Introduction to Particle Physics

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Lecture 1

Fundamental Concepts

- Any object which behaves as a point mass or charge. i.e. shows no evidence of substructure, is called a *particle*. Any object which shows no evidence of substructure in all known experiments is called an **elementary particle**.

- This is clearly dependent on the space resolution of the experiment(s) —

de Broglie relation: $\lambda = \frac{h}{p}$

High resolution $\Rightarrow$ small $\lambda \Rightarrow$ large $p \Rightarrow$ large E

  - Some 'particles' display substructure at high energies, e.g. atoms, nuclei, protons & neutrons,...
  - Some particles 'never' display substructure, e.g. electrons, photons, neutrinos,...

- The study of elementary particles and their interactions is called **Particle Physics**, or **High Energy Physics**.

- The main tools of Particle Physics are:
  - *special relativity*, because the momenta p are large
  - *quantum mechanics*, because the objects are small

Together, they give us

relativistic quantum field theory (RQFT)

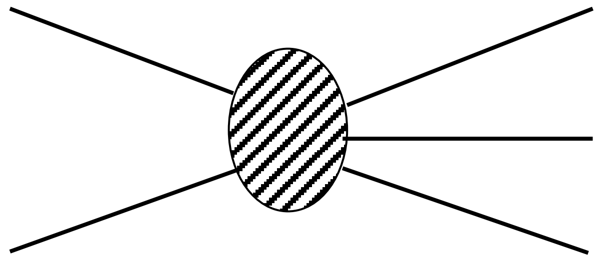
which is the language in which we express all of particle physics.

Thanks to Feynman, we can express the results of RQFT in terms of diagrams which are easy to understand physically. Thus, we can get away, up to a certain point, without learning RQFT.

What can elementary particles do?

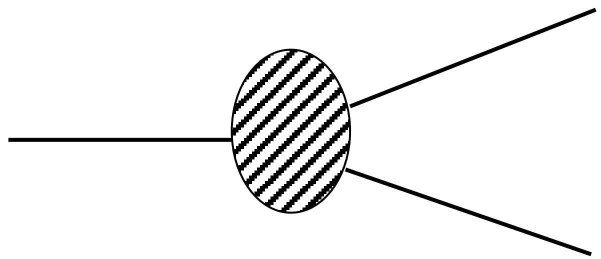
- Scattering processes

  cross-section σ



- Decay processes

  decay width Γ



What makes such processes happen?

Four fundamental interactions:

- gravitation $F \propto G_N m^2$
- electromagnetism $F \propto e^2/4\pi\varepsilon_0$
- strong (nuclear) interaction $F \propto g_s^2$
- weak (nuclear) interaction $F \propto G_F$

At high energies, electromagnetism and weak interactions unify to form the *electroweak interaction*.

Decay processes can be used to determine the nature of the interaction.

A decaying state evolves with time as $e^{-\Gamma t/\hbar}$

i.e. the (mean) lifetime is $\tau = \frac{\hbar}{\Gamma}$

From quantum mechanics, it can be shown that $\Gamma \propto g^2$ where

$$g^2 = \begin{cases} G_N & \text{for gravitation} \\ \dfrac{e^2}{4\pi\varepsilon_0} & \text{for electromagnetism} \\ g_s^2 & \text{for strong interactions} \\ G_F & \text{for weak interactions} \end{cases}$$

