Going beyond Mahabaleshwar: Search for CPT Violation

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PCPV 2013, Mahabaleshwar
Part I: Introduction to CPT violation
Parity violation can be incorporated through the current (Lee and Yang)

\[ P : \quad \bar{\psi} \gamma^\mu (1 - \gamma_5) \psi \rightarrow \bar{\psi} \gamma^\mu (1 + \gamma_5) \psi \]

Maximal P violation for weak interaction
This is not enough for CP violation, you need the coupling to be complex too (Kobayashi and Maskawa)

\[ \text{CP : } g \bar{\psi}_1 \gamma^\mu (1 - \gamma_5) \psi_2 + \text{h.c.} \implies g \bar{\psi}_2 \gamma^\mu (1 - \gamma_5) \psi_1 + \text{h.c.} \]

H.c. involves \( g^* \), but gauge couplings are real. Introduce quark mixing. Large CP violation for B systems, but too small to explain \( \frac{n_b}{n_\gamma} \).
CPT, taken in any order, is the *only* combination of C,P,T that is still conserved.

- **Pauli (1940):** Spin-statistics theorem, requires Lorentz invariance
- **Schwinger (1951):** Spin-statistics theorem, implicit use of CPT theorem
- **Lüders, Pauli, Bell (1954-55):** Proof of CPT theorem
- **Jost (1958):** General proof for axiomatic QFT

Why, then, should one look for CPT violation?

Motivation 1: George Mallory about Mt. Everest: Because it is there.
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CPT transformation on Fermion current:

\[ \bar{\psi}_a^{CPT}(t, x) \Gamma_i \psi_b^{CPT}(t, x) = \bar{\psi}_b(-t, -x) \Gamma_i^{CPT} \psi_a(-t, -x) \]

\[ \Gamma_i : \{1, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu \nu}\} \Rightarrow \{1, -\gamma_5, -\gamma_\mu, -\gamma_\mu \gamma_5, \sigma_{\mu \nu}\} \]

CPT is a good symmetry if

\[ (CPT) \mathcal{L}(t, x)(CPT)^{-1} = \mathcal{L}(-t, -x) \]

That gives you an idea of what terms can potentially violate CPT.
Theorem

CPT is a good symmetry of any local Lorentz-invariant axiomatic quantum field theory with a unique vacuum state.

You can never construct a Lorentz-invariant QFT with a hermitian Hamiltonian that violates CPT.

Proof of CPT theorem is not straightforward (see, e.g., Streater and Wightman)

Proof.

Consider real scalar field $\rightarrow$ $C$ is conserved
PT is $x^\mu \rightarrow -x^\mu$, proper LT, continuously connected to identity
In Euclidean space, just like a 4-d rotation — must be conserved
PT is always a good symmetry for real scalar field
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Consequences of CPT conservation

- Particle and antiparticle must have same mass and opposite electric charge
- Particle and antiparticle, if unstable, must have same decay width

Not true if stationary states are particle-antiparticle combinations

\[ K_L \approx \frac{1}{\sqrt{2}} (K^0 + \overline{K}^0), \quad K_S \approx \frac{1}{\sqrt{2}} (K^0 - \overline{K}^0), \]

\[ M_{K_L} \neq M_{K_S}, \quad \Gamma_{K_L} \neq \Gamma_{K_S} \]

- Particle and antiparticle must have equal and opposite mag. moment
- Hydrogen and antihydrogen must have identical spectra ....
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In a local QFT with well-defined asymptotic states, CPT violation needs Lorentz violation

[Greenberg, PRL 2002]

The reverse is not true.

Motivation 2: Strings are extended objects, so nonlocal. Critical dimensionality $d > 4$, Higher dimensional breaking of Lorentz covariance incorporated in a 4-d world? Lorentz symmetry can be broken in noncommutative FT too.
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Motivation 3: Asymptotic states for \( q, \bar{q} \) are not well-defined
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Lorentz violation

Lorentz breaking may be spontaneous (like SSB, but VEV to, say, a vector field) or explicit

The low-energy effective theory (SME) contains operators whose coefficients are Lorentz breaking

\[ \mathcal{L} = - (a_L)_{\mu ij} \bar{L}_i \gamma^\mu L_j - (a_R)_{\mu ij} \bar{R}_i \gamma^\mu R_j \]

Physics depends on direction! [Colladay and Kostelecky, PRD 1998]

- Lorentz transformations on the frame (observer) [passive] or on the fields (particle) [active]
- Should be inverse of each other if Lorentz symmetry is respected
- With LV, they are no longer so. Particle transformations are physically important.
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Observables that depend on sidereal time are smoking gun signals of Lorentz violation.

LV operators might be CPT-even or CPT-odd.
For each Dirac fermions, there are 44 possible observable LV terms in nonrelativistic limit. 20 among them are CPT-odd.

