Introduction to Particle Physics

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Lecture 5

Weak Interactions

Pauli's neutrino hypothesis

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Zürich

Zürich, 4 December 1930

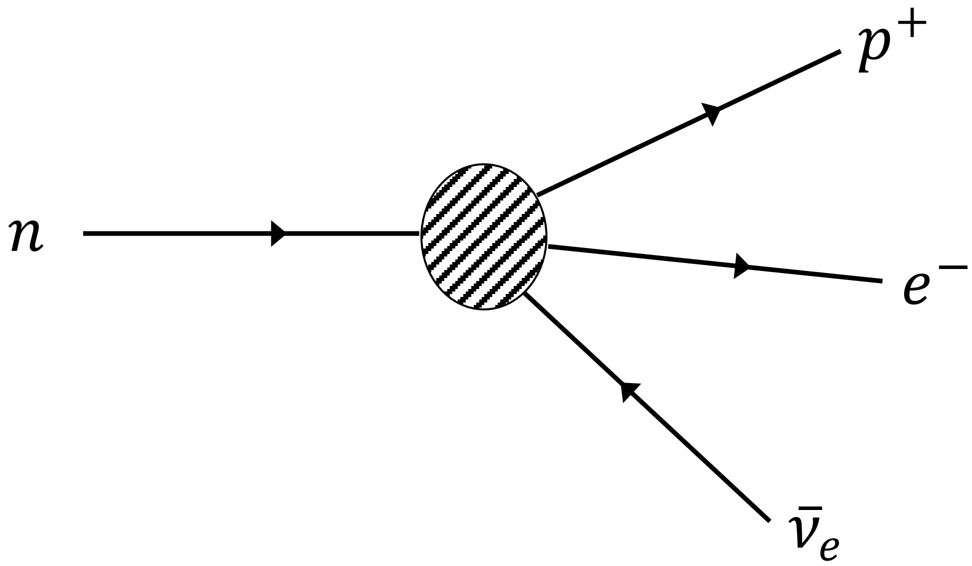
Dear Radioactive Ladies and Gentlemen,
As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the 'wrong' statistics of the N and ^{6}Li nuclei and the continuous β-spectrum, I have hit upon a desperate remedy to save the 'exchange theorem' of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have the spin $\frac{1}{2}$ and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses.—The continuous β-spectrum would then become understandable by the assumption that in β-decay, a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant. . . . . .

So, dear Radioactives, examine and judge it.—Unfortunately I cannot appear in Tübingen personally, since I am indispensable here in Zürich because of a ball on the night of 6/7 December.—With my best regards to you, and also Mr Back, your humble servant,

W Pauli

Fermi's theory of beta decay

$${}^{1}_{0}n \;\rightarrow\; {}^{1}_{1}p + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}_e$$



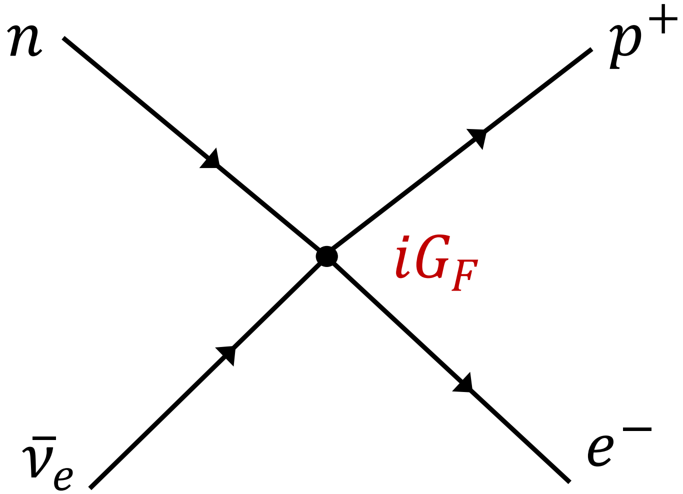
The decay must take place through weak interactions ($\tau = 887$ s).

Can we write down an interaction vertex?

First attempted by Fermi (1934)

Denote the Dirac fields: $\Psi_n = n, \;\; \Psi_p = p, \;\; \Psi_e = e$ and $\Psi_{\overline{\nu}_e} = \bar{\nu}_e$

<u>Fermi's first attempt</u>: try a four-fermion vertex



Weak interaction Hamiltonian:

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}} \, \overline{p} n \, \bar{e} \nu_e$$

dimension of G_F is M^{-2} : Fermi coupling constant

Simplest possible form of a four-fermion coupling

With this interaction, the probability for the transition, in the rest-frame of the neutron, comes out to be

$$|\mathcal{M}|^2 \approx 4G_F^2 M_n M_p E_e^2 (1 - \cos\theta_{e\bar{\nu}})$$

i.e. the electron and the antineutrino should tend to come out back-to-back...

Actual experiment showed that, instead, the electron and the antineutrino tended to come out *in the same direction*!

More as if we have $|\mathcal{M}|^2 \propto (1 + \cos\theta_{e\bar{\nu}})$

Fermi's second attempt: try a vertex modelled on e.m. interactions,

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}} \bar{p}\gamma^\mu n \;\; \bar{e}\gamma_\mu \nu_e = \frac{G_F}{\sqrt{2}} J_{\text{had}}^\mu \, J_\mu^{\text{lep}}$$

Current-current form of the weak interaction

With this interaction, the probability for the transition, in the rest-frame of the neutron, comes out to be

$$|\mathcal{M}|^2 \approx 8G_F^2 M_n M_p E_e^2 (1 + \cos\theta_{e\bar{\nu}})$$

This fits the experimental data much better...