Thus, the lifetime of a process satisfies

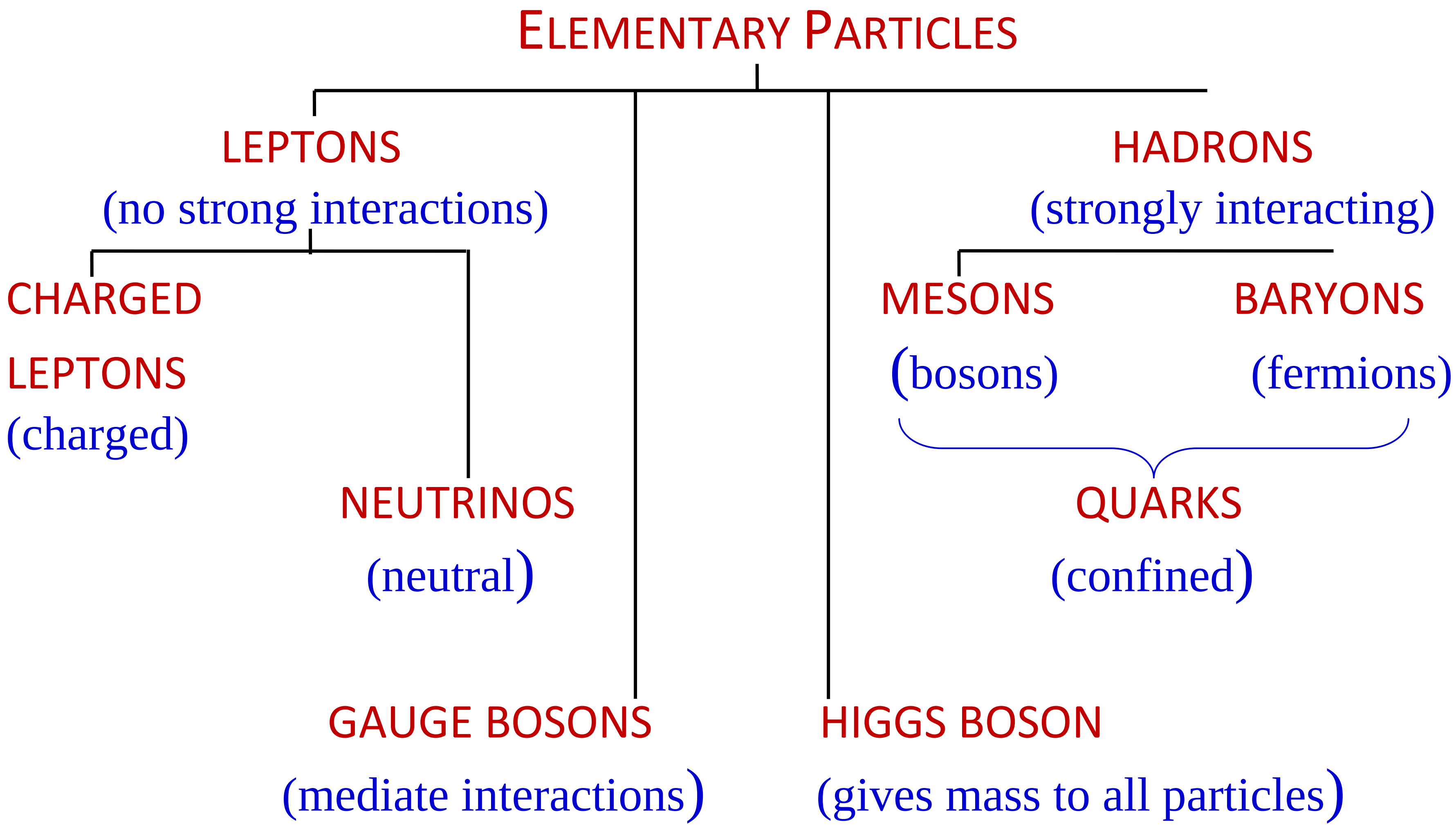
$$\tau \propto \frac{\hbar}{g^2}$$

i.e. the stronger the interaction, the shorter the lifetime.

Interaction	$\boldsymbol{\tau}$	$\boldsymbol{\ell = c\tau}$
Strong interaction	$\sim 10^{-23}$ s	$\sim 10^{-13}$ cm
Electromagnetic interaction	$\sim 10^{-16}$ s	$\sim 10^{-6}$ cm
Weak interactions	$\sim 10^{-9}$ s	~ 10 cm
Gravitational interaction	$\sim 10^{+22}$ s	$\sim 10^{+34}$ cm

Only weakly-decaying particles will leave observable tracks

Classification of particles according to interactions:



Some well-known particles:

CHARGED LEPTONS:	electron (e^-), muon (μ^-), tau (τ^-)
NEUTRINOS:	electron-neutrino (ν_e), muon neutrino (ν_μ), tau neutrino (ν_τ)
MESONS:	pions (π^+, π^0), kaons (K^+, K^0), rho (ρ^+, ρ^0), eta (η^0), etc.
BARYONS:	proton (p^+), neutron (n^0), Delta ($\Delta^{++}, \Delta^+, \Delta^0$), Lambda ($\Lambda^0$), Sigma ($\Sigma^+, \Sigma^0$), cascade ($\Xi^+, \Xi^0$), Omega-minus ($\Omega^-$)
GAUGE BOSONS:	photon (γ), W-boson (W^+), Z-boson (Z^0), gluons (g)

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GAUGE BOSONS:	photon (γ), W-boson (W^+), Z-boson (Z^0), gluons (g)

Each has its own antiparticle:

ANTI-LEPTONS:	positron (e^+), anti-muon (μ^+), anti-tau (τ^+)
NEUTRINOS:	electron-antineutrino ($\bar{\nu}_e$), muon antineutrino ($\bar{\nu}_\mu$), tau antineutrino ($\bar{\nu}_\tau$)
ANTI-MESONS:	pions (π^-, π^0), Kaons (K^-, $\bar{K}^0$), rho (ρ^-, ρ^0), eta (η^0), etc.
ANTI-BARYONS:	antiproton (p^-), antineutron ($\bar{n}^0$), Delta ($\Delta^{--}, \Delta^-, \Delta^0$), Lambda ($\Lambda^0$), Sigma ($\Sigma^-, \Sigma^0$), Cascade ($\Xi^-, \bar{\Xi}^0$), Omega-plus ($\Omega^+$)
GAUGE BOSONS:	photon (γ), W-boson (W^-), Z-boson (Z^0), gluons (g)

Nöther's Theorem and conserved quantum numbers

Consider a system with a Lagrangian $L = L(q_1, q_2, \dots, q_n, \dot{q}_1, \dot{q}_2, \dots, \dot{q}_n)$ such that under a transformation $q_i \to q_i + \varepsilon\eta_i$, for all i, the Lagrangian remains unchanged. We call this a symmetry of the system.

It follows that, treating ε as a parameter: $\frac{\partial L}{\partial \varepsilon} = 0$

or, more explicitly,

$$\sum_{i=1}^{n} \left(\frac{\partial L}{\partial q_i} \eta_i + \frac{\partial L}{\partial \dot{q}_i} \dot{\eta}_i \right) = 0$$

Substituting the Euler-Lagrange equations

$$\frac{\partial L}{\partial q_i} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)$$

we get

$$\sum_{i=1}^{n}\left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)\eta_i + \frac{\partial L}{\partial \dot{q}_i}\frac{d\eta_i}{dt}\right] = 0 \qquad \text{or,} \qquad \sum_{i=1}^{n}\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\eta_i\right] = 0$$

or,

$$\frac{d}{dt}\left(\sum_{i=1}^{n}\frac{\partial L}{\partial \dot{q}_i}\eta_i\right) = 0$$

i.e. we have a conserved quantity

$$Q = \sum_{i=1}^{n}\frac{\partial L}{\partial \dot{q}_i}\eta_i$$

Thus we have proved Nöther’s Theorem: *to every symmetry of a Lagrangian system there corresponds a conserved quantity*. The conserved quantity Q is called the Nöther charge.

