$K \to \pi\pi$ Decays from Lattice QCD.

Andrew Lytle (for RBC-UKQCD)
TIFR, Mumbai

PCPV 2013
Mahabaleshwar, India
21.02.13

BROOKHAVEN NATIONAL LABORATORY

RIKEN

Columbia University

University of Southampton

The University of Edinburgh

CP-violation & Kaon Decay.

- CP-violation was discovered in 1964 (Cronin & Fitch), in the kaon system.
- Subsequently observed in the B-meson (2001), and very recently, D-meson (2011) systems.
- The Standard Model predicts this stems from a complex phase in the CKM matrix.

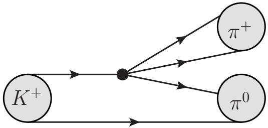
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

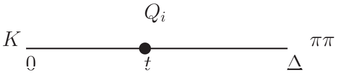
$$\boxed{\text{Does } \delta \in V_{CKM} \to \epsilon, \epsilon' \,??}$$

Does the SM account for the CP-violation in these systems?

Outline

- Phenomenology of Kaon decays.
- $\Delta S = 1$ Effective Hamiltonian.
- Numerical Simulation.
- Structure of the Calculation.
- Theoretical Challenges.
- Results/Status.
- Future.





Phenomenology of Kaon Decay

Kaons are *strange mesons* that decay via the weak interaction.

$$K^+ = \bar{s}\gamma_5 u \qquad K^- = \bar{u}\gamma_5 s$$

$$K^0 = \bar{s}\gamma_5 d \qquad \overline{K}^0 = \bar{d}\gamma_5 s$$

They also *mix* under the weak interaction.

- CP-eigenstates: $|K^0_\pm\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle \mp |\overline{K}^0\rangle\right]$
- Weak eigenstates: $|K_{\mathrm{L,S}}\rangle = |K^0_\mp\rangle + \bar{\epsilon}|K^0_\pm\rangle$

If CP is conserved, then $\bar{\epsilon} = 0$.

$$\boxed{CP \to \bar{\epsilon} = 0}$$

Phenomenology of Kaon Decay

The K_L decays predominantly to 3 pions, while the K_S decays to two pion final states. The following ratios are used to characterize CP-violation:

$$\eta_{+-} = \frac{\langle \pi^+ \pi^- | H_w | K_L \rangle}{\langle \pi^+ \pi^- | H_w | K_S \rangle} \qquad \eta_{00} = \frac{\langle \pi^0 \pi^0 | H_w | K_L \rangle}{\langle \pi^0 \pi^0 | H_w | K_S \rangle}$$

$$\eta_{+-} = \epsilon + \epsilon' \qquad \eta_{00} = \epsilon - 2\epsilon'$$

The parameter ϵ' is a unique measure of direct ($\Delta S = 1$) CP-violation.

$$\eta_{+-} = \epsilon + \epsilon'$$
$$\eta_{00} = \epsilon - 2\epsilon'$$

Phenomenology of Kaon Decay

We decompose the two-pion final state in terms of isospin:

$$\frac{1}{\sqrt{2}}\langle(\pi\pi)_{I=2}|H_w|K^0\rangle = A_2\, e^{i\delta_2}$$
$$\frac{1}{\sqrt{2}}\langle(\pi\pi)_{I=0}|H_w|K^0\rangle = A_0\, e^{i\delta_0}$$

In terms of these quantities,

$$\mathrm{Re}(\epsilon'/\epsilon) = \frac{w}{\sqrt{2}\,|\epsilon|}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]$$

Experimental value: $1.65(26) \times 10^{-3}$

Challenges

A quantitative calculation requires:

- Control over several scales from EW to QCD.
- Calculating in a strongly coupled field theory (QCD).

Correspondingly the two main ingredients of our calculation are:

1. An effective theory of the weak interaction valid at scales $\sim \Lambda_{\mathrm{QCD}}$.
2. A non-perturbative definition of QCD amenable to numerical evaluation.