Total decay width (rough estimate):

$$\Gamma_\beta \approx \frac{G_F^2\,\Delta^5}{80\,\pi^3} \approx \frac{1}{887\ s} \qquad \text{where} \quad \Delta = M_n - M_p$$

From this we can estimate

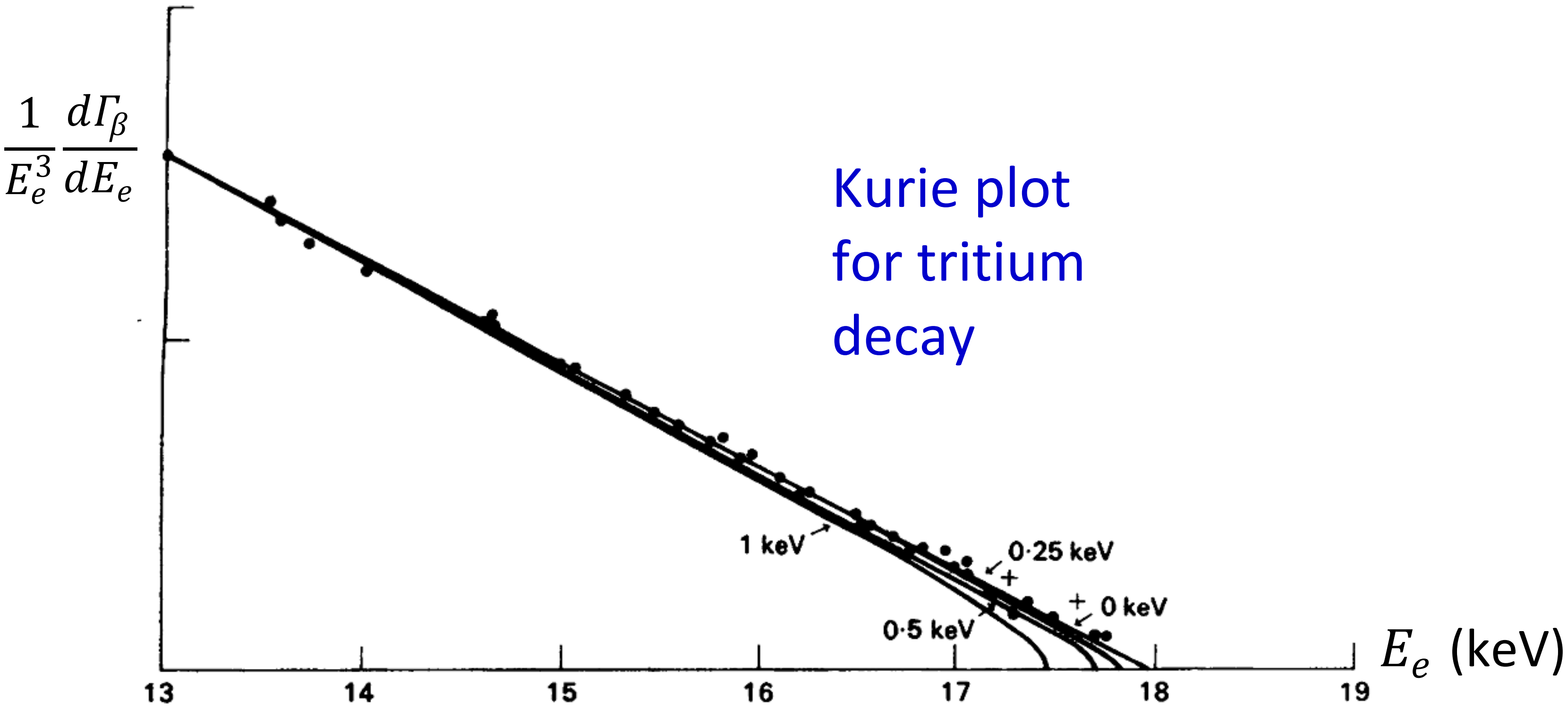
$$G_F \approx 1.8 \times\ 10^{-5}\ \ \mathrm{GeV}^{-2}$$

Given the crudeness of the approximation, this is not a bad estimate...

Current value: $G_F = 1.166 \times\ 10^{-5}\ \ \mathrm{GeV}^{-2}$

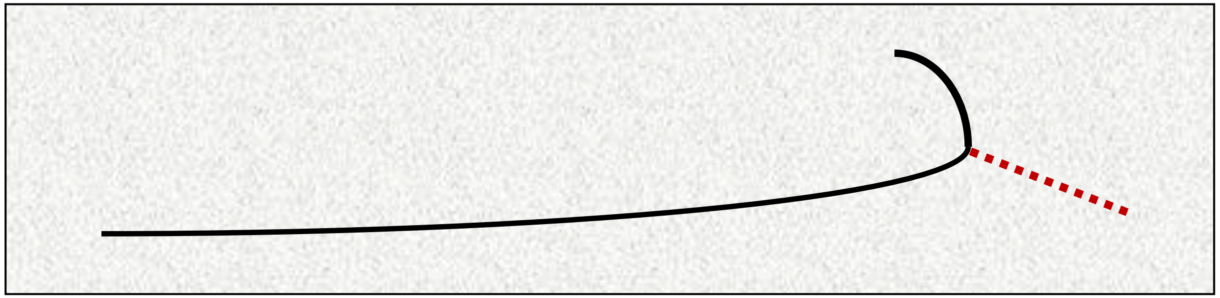
More important:

$$\frac{d\Gamma_\beta}{dE_e} \approx \frac{G_F^2}{2\pi^3} E_e^3 \, [\Delta - E_e] \quad \Rightarrow \quad \frac{1}{E_e^3}\frac{d\Gamma_\beta}{dE_e} \approx \frac{G_F^2}{2\pi^3} (\Delta - E_e)$$



Fermi's theory is spectacularly successful in explaining beta energy spectrum

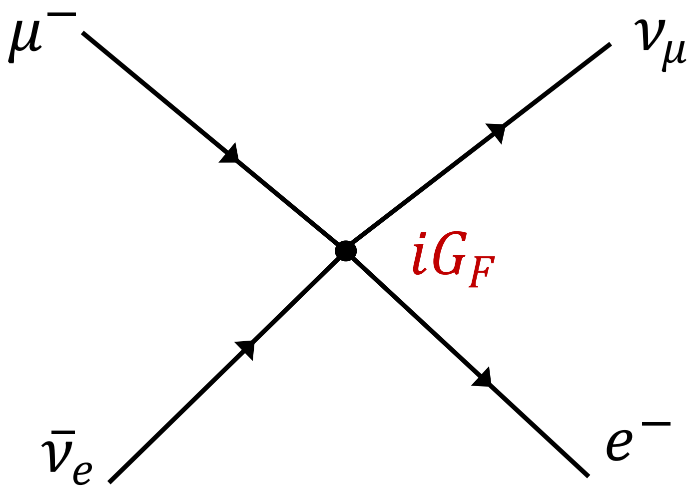
In 1937, the muon was discovered... it decays to electron...