Typical conserved quantities and the corresponding symmetries:

	Symmetry under	Conserved quantity	Symbol	S	E	W
1.	Space translation	Linear momentum	$\vec{p}$	√	√	√
2.	Time translation	Energy	E	√	√	√
3.	Spatial rotations	Angular mom, spin	$\vec{J}$	√	√	√
4.	Gauge transfns.	Electric charge	q	√	√	√
5.	Space Inversion	Parity	P	√	√	×
6.	Charge conjugation	C-parity	C	√	√	×
7.	Time inversion	T-parity	$T = CP$	√	√	×
8.	U(1) phase change	Baryon number	B	√	√	×
9.	U(1) phase change	Lepton numbers	L_e, L_μ, L_τ	√	√	×
10.	SU(2) 'rotation'	Isospin	$\vec{I}$	√	√	×
11.	U(1) phase changes	Strangeness, Charm, Beauty, Truth	$S, C, \mathcal{B}, \mathcal{T}$	√	√	×

A scattering or decay process can only take place if all these conservation laws are obeyed.

Example 1: $\Delta^{+} \rightarrow p^{+} + \pi^{0}$

Quantum No	Δ^{+}	p^{+}	π^{0}	Conclusion
mass M (MeV/c^2)	1232	938	135	√
charge Q	+1	+1	0	√
spin J	½	½	1	√
baryon no B	+1	+1	0	√
Isospin I_3	½	½	0	√
all others	0	0	0	√

permitted through all interactions

Example 2: $\Sigma^+ \rightarrow p^+ + \pi^0$

Quantum No	Σ^+	p^+	π^0	Conclusion
mass M (MeV/c^2)	1190	938	135	√
charge Q	+1	+1	0	√
spin J	½	½	1	√
baryon no B	+1	+1	0	√
isospin I_3	½	½	0	√
strangeness S	+1	0	0	×
all others	0	0	0	√

permitted only through weak interactions

Example 3: $n^0 \to p^+ + e^- + \bar{\nu}_e$

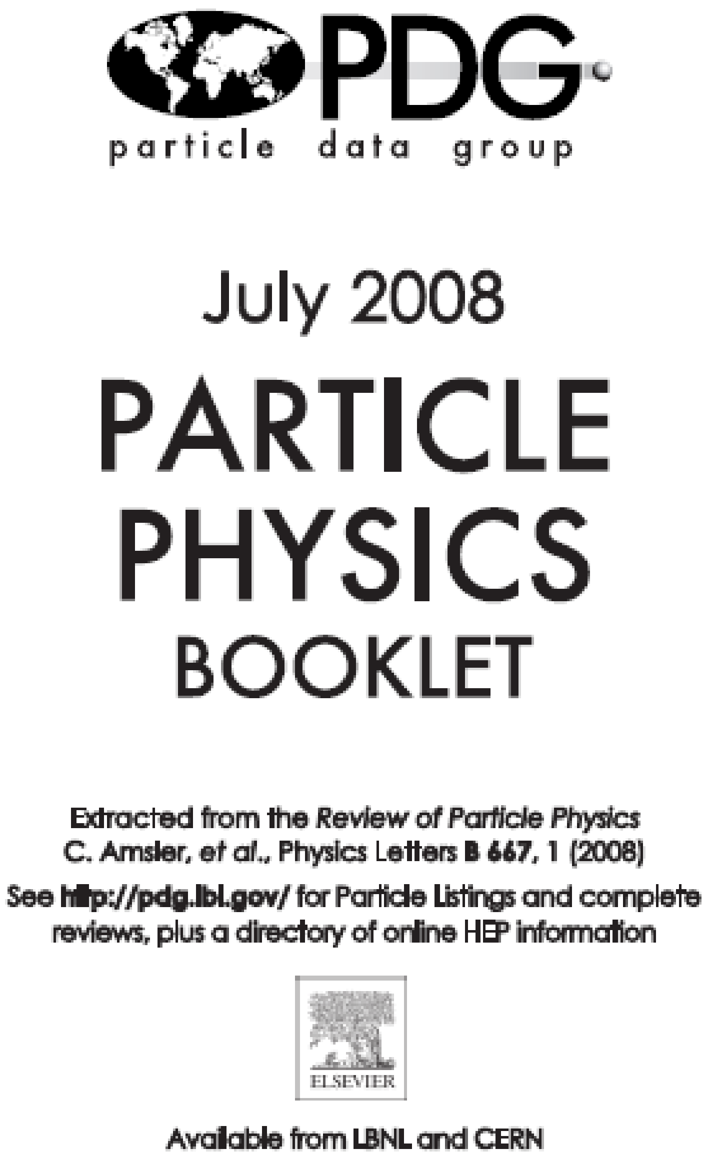
Quantum No	n^0	p^+	e^-	$\bar{\nu}_e$	Conclusion
mass M (MeV/c^2)	939.5	938.2	0.5	0	√
charge Q	0	+1	-1	0	√
spin J	½	½	½	½	√
baryon no B	+1	+1	0	0	√
lepton no L_e	0	0	+1	-1	√
isospin I_3	-½	½	0	0	×
all others	0	0	0	0	√

permitted only through weak interactions

Example 4: $p^+ \rightarrow \pi^0 + e^+ + \nu_e$

Quantum No	p^+	π^0	e^+	ν_e	Conclusion
mass M (MeV/c^2)	938.2	135	0.5	0	√
charge Q	+1	0	+1	0	√
spin J	½	0	½	½	×
baryon no B	+1	0	0	0	×
lepton no L_e	0	0	-1	+1	√
isospin I_3	½	0	0	0	×
all others	0	0	0	0	√

not permitted



Every alternate year the PDG brings out a fresh version of this handbook

Contains all the basic information about all the known particles, with all the updates

Also contains short reviews of basic topics...

