
182     %
```



```
180      % chosing the parametrs of ellipse; a, b, ...  
        size of ellipse, origin  
181      a = ones(size(s0,1),size(s0,2)); % setting ...  
        the length of semi major axis =1  
182      b=tan(avchi); % calulating the length of ...  
        semi minor axis  
183      soe=avgfact*0.4; %size of ellipse  
184  
185      orign=avgfact/2;  
186  
187      % calculating the pol. ellipses and then ...  
        drawing them on the axes provided above  
188      xp=0; yp=0; x0=0; y0=0; x=0; y=0;  
189      for i=1:size(avchi,1)  
190          for j=1:size(avchi,2)  
191  
192              theta=linspace(0,2*pi,500); ...  
                  %genearating values for theta ...  
                  to draw an  
193              %ellipse using parametric form  
194              xp=a(i,j).*cos(theta).*soe;  
195              yp=b(i,j).*sin(theta).*soe;  
196  
197              x0=((i-1)*avgfact)+orign;  
198              y0=(avgfact*(j-1))+orign; ...  
                  %calculating the X and y coord. ...
```

```
                                of center of an ellipse
199
200
201     x=x0+(xp.*sin(avphi(i,j))-yp*cos(avphi(i,j)));
202     y=y0+(xp.*cos(avphi(i,j))+yp*sin(avphi(i,j)));
203
204     if avchi(i,j)> 0
205         line(y,x,'color','g','linewidth',1.2);
206     end
207
208     if avchi(i,j)< 0
209         line(y,x,'color','w','linewidth',1.2);
210     end
211
212     if avchi(i,j)==0
213         line(y,x,'color','[.2 .2 .2 ...
214             ]','linewidth',1.2,'linewidth',1.2);
215     end
216
217     end
218
219     axis on
220 end
221 %% Calculating final results
222 % Plotting poincare sphere
```

```
223     Vs1 = reshape(NORMs1,[],1);
224     Vs2 = reshape(NORMs2,[],1);
225     Vs3 = reshape(NORMs3,[],1);
226
227     figure('NumberTitle','off','Name','Poincare ...
           sphere Experiment');
228     sphere(100);
229     colormap gray
230     shading flat
231     hold
232     scatter3(Vs1,Vs2,Vs3,'.','b')
233     xlabel('S1')
234     ylabel('S2')
235     zlabel('S3')
236     hold off
237
238     % Calculating Polarization coverage
239     sample1=pi/(31);
240     sample2=2*pi/(31);
241     x =2* reshape(chi,[],1);
242     y =2* reshape(phi,[],1);
243     data = [x,y];
244     counts = ...
           hist3(data,'Ctrs',{-0.5*pi:sample1:0.5*pi ...
           0:sample2:2*pi});
245     out=nnz(counts);
```

```
246     coverage=out*100/(size(counts,1)*size(counts,2));  
247     plotx(n,1)=test;  
248     ploty(n,1)=coverage;
```

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List of Publications

Journals

1. Evolution of C-point singularities and polarization coverage of Poincare-Bessel beam in self-healing process, Subith Kumar, Anupam Pal, Arash Shiri, G. K. Samanta, and Greg Gbur, Scientific Reports 14 , 16647 (2024).
2. Simple experimental realization of optical Hilbert Hotel using scalar and vector fractional vortex beams (Cover page, Featured) , Subith Kumar, Anirban Ghosh, Chahat Kaushik, Arash Shiri, Greg Gbur, Sudhir Sharma, and G. K. Samanta APL Photonics 8, 066105 (2023).
3. Controlling the coverage of full Poincare beams through second-harmonic generation, Subith Kumar, Ravi K. Saripalli, Anirban Ghosh, Wagner T. Buono, Andrew Forbes, and G.K. Samanta, Physical Review Applied 19, 034082 (2023)

Conference proceedings

1. Hilbert hotel realization using Poincare Bessel beam, Subith Kumar, Anupam Pal, Arash Shiri, G. K. Samanta, and Greg Gbur, Frontiers in Optics, JTu5B. 15
2. Frequency-doubling characteristics of non-collinear Poincaré beams, Subith Ku-

mar, Ravi K. Saripalli, Anirban Ghosh, and G.K. Samanta, *Frontiers in Optics*, FM7C. 2

3. Generation of non-collinear Poincaré beam through the spin-orbit interaction in epsilon-near-zero film, Anirban Ghosh, RK Saripalli, K Subith, N Apurv Chaitanya, Varun Sharma, Israel De Leon and GK Samanta, *Frontiers in Optics*, FM7C. 3