Decay must be through weak interactions (tracks are seen)...

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

Fermi's guess: universality of weak interactions



Use this to calculate the muon lifetime:

$$\tau_\mu \approx \frac{192\,\pi^3}{G_F^2\,M_\mu^5} \approx 2.25 \times 10^{-6}\ \mathrm{s}$$

Spectacular agreement with the experimental value $2.197 \times 10^{-6}\ \mathrm{s}$

Vindicates Fermi's hypothesis about universality of weak interactions... today we have many more proofs...

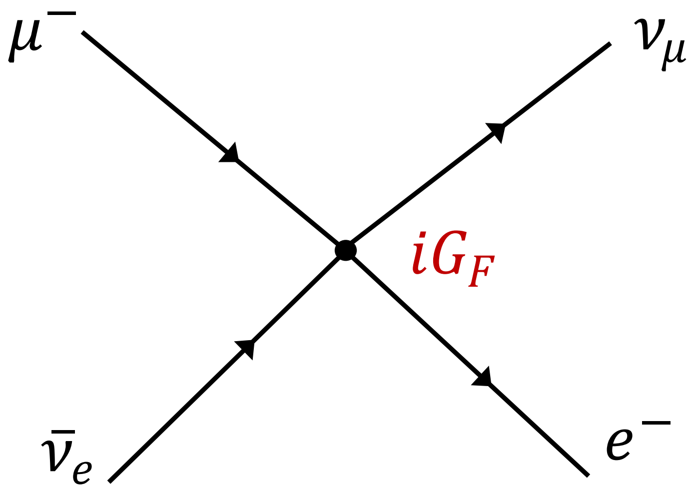
Interestingly, Fermi could have written several forms of the interaction, e.g.

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}}\,\overline{p}\gamma^\mu\gamma_5 n\ \bar{e}\gamma_\mu\gamma_5\nu_e \quad \text{or} \quad \mathcal{H}_I = \frac{G_F}{\sqrt{2}}\,\overline{p}\sigma^{\mu\nu} n\ \bar{e}\sigma_{\mu\nu}\nu_e$$

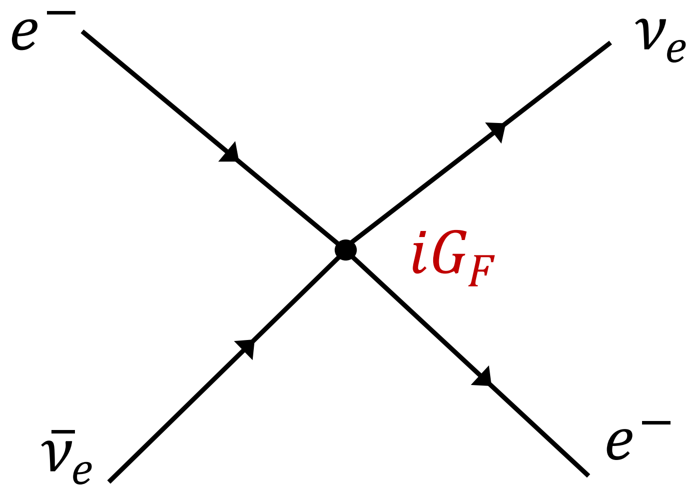
The choice of the vector-vector form turned out to be a stroke of genius, for that is exactly what we predict in the gauge theory of weak interactions – which is what the Fermi theory ultimately leads to...

Weak scattering processes: the unitarity problem

If we have a vertex



as Fermi postulated, then, by universality, we should also have



and it should be possible to have a scattering process

$$e^- + \nu_e \to e^- + \nu_e$$

Cross-section:

$$\sigma \approx \frac{G_F^2}{\pi} s \left(1 - \frac{M_e^2}{s}\right)$$

where $s = (p_e + p_{\nu_e})^2 = E_{\text{cm}}^2$.

Clearly, as $s \uparrow$, $\sigma \uparrow$ ...

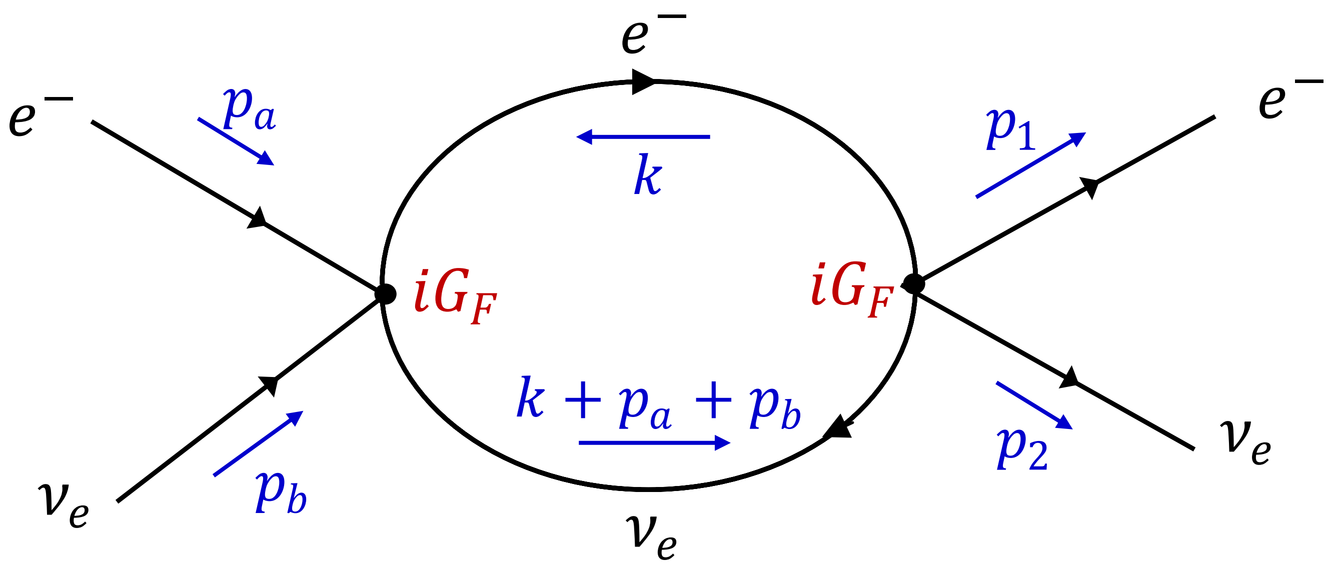
unitarity violation

Perhaps this arises because we took only the LO diagram... ?