Every particle physicist should possess a copy!

Natural units:

we shall use relativity + quantum mechanics: convenient to set

$$c = 1 \quad \text{and} \quad \hbar = 1$$

Dimensions:

$$[c] = LT^{-1} \Rightarrow \mathrm{L} = \mathrm{T}$$

$$E = \hbar\omega \Rightarrow [E] = [T^{-1}]$$

$$E = mc^2 \Rightarrow [E] = M$$

$$\mathrm{L} = \mathrm{T} = \mathrm{M}^{-1} = [E]^{-1}$$

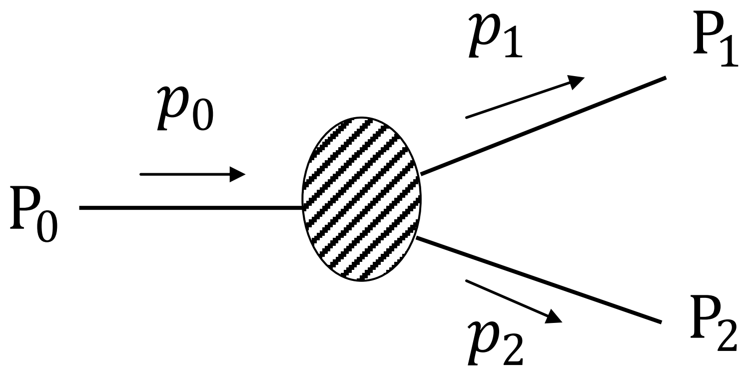
conventional to write everything in units of energy (GeV)

1 GeV^{-1} = 0.1973 fm (1 femtometre = 10^{-15} m)

1 GeV^{-2} = 0.3894 mb (1 millibarn = 10^{-27} cm^2)

1 GeV^{-1} = 0.6582×10^{-22} s

Two-body decay:



conservation of four-momentum:

$$p_0 = p_1 + p_2$$

Hence, in the rest frame of P_0, we have

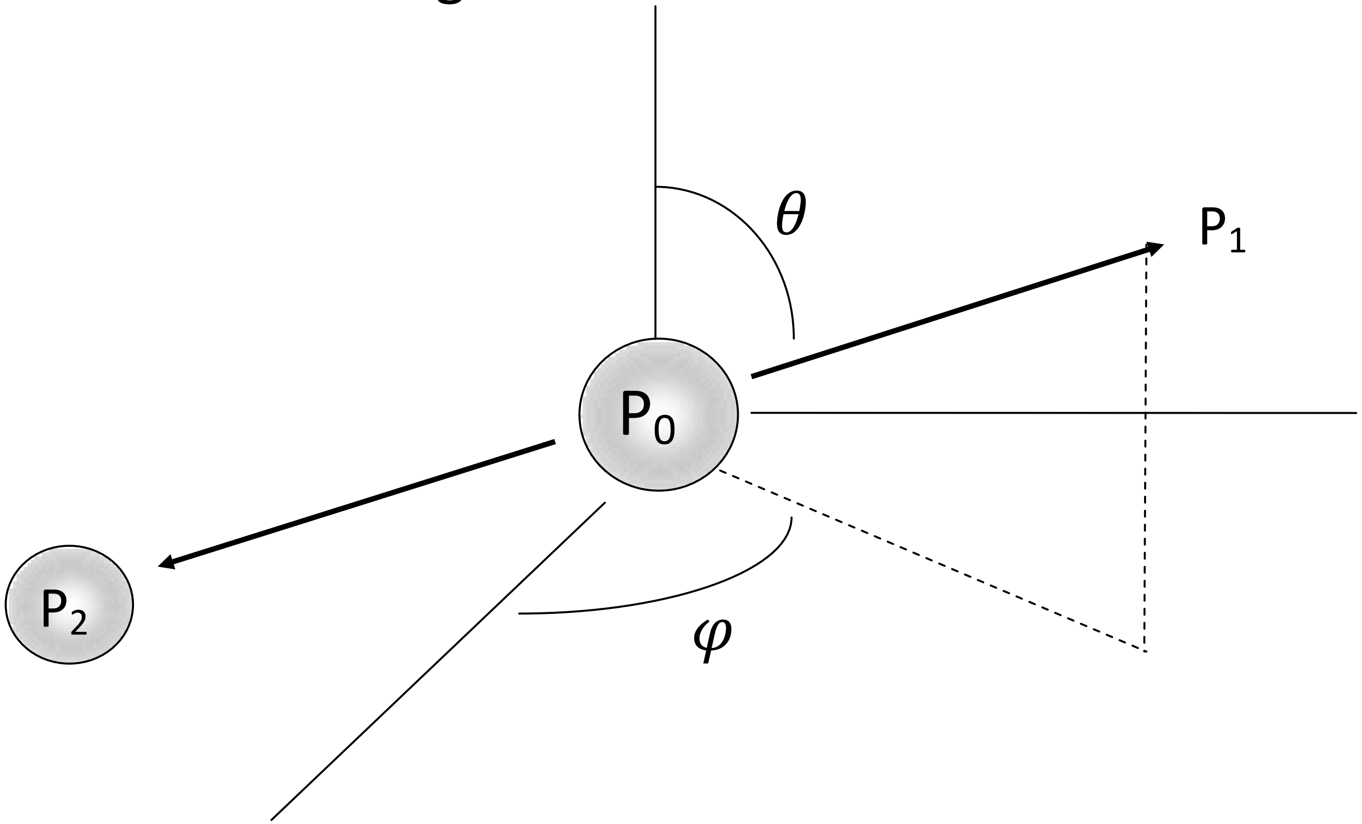
$$p_0 = (M_0, \vec{0}\ ) \qquad p_1 = (E_1, \vec{k}\ ) \qquad p_2 = (E_2, -\vec{k}\ )$$