The Effective Hamiltonian

The Effective Hamiltonian, $H_w^{\Delta S=1}$

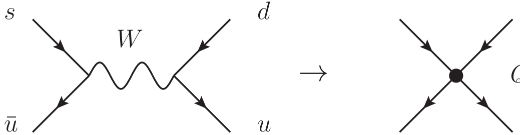
- It is difficult for a lattice simulation to span many scales directly; scales $\sim (L/a)^4$ (or worse).
- Must bridge orders of magnitude between the electroweak and strong scales.
- Using the OPE, effects of the weak interaction are parameterized in the effective theory.

The Effective Hamiltonian, $H_w^{\Delta S=1}$

Kaons decay via the $\Delta S = 1$ weak interaction:

$$H_w^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} C^i(\mu) Q_i(\mu)$$

The Q_i are four-quark operators generated from integrating the theory down to a scale $\mu \sim 2$ GeV, e.g.



$$Q_1 = (\bar{d}\gamma_L^{\mu} u)\,(\bar{u}\gamma_{\mu L} s)$$

$\Delta S = 1$ Operators.

current-current

$$Q_1 = (\bar{s}\,\gamma^\mu_{\rm L}\,u)(\bar{u}\,\gamma^\mu_{\rm L}\,d)_{\rm mx}$$
$$Q_2 = (\bar{s}\,\gamma^\mu_{\rm L}\,u)(\bar{u}\,\gamma^\mu_{\rm L}\,d)$$

QCD penguins

$$Q_3 = (\bar{s}\,\gamma^\mu_{\rm L}\,d)\sum_q(\bar{q}\,\gamma^\mu_{\rm L}\,q)$$
$$Q_4 = (\bar{s}\,\gamma^\mu_{\rm L}\,d)\sum_q(\bar{q}\,\gamma^\mu_{\rm L}\,q)_{\rm mx}$$
$$Q_5 = (\bar{s}\,\gamma^\mu_{\rm L}\,d)\sum_q(\bar{q}\,\gamma^\mu_{\rm R}\,q)$$
$$Q_6 = (\bar{s}\,\gamma^\mu_{\rm L}\,d)\sum_q(\bar{q}\,\gamma^\mu_{\rm R}\,q)_{\rm mx}$$

electroweak penguins

$$Q_7 = \frac{3}{2}(\bar{s}\,\gamma^\mu_{\rm L}\,d)\sum_q e_q(\bar{q}\,\gamma^\mu_{\rm R}\,q)$$
$$Q_8 = \frac{3}{2}(\bar{s}\,\gamma^\mu_{\rm L}\,d)\sum_q e_q(\bar{q}\,\gamma^\mu_{\rm R}\,q)_{\rm mx}$$
$$Q_9 = \frac{3}{2}(\bar{s}\,\gamma^\mu_{\rm L}\,d)\sum_q e_q(\bar{q}\,\gamma^\mu_{\rm L}\,q)$$
$$Q_{10} = \frac{3}{2}(\bar{s}\,\gamma^\mu_{\rm L}\,d)\sum_q e_q(\bar{q}\,\gamma^\mu_{\rm L}\,q)_{\rm mx}$$

Lattice Simulation

Simulating QCD

Regulate QCD using a (Euclidean) spacetime lattice.
Integrate out fermionic degrees of freedom.

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-\int (\mathcal{L}_{\mathrm{YM}} + \bar{\psi} D \psi)} \\ &= \int \mathcal{D}U \, (\det D) \, e^{-\int \mathcal{L}_{\mathrm{YM}}} \end{aligned}$$

Generate configurations using Monte Carlo techniques.

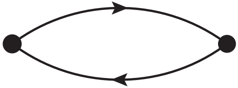
Simulating QCD

Calculate quark propagators on gauge backgrounds.

$$D^{-1} =$$

Use Wick's theorem to evaluate correlation functions.