... inclusion of higher orders may soften the growth with energy...

...but this leads to a new problem: renormalisability

Consider the simplest one-loop contribution to $e^-\nu_e \to e^-\nu_e$:



The effective coupling due to this would be

$$\frac{iG_F}{\sqrt{2}} \to \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{2}\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{\not{k} - M_e}\,\frac{1}{\not{k} + \not{p}_a + \not{p}_b}$$

Since k is integrated over all values, the dominant contribution will come from $k \to \infty$, i.e.

$$\frac{iG_F}{\sqrt{2}} \to \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{2}\int_0^\infty \frac{2\pi^2 k^3 dk}{(2\pi)^4}\,\frac{1}{\not{k}\not{k}}$$

$$= \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{2}\int_0^\infty \frac{2\pi^2 k^3 dk}{(2\pi)^4}\,\frac{1}{k^2}$$

$$= \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{16\pi^2}\int_0^\infty k\,dk$$

This extra contribution is quadratically divergent, i.e. if we put a momentum cutoff $k \leq \Lambda$ then,

$$\frac{iG_F}{\sqrt{2}} \to \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{16\pi^2}\int_0^\Lambda k\,dk$$

$$= \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{32\pi^2}\Lambda^2$$

If the NLO contribution $\gg$ LO contribution, perturbation theory fails...

Such problems arise in QED as well for e, but there the divergences are logarithmic, i.e. proportional to $\log \Lambda$. Moreover, in every order (NLO, NNLO, NNNLO, ...) we always get a similar logarithmic divergence.

These can be summed up, and the result absorbed into the definition of e -- this process is called renormalisation

In the Fermi theory, however, higher and higher powers of Λ^2 keep coming with higher and higher orders, and there is no scope for renormalisation...

Does this mean that the Fermi theory is wrong?

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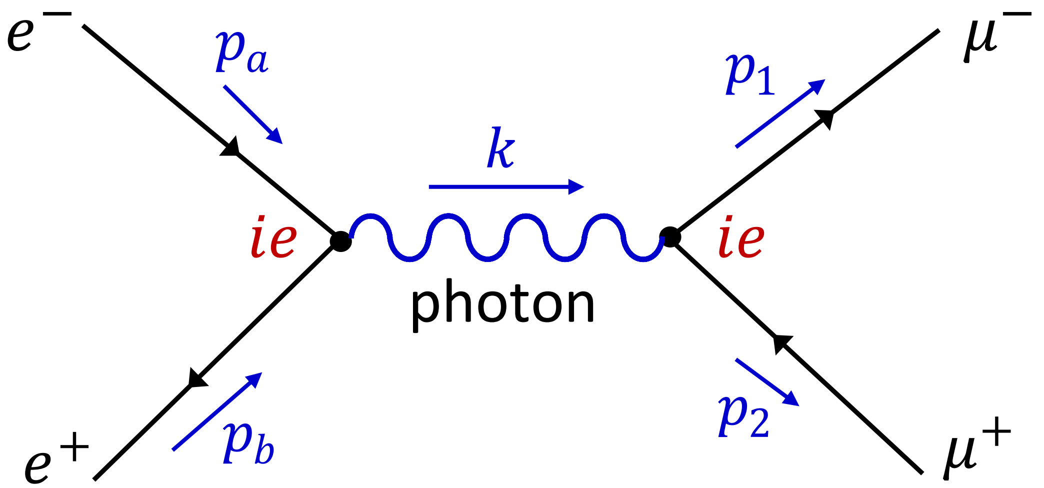
Correspondence Principle: every new theory should reduce to the old theory in the range of parameters where that theory was successful

Fermi theory must be a low-energy effective theory...

Intermediate Vector Bosons (IVB):

Schwinger (1953): if renormalisation is possible in QED, can we make it possible in weak interactions by copying the same form?

Consider the following process in QED: $e^- + e^+ \to \mu^- + \mu^+$



Taking Fermi's idea a step further...

$$i\mathcal{M} = \bar{v}(p_b)\, ie\gamma^\mu u(p_a)\, \frac{-ig_{\mu\nu}}{k^2}\, \bar{u}(p_1)\, ie\gamma^\nu v(p_2)$$

$$= \frac{ie^2}{k^2} \bar{v}(p_b)\, \gamma^\mu u(p_a)\ \bar{u}(p_1)\, \gamma_\mu v(p_2)$$

$$\frac{G_F}{\sqrt{2}} \to \frac{e^2}{k^2}$$

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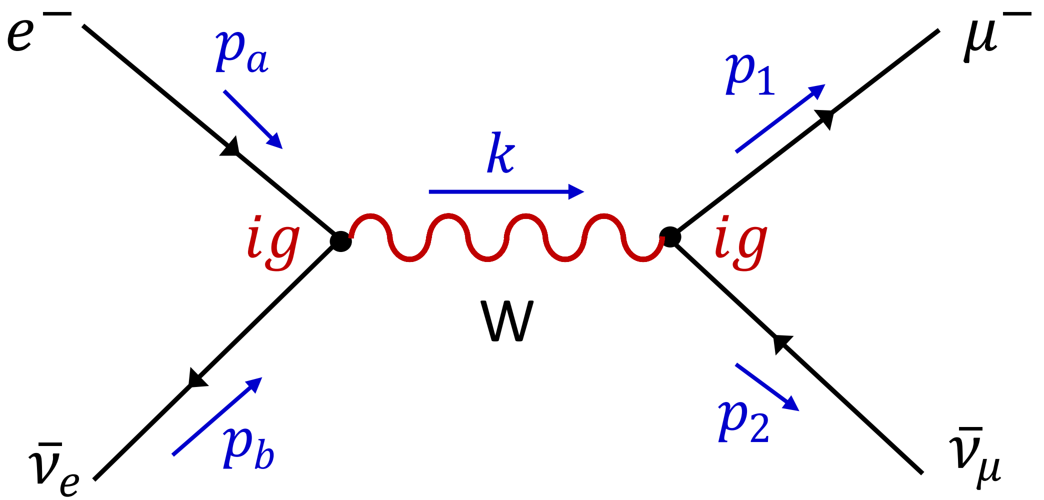