i.e. $\quad M_0 = E_1 + E_2 \qquad E_1 = \sqrt{\vec{k}^2 + M_1^2} \qquad E_2 = \sqrt{\vec{k}^2 + M_2^2}$

Solving these three equations:

$$|\vec{k}| = \frac{\sqrt{\lambda(M_0^2, M_1^2, M_2^2)}}{2M_0}\ , \quad E_1 = \frac{M_0^2 + M_1^2 - M_2^2}{2M_0}\ , \quad E_2 = \frac{M_0^2 - M_1^2 + M_2^2}{2M_0}$$

Only unknowns are the angles...



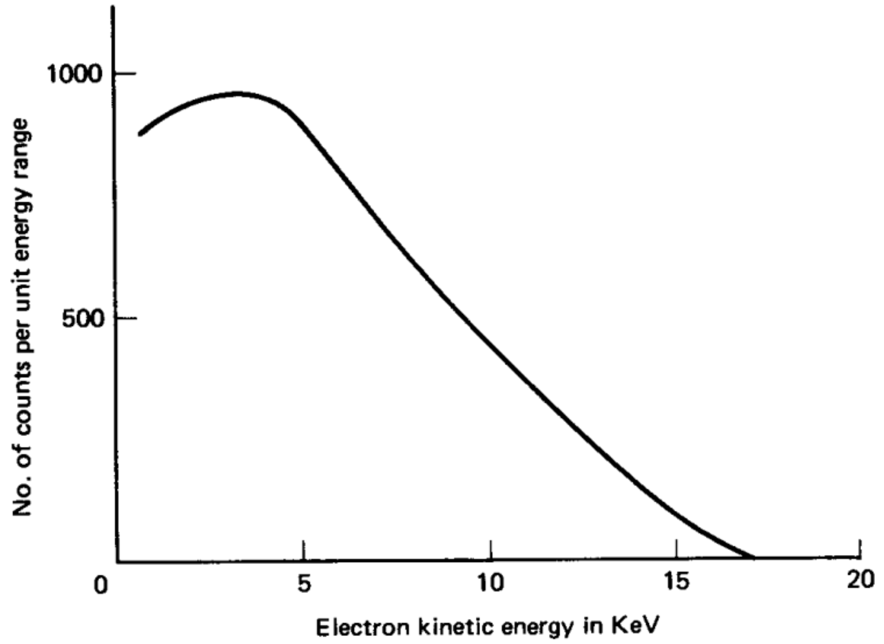
The distribution of angles will depend on the dynamics of the decay

e.g. if the decaying particle has a spin aligned in some direction, the daughter particles may emerge parallel/perpendicular to that direction

Classic story:

nuclear beta decay: originally thought to be $n^0 \rightarrow p^+ + e^-$

but in this case, $E_e = \frac{M_n^2 + M_e^2 - M_p^2}{2M_n} = 1.3\ \text{MeV}$



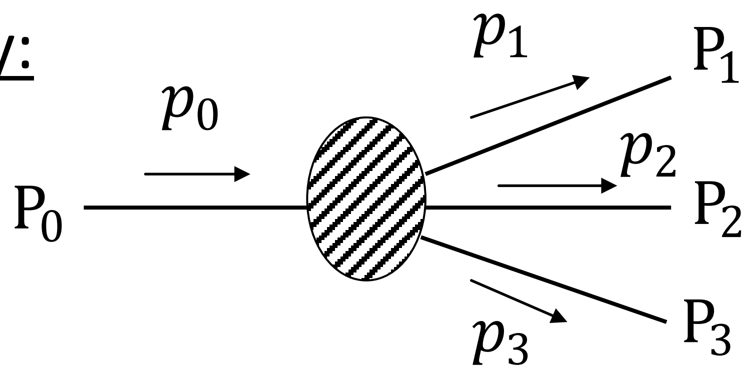
The beta decay spectrum of tritium ($^3_1H \rightarrow ^3_2He$). (*Source:* G. M. Lewis, London: Wykeham, 1970), p. 30.)

Such a distribution is only possible if an invisible particle is also being emitted and it is actually a *three-body* decay... also demanded by the conservation of spin...