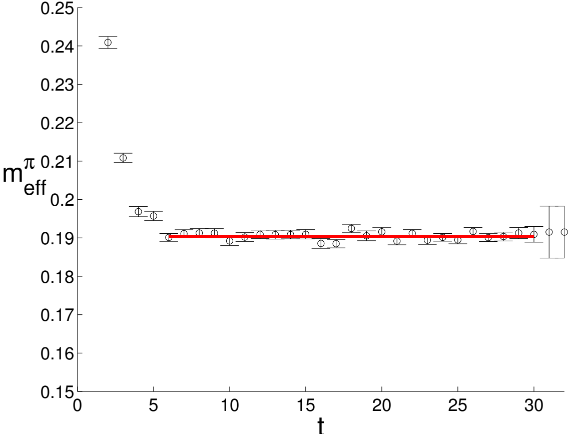
$$\langle \pi \ \pi^\dagger \rangle = \langle \quad \rangle$$



Example - m_π

Energies and matrix elements can be determined by fitting exponentials.

$$\langle \pi(t)\ \pi^\dagger(0) \rangle \sim |\langle 0|\pi|\pi\rangle|^2 e^{-m_\pi t}$$

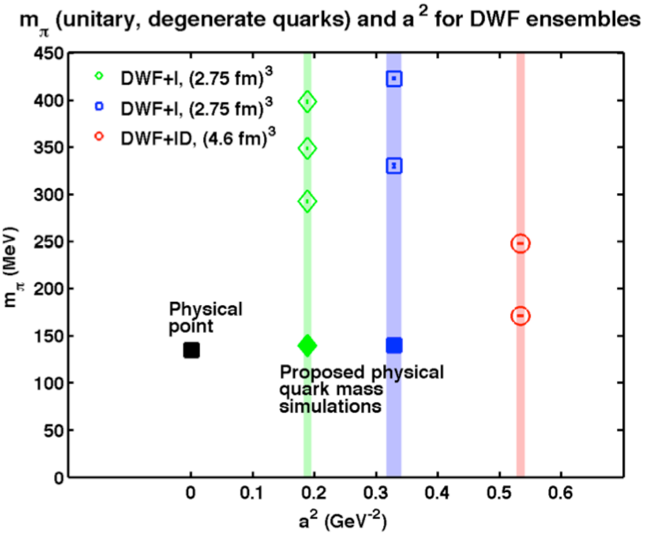


RBC-UKQCD ensembles

Three datasets of $N_f = 2 + 1$ DWF with Iwasaki Gauge Action.

1. $a \simeq 0.114$ fm, $16^3 \times 32 \times 16$ and $24^3 \times 64 \times 16$ `arXiv:0804.0473`
   - Four light-quark masses $m_\pi \simeq 330, 415, 555, 670$ MeV.
   - Lightest partially quenched pion $m_\pi \simeq 240$ MeV.
2. $a \simeq 0.086$ fm, $32^3 \times 64 \times 16$ `arXiv:1011.0892`
   - Three light-quark masses $m_\pi \simeq 290, 343$, and 390 MeV.
   - Lightest partially quenched pion $m_\pi \simeq 240$ MeV.
3. $a \simeq 0.14$ fm, $32^3 \times 64 \times 32$ `arXiv:1208.4412`
   - Two light-quark masses $m_\pi \simeq 170$ and 250 MeV.
   - Lightest partially quenched pion $m_\pi \simeq 142$ MeV.

RBC-UKQCD ensembles



$$K \to \pi\pi$$

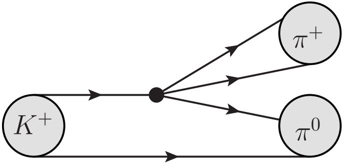
$K \to \pi\pi$ "Master Formula"

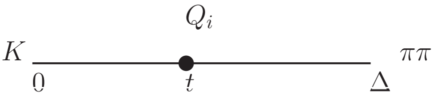
$$A_{2/0} = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} \sum_{j=1}^{7} \left[\left(z_i(\mu) + \tau y_i(\mu) \right) Z_{ij}^{\mathrm{lat} \to \overline{\mathrm{MS}}} M_j^{\frac{3}{2}/\frac{1}{2}, \mathrm{lat}} \right]$$

We want to calculate

$$M_i^{\frac{3}{2}/\frac{1}{2}, \mathrm{lat}} \equiv \langle (\pi\pi)_{I=2/0} | Q_i^{\mathrm{lat}} | K \rangle \,.$$

Correlation functions





We construct correlation functions $C_{I,i}(\Delta, t)$.