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$$i\mathcal{M} = \bar{v}(p_b)\, ig\gamma^\mu u(p_a)\, \frac{-ig_{\mu\nu}}{k^2}\, \bar{u}(p_1)\, ig\gamma^\nu v(p_2)$$

$$= \frac{ig^2}{k^2} \bar{v}(p_b)\, \gamma^\mu u(p_a)\; \bar{u}(p_1)\, \gamma_\mu v(p_2)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{k^2}$$

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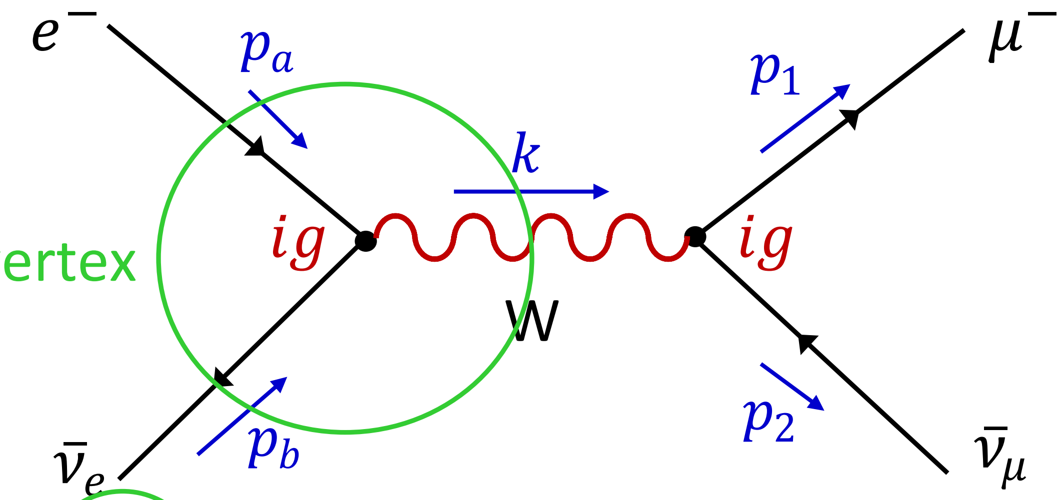
W boson vertex



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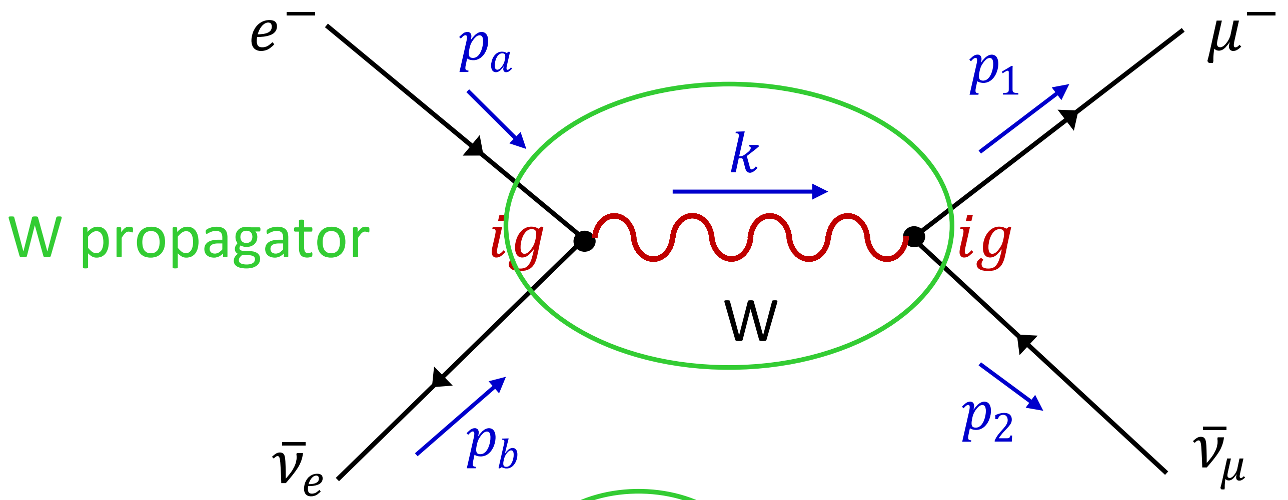

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$$= \frac{ig^2}{k^2} \bar{v}(p_b)\, \gamma^\mu u(p_a)\;\; \bar{u}(p_1)\, \gamma_\mu v(p_2)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{k^2}$$

<u>Objection:</u> The Fermi coupling constant does not show significant variation with energy as $k^2 \to 0$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{k^2}$$

<u>Schwinger's solution</u>: make the W boson massive W:

$$\frac{-ig_{\mu\nu}}{k^2} \to \frac{-ig_{\mu\nu} + k_\mu k_\nu / M_W^2}{k^2 - M_W^2}$$

$$i\mathcal{M} = \bar{v}(p_b)\, ig\gamma^\mu u(p_a)\, \frac{-ig_{\mu\nu} + k_\mu k_\nu / M_W^2}{k^2 - M_W^2}\, \bar{u}(p_1)\, ig\gamma^\nu v(p_2)$$