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$$

neutrino hypothesis
(Pauli 1930)

Three-body decay:



conservation of four-momentum:

$$p_0 = p_1 + p_2 + p_3$$

Hence, in the rest frame of P_0, we have

$p_0 = \left(M_0, \vec{0}\ \right)$ $\quad p_1 = (E_1, \vec{p}_1\ )$ $\quad p_2 = (E_2, \vec{p}_2\ )$ $\quad p_2 = (E_2, \vec{p}_3\ \ )$

with $\quad E_1 = \sqrt{\vec{p}_1^2 + M_1^2}$ $\quad E_2 = \sqrt{\vec{p}_2^2 + M_2^2}$ $\quad E_3 = \sqrt{\vec{p}_3^2 + M_3^2}$

i.e. we have 4 equations $E_1 + E_2 + E_2 = M_0$

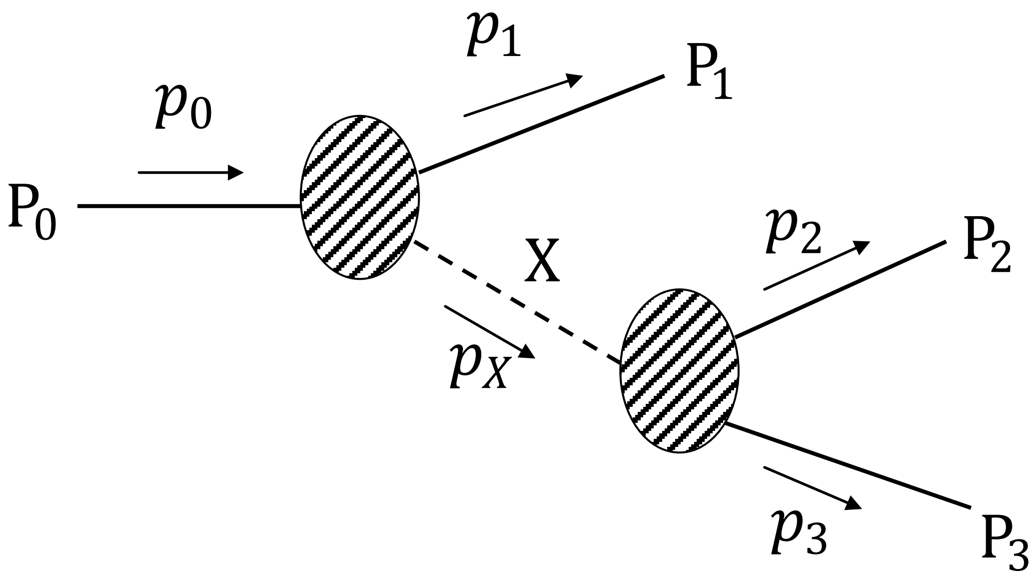
and $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{0}$

for 9 unknowns $\vec{p}_1, \vec{p}_2, \vec{p}_3$

The system has 5 degrees of freedom, which can be chosen in different ways.

Most useful convention:

consider the decay as a two-stage process:



$P_0 \to P_1 + X$

$X \to P_2 + P_3$

Each is a 2-body decay

X is a virtual particle

i.e. M_X is variable

Hence, in the rest frame of P_0, we have

$p_0 = (M_0, \vec{0}\ )$ $\quad$ $p_1 = (E_1, \vec{k}\ )$ $\quad$ $p_X = (E_X, -\vec{k}\ )$

$E_1 = \sqrt{\vec{k}^2 + M_1^2}$ $\quad$ $E_X = \sqrt{\vec{k}^2 + M_X^2}$

and one can calculate

$$\left|\vec{k}\right| = \frac{\sqrt{\lambda(M_0^2, M_1^2, M_X^2)}}{2M_0} \ , \quad E_1 = \frac{M_0^2 + \ M_1^2 - \ M_X^2}{2M_0} \ , \quad E_X = \frac{M_0^2 - \ M_1^2 + \ M_X^2}{2M_0}$$

All of these vary because of the <u>variation of M_{X}</u>

<u>Two other variables</u> $\theta, \varphi \Rightarrow$ direction of $\vec{k}$ in the rest frame of P_0

We can now construct the full momentum $\vec{p}_1$ in the rest frame of P_0

$$p_{1x} = \left|\vec{k}\right| \sin\theta \cos\varphi$$
$$p_{1y} = \left|\vec{k}\right| \sin\theta \sin\varphi$$
$$p_{1z} = \left|\vec{k}\right| \cos\theta$$

which vary with $M_{\mathrm{X}}, \theta, \varphi$. As usual $0 \leq \theta \leq \pi$ and $0 \leq \varphi \leq 2\pi$.

We also have $M_{\mathrm{X}} \leq M_0 - M_1$ with the equality corresponding to $\left|\vec{k}\right| = 0$.

Again, in the rest frame of X, we have

$$p_X = (M_X, \vec{0}\ ) \qquad p_2 = (E_2, \vec{\ell}\ ) \qquad p_3 = (E_3, -\vec{\ell}\ )$$

$$E_2 = \sqrt{\vec{\ell}^2 + M_2^2} \qquad E_X = \sqrt{\vec{\ell}^2 + M_3^2}$$

and one can calculate

$$|\vec{\ell}| = \frac{\sqrt{\lambda(M_X^2, M_2^2, M_3^2)}}{2M_X}\ , \quad E_2 = \frac{M_X^2 + M_2^2 - M_3^2}{2M_X}\ , \quad E_X = \frac{M_X^2 - M_2^2 + M_3^2}{2M_X}$$

Two other variables $\theta', \varphi' \Rightarrow$ direction of $\vec{\ell}$ in the rest frame of X

We can now construct the full momentum $\vec{p}_2$ in the rest frame of X

$$p_{2x} = |\vec{\ell}| \sin\theta' \cos\varphi' \qquad p_{2y} = |\vec{\ell}| \sin\theta' \sin\varphi' \qquad p_{2z} = |\vec{\ell}| \cos\theta'$$

which vary with M_X, θ', φ' . As usual $0 \leq \theta' \leq \pi$ and $0 \leq \varphi' \leq 2\pi$.