Fitting correlation functions

We fit the correlation functions $C_{I,i}(\Delta, t)$,

$$C_{2/0,i}(\Delta, t) \approx M_i^{\frac{3}{2}/\frac{1}{2}} N_{\pi\pi} N_K e^{-E_{\pi\pi}\Delta} e^{-(m_K - E_{\pi\pi})t}$$

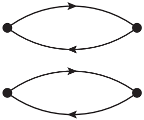
for $t \ll 0 \ll \Delta$, using a one parameter exponential fit to determine the matrix elements $M_i^{\frac{3}{2}/\frac{1}{2}}$. Requires:

- $E_{\pi\pi}$, $N_{\pi\pi}$
- m_K, N_K

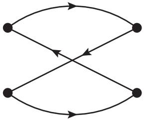
determined from two-pion and kaon correlators.

$E_{\pi\pi}$

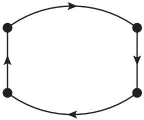
Two-pion correlation functions can be expressed in terms of four contractions:



D



C

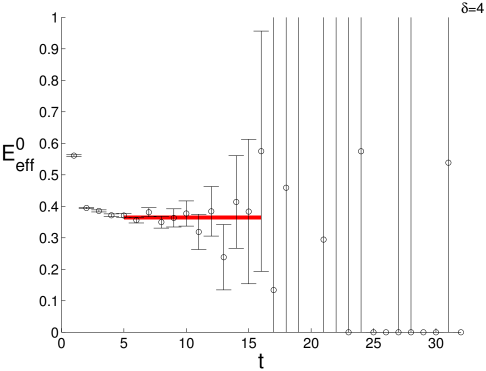


R

V

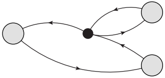
$E_{\pi\pi}$

$I = 0$ combination: 2D + C - 6R +3V

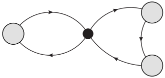


$K \to \pi\pi$

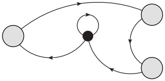
The $K \to \pi\pi$ correlators have four contraction topologies.



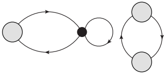
$type1$



$type2$



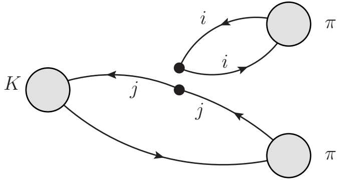
$type3$



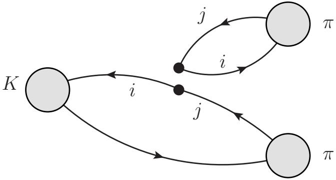
$type4$

$K \to \pi\pi$

48 separate contractions (① – ㊽) appear amongst the set of matrix elements $\langle(\pi\pi)_{I=2/0}|Q_i^{\mathrm{lat}}|K\rangle$. e.g.