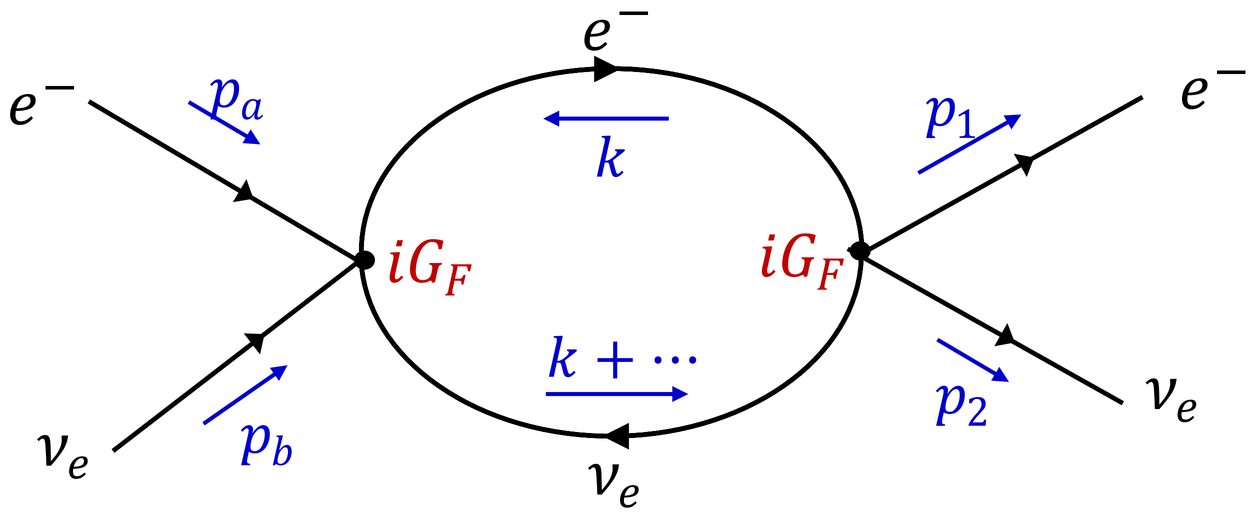
$$= \bar{v}(p_b)\, ig\gamma^\mu u(p_a)\, \frac{-ig_{\mu\nu}}{k^2 - M_W^2}\, \bar{u}(p_1)\, ig\gamma^\nu v(p_2)$$

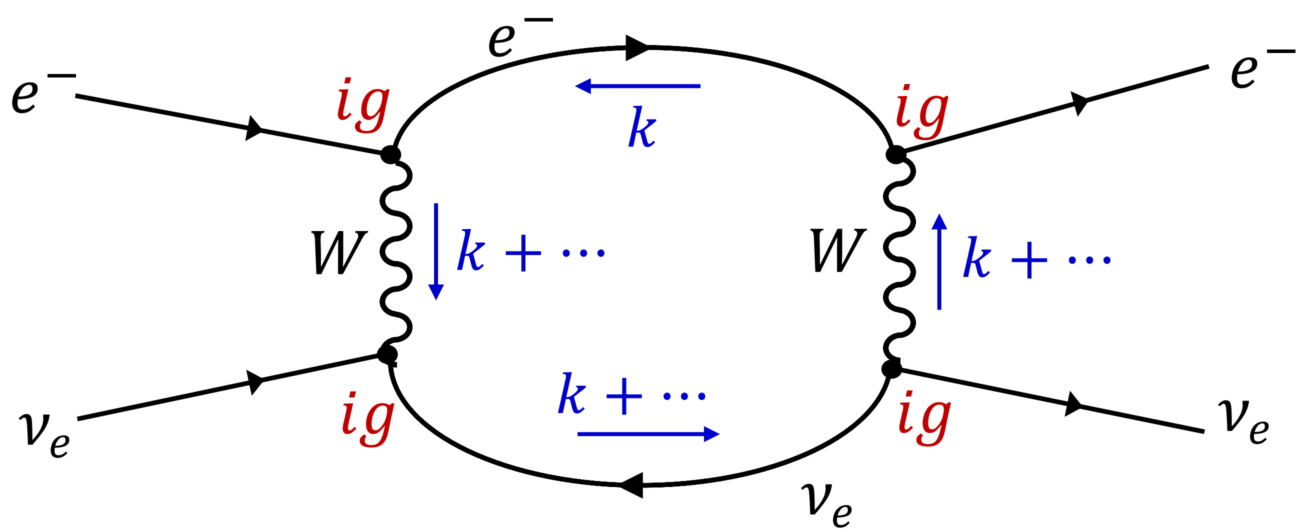
$$= \frac{ig^2}{k^2 - M_W^2} \bar{v}(p_b)\, \gamma^\mu u(p_a)\ \bar{u}(p_1)\, \gamma_\mu v(p_2)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{k^2 - M_W^2}$$

In the low energy limit, $k^2 \to 0$ we get: $\frac{G_F}{\sqrt{2}} = -\frac{g^2}{M_W^2}$ constant!!

Q. How does this help?





Rewrite the loop integral...

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{k}} \frac{1}{\not{k}+\cdots} \rightarrow \int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{k}} \frac{1}{k^2 - M_W^2} \frac{1}{\not{k}+\cdots} \frac{1}{k^2 - M_W^2}$$

$$\propto \int k^3 dk \frac{1}{k^2} \frac{1}{k^4}$$

$$\propto \int \frac{dk}{k^3} \qquad \text{finite!}$$

As it would be in QED...

Rewrite the loop integral...

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{k}} \frac{1}{\not{k} + \cdots} \rightarrow \int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{k}} \frac{1}{k^2 - M_W^2} \frac{1}{\not{k} + \cdots} \frac{1}{k^2 - M_W^2}$$

$$\propto \int k^3 dk \frac{1}{k^2} \frac{1}{k^4}$$

$$\propto \int \frac{dk}{k^3} \qquad \text{finite!}$$

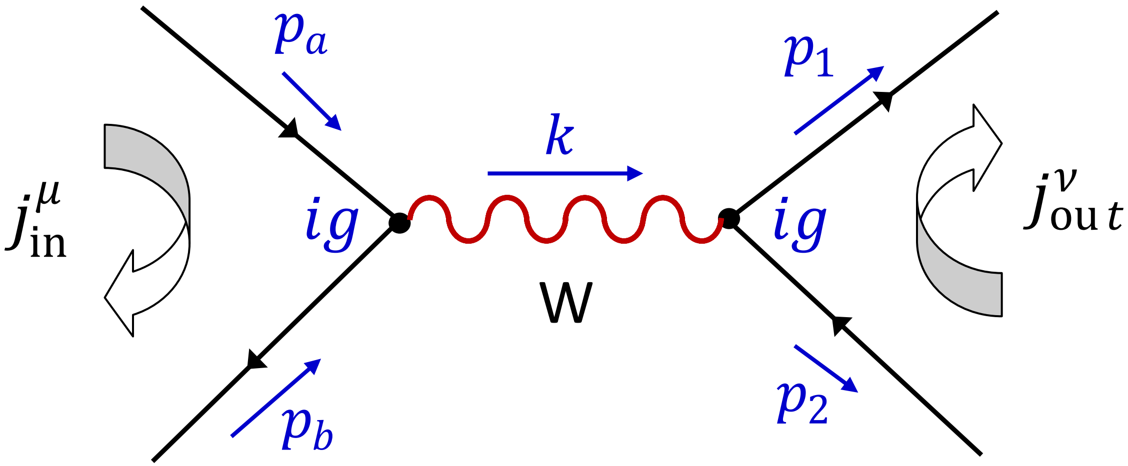
As it would be in QED...