We also have $M_X \geq M_2 + M_3$ with the equality corresponding to $|\vec{\ell}| = 0$.

Thus, $M_2 + M_3 \leq M_\mathrm{X} \leq M_0 - M_1$ is the full range of M_X.

Note that X is boosted in the rest frame of P_0 by the factor

$$\beta_X = \frac{p_X}{E_X} = \frac{|\vec{k}|}{E_X} = \frac{\sqrt{\lambda(M_0^2, M_1^2, M_X^2)}}{M_0^2 - M_1^2 + M_X^2}$$

and the direction of the boost is along $-\vec{p}_1$, which we have already constructed.

Thus, to get the momentum $\vec{p}_2$ in the rest frame of P_0, we require to boost back by the factor β_X, i.e. boost by β_X in the direction of $+\vec{p}_1$.

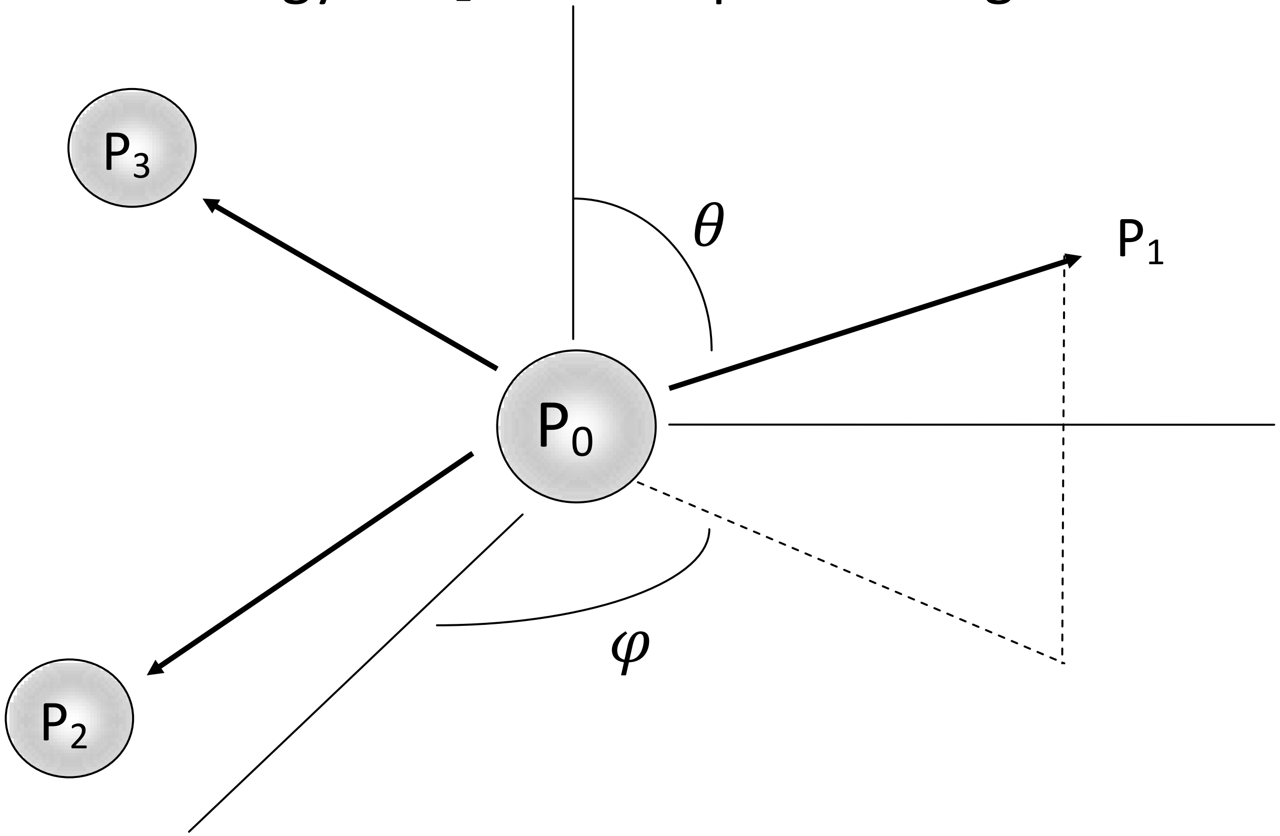
The formula for this general Lorentz boost is

$$\vec{p}_2 \rightarrow \vec{p}_2 + \gamma_X \vec{\beta}_X E_2 - (\gamma_X - 1)\,\frac{\vec{\beta}_X . \vec{p}}{\beta_X^2} \vec{\beta}_X$$

Finally, we can simply construct

$$\vec{p}_3 = -\vec{p}_1 - \vec{p}_2$$

Unknowns are the energy of P_1 and two pairs of angles...