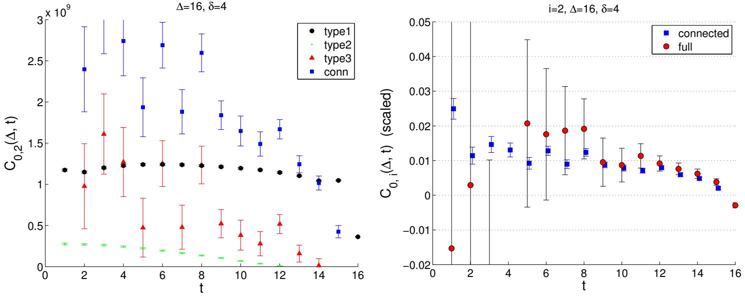
①



②

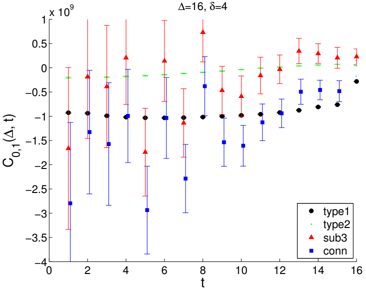
$K \to \pi\pi$

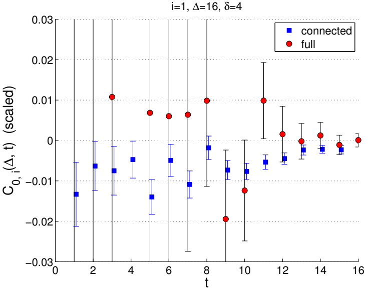
Contributions of the four topologies. Q_2



$K \to \pi\pi$

Contributions of the four topologies. Q_1





Results

Results for A_2

- Realistic kinematics.
- 146 configurations.
- 140 MeV partially quenched pions (170 MeV unquenched).
- Single lattice spacing ($a \simeq .14$ fm).

	m_{K^+}	m_{π^+}	$E_{\pi\pi}$	$m_K - E_{\pi\pi}$
Simulated	511.3(3.9)	142.9(1.1)	492.6(5.5)	18.7(4.8)
Physical	493.677(0.016)	139.57018(0.00035)	m_{K^+}	0

Results for A_2

$$\mathrm{Re}\,A_2 = (1.381 \pm 0.046_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}}) \times 10^{-8}\ \mathrm{GeV}$$

$$(\mathrm{Re}\,A_2)_{\mathrm{exp}} = (1.479 \pm 0.004) \times 10^{-8}\ \mathrm{GeV}$$

$$\mathrm{Im}\,A_2 = -(6.54 \pm 0.46_{\mathrm{stat}} \pm 1.20_{\mathrm{syst}}) \times 10^{-13}\ \mathrm{GeV}$$

Estimated 54 million BG/P processor hours.

Results for A_0

The A_0 calculation is significantly more challenging.

- All 10 operators contribute.
- All 48 diagrams contribute.
- *type*3 and *type*4 diagrams require subtractions.
- *type*4 are disconnected and therefore *noisy*.

Nevertheless significant progress has been made.

Results for A_0

Calculations tuned to be at threshold, $m_K \approx 2m_\pi$.

422 MeV pions:

- $\mathrm{Re}\, A_0 = 3.80(82) \times 10^{-7}$ GeV
- $\mathrm{Im}\, A_0 = -2.5(2.2) \times 10^{-11}$ GeV

330 MeV pions:

- $\mathrm{Re}\, A_0 = 3.21(45) \times 10^{-7}$ GeV
- $\mathrm{Im}\, A_0 = -3.3(1.5) \times 10^{-11}$ GeV

For comparison, $(\mathrm{Re}\, A_0)_{\mathrm{exp}} = 3.320(2) \times 10^{-7}$ GeV.

$\Delta I = 1/2$

For our heavy threshold calculations, we have looked at the quantity

$$w^{-1} = \frac{\mathrm{Re}\, A_0}{\mathrm{Re}\, A_2} \approx 22.5$$

of "$\Delta I = 1/2$ rule" fame.

We find enhancement factors of 9.1(2.1) and 12.0(1.7) for the 422 MeV and 330 MeV pions, respectively.

Future

- A_2 calculation needs to be repeated on a second lattice spacing (this ensemble is currently being generated).
- A_0 will benefit from improved algorithmic techniques (A2A propagators). Dedicated G-parity ensembles are being generated specifically for this calculation.
- Isospin effects, ...

Recap

Realistic $K \to \pi\pi$ decays are finally achievable!

- Full calculation of A_2 with realistic masses and kinematics (at one lattice spacing).
- $\mathrm{Re}\, A_2$ is consistent w/ experiment.
- A_0 significantly more challenging but "proof of principle" calculations look promising.
- We've seen $\Delta I = 1/2$ enhancements of 9 and 12 as the pion mass is lowered.

Thank you!