But we have cheated... $\frac{-ig_{\mu\nu}}{k^2} \rightarrow \frac{-ig_{\mu\nu} + k_\mu k_\nu / M_W^2}{k^2 - M_W^2}$

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{k}} \frac{k_\mu k_\nu}{k^2 - M_W^2} \frac{1}{\not{k} + \cdots} \frac{k_\mu k_\nu}{k^2 - M_W^2} \propto \int k^3 dk \frac{1}{k^2} \frac{k^4}{k^4} \propto \int k dk$$

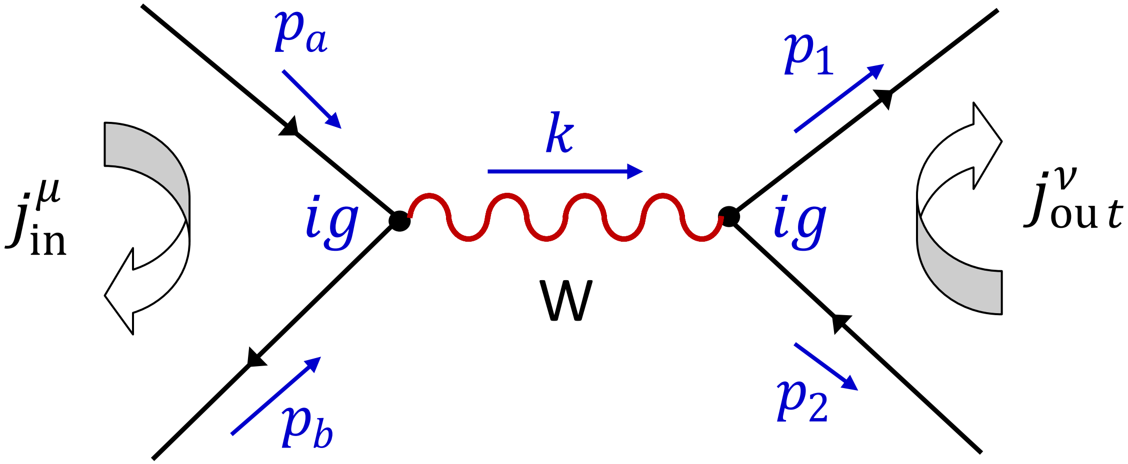
Only way to make IVB work is to get rid of the $k_\mu k_\nu / M_W^2$ term...

What does the propagator couple to?



$$\mathcal{M} \propto j^{\mu}_{\text{in}} \; ig \; \frac{-ig_{\mu\nu} + k_{\mu}k_{\nu}/M_W^2}{k^2 - M_W^2} \; ig \; j^{\nu}_{\text{out}}$$

What does the propagator couple to?



$$\mathcal{M} \propto j_{\text{in}}^{\mu} \, ig \, \frac{-ig_{\mu\nu} + k_\mu k_\nu / M_W^2}{k^2 - M_W^2} \, ig \, j_{\text{out}}^{\nu}$$

The offending term will go away if $j_{\text{in}}^{\mu} k_\mu = 0$ and/or $k_\nu j_{\text{out}}^{\nu} = 0$

To have conserved currents, there must be a gauge symmetry...

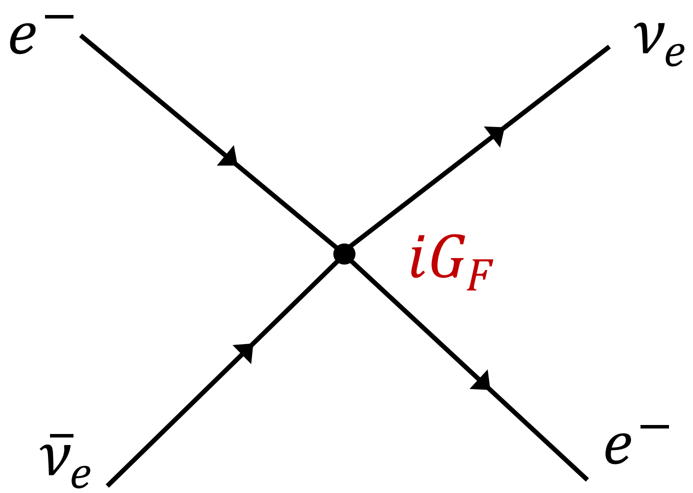
But this cannot be a U(1) gauge symmetry, like QED

Why not? Because the W boson is charged,
i.e. there are two W bosons

$$W_\mu^+ = \frac{1}{\sqrt{2}}(W_1^+ + iW_2^+)$$
$$W_\mu^- = \frac{1}{\sqrt{2}}(W_1^+ - iW_2^+)$$

i.e. the group of gauge symmetries must have at least two generators

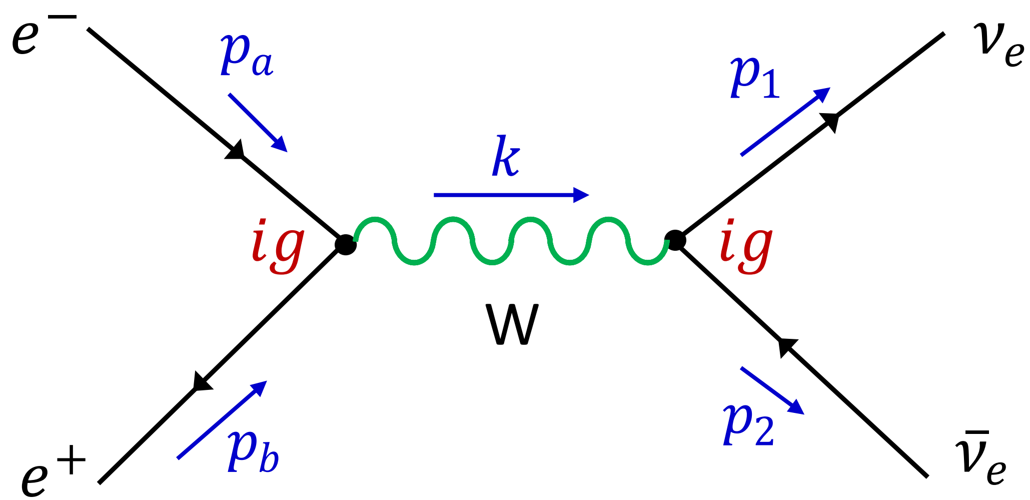
In fact, if we have a four-fermion theory with the vertex



there is nothing, in principle, to prevent a process like

$$e^- + e^+ \to \nu_e + \bar{\nu}_e$$

How will this look in the IVB theory?