The distribution of angles will depend on the dynamics of the decay

e.g. if the decaying particle has a spin aligned in some direction, the daughter particles may emerge parallel/perpendicular to that direction

<u>Back to the neutrino story</u>:

The three-body decay

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$$

can be written

$$n^0 \rightarrow p^+ + X$$
$$X \rightarrow e^- + \bar{\nu}_e$$

Now, we can write

$$E_e = \frac{M_X^2 - M_e^2 + M_\nu^2}{2M_X} = \frac{1}{2} M_X \left(1 - \frac{M_e^2 - M_\nu^2}{M_X^2}\right)$$

where

$$M_e + M_\nu \leq M_X \leq M_n - M_p$$

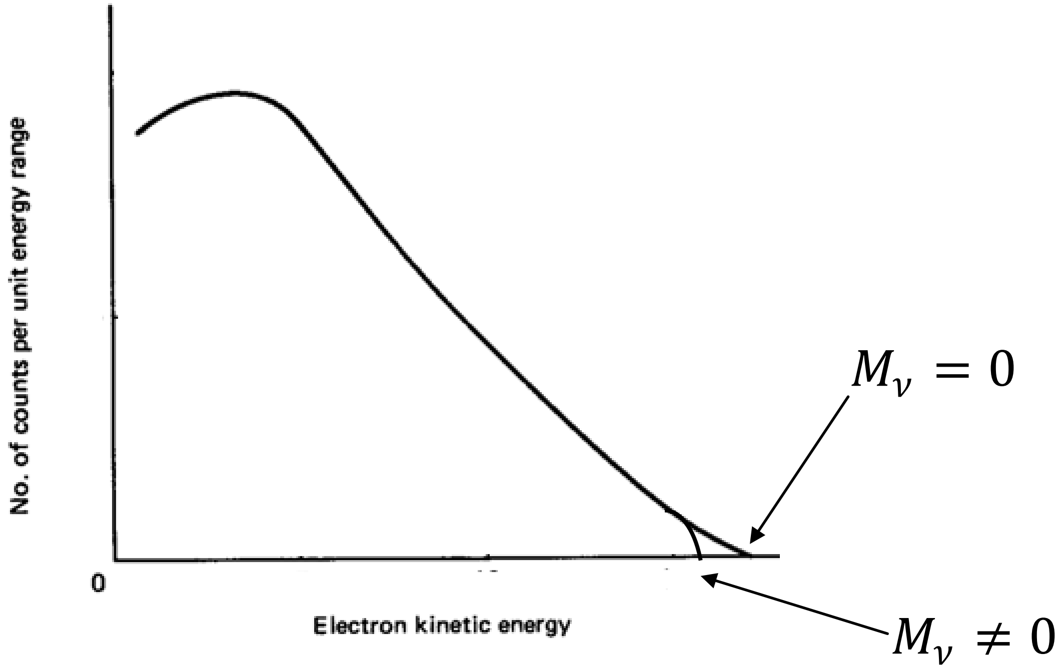
i.e.

$$0.5 \text{ MeV} + M_\nu \leq M_X \leq 1.3 \text{ MeV}$$

Thus, the maximum value of the beta particle energy will be

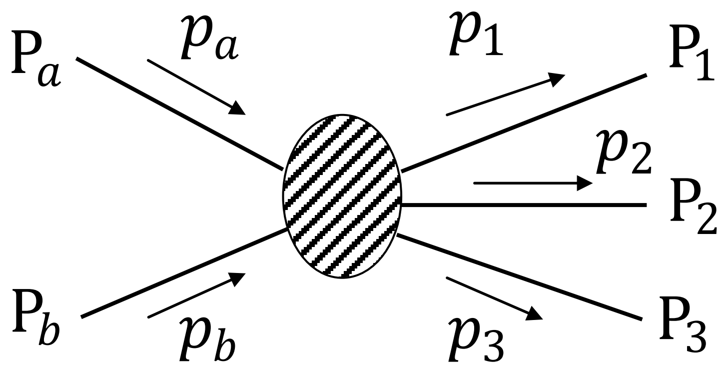
$$E_e|_{\max} = \frac{1}{2}(M_n - M_p)\left[1 - \frac{M_e^2 - M_\nu^2}{(M_n - M_p)^2}\right] = \left[1.3 - 0.1\left(\frac{M_\nu}{M_e}\right)^2\right] \text{MeV}$$

The neutrino mass can be calculated from the endpoint of the β spectrum.

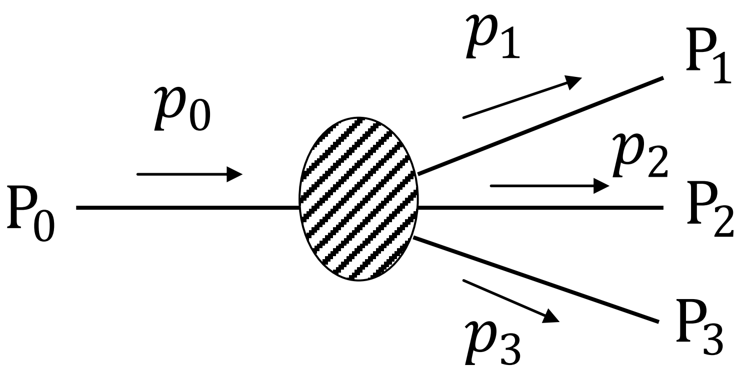


All beta decay experiments are consistent with $M_{\nu_e} < 2.2$ eV

Scattering processes: $2 \rightarrow 3$



Treat P_a and P_b together as a composite particle, i.e. $\mathrm{P}_0 \equiv \mathrm{P}_a$ and P_b



$p_0 = p_a + p_b$

$M_0 = \sqrt{(p_a + p_b)^2} = E_{cm}$

Treat it as a 3-body decay

In a fixed target experiment

$$p_a = (k, \vec{k}) \qquad\qquad p_b = (M, \vec{0})$$

we have

$$M_0 = E_{cm} = \sqrt{2Mk}$$

i.e. available energy grows only as $\sqrt{k}$

In a colliding beam experiment

$$p_a = (k, \vec{k}) \qquad\qquad p_b = (k, -\vec{k})$$

we have

$$M_0 = E_{cm} = 2k$$

i.e. available energy grows as k

$\Rightarrow$ high energy machines are of this type