So, perhaps we have a neutral W boson also – a W_μ^0

This W_μ^0 cannot be the photon because it couples to neutrinos...

i.e. the group of gauge symmetries must have three generators

after U(1), the next unitary group is SU(2), which has 3 generators...

Parity violation : the $\theta - \tau$ puzzle

Consider the Fermi form of the current-current interaction:

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}}(\overline{p}\gamma^\mu n)(\bar{e}\gamma_\mu \nu_e)$$

Under parity:

$$(\overline{p}\gamma^\mu n) \rightarrow -(\overline{p}\gamma^\mu n)$$
$$(\bar{e}\gamma_\mu \nu_e) \rightarrow -(\bar{e}\gamma_\mu \nu_e)$$

i.e. parity is conserved in the Fermi theory

Before the 1950s, it was thought that parity is as sacred as energy, momentum and angular momentum...

But it was known that some particle are <u>pseudoscalars</u>,

e.g. pions and Kaons have intrinsic parity $P = -1$

This led to the famous $\theta - \tau$ puzzle

Through the early 1950s, cosmic ray experiments showed the existence of two degenerate particles θ and τ, each with mass around 483 MeV and lifetime around 12 ns.

However, it was seen that

$$\theta^+ \rightarrow \pi^+ + \pi^0$$

$$\tau^+ \rightarrow \pi^+ + \pi^- + \pi^+$$

indicating that $P_\theta = +1$ and $P_\tau = -1$.

Note that the phase space for these decays is very different:

$$M(\theta^+) - M(\pi^+) - M(\pi^0) = 493 - 140 - 135 = 218 \text{ MeV}$$

$$M(\tau^+) - M(\pi^+) - M(\pi^-) - M(\pi^+) = 493 - 3 \times 140 = 73 \text{ MeV}$$

Since the lifetimes are identical, the strength of weak interactions must be different for these different decays $\Rightarrow$ universality is violated

Yang & Lee (1956)

Maybe parity is not conserved in weak interactions, i.e.

$\theta^+ \to \pi^+ + \pi^0$ is really $K^+ \to \pi^+ + \pi^0$

parity-violating channel

$\tau^+ \to \pi^+ + \pi^- + \pi^+$ is really $K^+ \to \pi^+ + \pi^- + \pi^+$

parity-conserving channel

They also showed that none of the earlier experiments had really tested intrinsic parity violation...

Suggested that if the mean value of a party-odd variable, e.g. $\vec{S}.\vec{p}$ could be found to be nonzero, this would be a 'smoking gun' signal for parity violation

Experiment was actually performed by Wu et al (1957)...

Maximal parity violation:

In 1956, Marshak & Sudarshan, and separately, Feynman & Gell-Mann, assumed the parity-violating weak interactions to be of the form

$$\mathcal{H}_I = \frac{G_F}{4\sqrt{2}}\ \overline{p}\gamma^{\mu}(1-\lambda\gamma_5)n\,.\,\bar{e}\gamma_{\mu}(1-\lambda\gamma_5)\nu_e$$

Parity is <u>conserved</u> when $\lambda = 0$ (V current), $\lambda \to \infty$ (A current)

Parity is <u>maximally</u> violated when $\lambda = 1$ (V-A currents)

Parity is partially violated for other values of λ....

Rewrite the leptonic current as

$$\begin{aligned} J^{\mu}_{\text{lep}} &= \bar{e}\gamma_{\mu}(1-\lambda\gamma_5)\nu_e \\ &= (1+\lambda)\overline{e_L}\gamma_{\mu}\nu_{eL} + (1-\lambda)\overline{e_R}\gamma_{\mu}\nu_{eR} \end{aligned}$$

If we can measure the chirality of neutrinos emitted in beta decay, then we should have

$$P(\nu_{eL}) = \frac{(1+\lambda)^2}{(1+\lambda)^2 + (1-\lambda)^2}$$

$$P(\nu_{eR}) = \frac{(1-\lambda)^2}{(1+\lambda)^2 + (1-\lambda)^2}$$

and hence

$$\frac{P(\nu_{eR})}{P(\nu_{eL})} = \left(\frac{1-\lambda}{1+\lambda}\right)^2$$

Goldhaber et al did an experiment in 1957 with the electron capture process

$$e^- + {}^{152}_{63}Eu \to {}^{152}_{62}Sm^* + \nu_e$$

and found that the neutrino is always left-chiral... It follows that $\lambda = 1$.

Thus the weak interactions do have the form V – A and the W boson vertex for electrons is of the form

$$\mathcal{H}_I = \frac{g}{2\sqrt{2}}\bar{e}\gamma^{\alpha}(1-\gamma_5)\nu_e W_{\alpha}^{-} + \frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^{\alpha}(1-\gamma_5)eW_{\alpha}^{+}$$

We will have similar interactions for the muon

$$\mathcal{H}_I = \frac{g}{2\sqrt{2}}\bar{\mu}\gamma^{\alpha}(1-\gamma_5)\nu_{\mu} W_{\alpha}^{-} + \frac{g}{2\sqrt{2}}\bar{\nu}_{\mu}\gamma^{\alpha}(1-\gamma_5)\mu W_{\alpha}^{+}$$

and for the nucleons

$$\mathcal{H}_I = \frac{g}{2\sqrt{2}}\bar{p}\gamma^{\alpha}(1-\gamma_5)nW_{\alpha}^{-} + \frac{g}{2\sqrt{2}}\bar{n}\gamma^{\alpha}(1-\gamma_5)pW_{\alpha}^{+}$$

and for the quarks

$$\mathcal{H}_I \approx \frac{g}{2\sqrt{2}}\bar{d}\gamma^{\alpha}(1-\gamma_5)uW_{\alpha}^{-} + \frac{g}{2\sqrt{2}}\bar{u}\gamma^{\alpha}(1-\gamma_5)dW_{\alpha}^{+}$$