[Kostelecky and Russell, RMP 2008, 0801.0287]

\[
\mathcal{L} = \bar{\psi}_i (i \Gamma_\mu D^\mu - M_{ij}) \psi_j
\]

\[
\Gamma_\mu = \gamma_\mu \delta_{ij} + c^{ij}_{\mu \nu} \gamma^\nu + d^{ij}_{\mu \nu} \gamma^\mu \gamma_5 + e_{ij}^\mu + if_{ij}^\mu \gamma_5 + \frac{1}{2} g^{ij}_{\mu \kappa \rho} \sigma^{\kappa \rho}
\]

\[
M_{ij} = m_{ij} + im_{5ij} \gamma_5 + a^{ij}_{\mu} \gamma^\mu + b^{ij}_{\mu} \gamma^\mu \gamma_5 + \frac{1}{2} h^{ij}_{\mu \nu} \sigma^{\mu \nu}
\]

Blue terms are CPT-odd. Red terms are LV but CPT-even

CPT (and LV) tests have been carried out in gravity, photon, charged lepton, neutrino, proton, neutron, and meson sectors.
CPT and LV tests

- **Astrophysics**
  - Pulsar rates
  - CMB polarization
  - Birefringence

- **Atomic physics**
  - K/He magnetometer
  - H maser
  - QED tests with Penning trap
CPT and LV tests

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- **Optics**
  - Optical and microwave resonators
  - Atomic clocks
  - Lunar laser ranging
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▶ Optics
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Several particle physics experiments too.

1. Neutrino oscillation
2. \((g - 2)_{e,\mu}\)
3. \(e^+e^-\) annihilation
4. Particle and antiparticle mass measurement
5. Neutral meson oscillation
6. and others ...

Sidereal time variation for observables \(\rightarrow\) LV
Difference between particle and antiparticle \(\rightarrow\) LV + CPTV
Not all measurements are equally precise.

Define \( p = \frac{\text{Obs}(\text{particle}) - \text{Obs}(\text{antiparticle})}{\text{Obs}(\text{average})} \)

| \( p(m_W) \) | \(-0.002 \pm 0.007 \) | \( (2 \pm 5) \times 10^{-4} \) |
| \( p(m_p) \) | \(< 2 \times 10^{-9} \) | \( (9 \pm 6) \times 10^{-5} \) |
| \( p(g_{e^+}) \) | \((-0.5 \pm 2.1) \times 10^{-12} \) | \((-0.11 \pm 0.12) \times 10^{-8} \) |
| \( m_t - m_{\bar{t}} \) | \((-1.4 \pm 2.0) \text{ GeV} \) | |
Take, as an example, the charge equality of electron and positron

$$e^+ + e^- \rightarrow \gamma + \gamma$$

Direct PDG: $$(Q_{e^+} + Q_{e^-})/|Q_{e^-}| < 4 \times 10^{-8}$$

Much better bound assuming charge conservation

$$(Q_{e^+} + Q_{e^-})/|Q_{e^-}| \sim Q_{\gamma}/|Q_{e^-}| < 10^{-33}$$
Muons are stored in the storage ring — their spin precession frequency $\omega$ can be measured very precisely

- Is $\omega_+ = \omega_-$?
- Is there a sidereal variation?

Both the answers are consistent with zero — one of the most precise measurements. [Muon $g-2$ Collab., PRL 2008]

However, this does not say anything, for example, about the CPT violating parameters in the $\tau$ sector — CPT violation can be a flavour dependent thing.
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Part II: CPT violation in the K and B systems
$K^0 - \bar{K}^0$ mixing and CPT violation

Beam of neutral Kaon in its rest frame

$$|K(t)\rangle = a_1(t)|K^0\rangle + a_2(t)|\bar{K}^0\rangle, \quad \langle K^0|\bar{K}^0\rangle = 0$$

The evolution is given by

$$i \frac{\partial}{\partial t} \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2} \Gamma_{11} & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M_{22} - \frac{i}{2} \Gamma_{22} \end{pmatrix} \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix}$$
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The Hamiltonian matrix $H$ can be parametrized by 7 parameters as relative phase between $H_{12}$ and $H_{21}$ is meaningless. Diagonalize with eigenvalues $\lambda_1$ and $\lambda_2$:

$$
\lambda_1 - \lambda_2 = \Delta M + \frac{i}{2} \Delta \Gamma , \quad \Delta M = M_1 - M_2 , \quad \Delta \Gamma = \Gamma_2 - \Gamma_1 .
$$

$$
M = (M_{11} + M_{22})/2 , \quad \Gamma = (\Gamma_{11} + \Gamma_{22})/2 , \quad \Delta M , \quad \Delta \Gamma
$$

$$
\theta = \frac{H_{22} - H_{11}}{\Delta M - \frac{i}{2} \Delta \Gamma} , \quad \chi = \frac{|H_{12}|^2 - |H_{21}|^2}{|H_{12}|^2 + |H_{21}|^2}
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$\theta$ is CPT violating $\implies$ CPT conservation: $M_{11} = M_{22}, \quad \Gamma_{11} = \Gamma_{22}$

$\chi$ is CPT conserving but T violating.

Mass eigenstates: $K_L$ and $K_S$
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Mass eigenstates: $K_L$ and $K_S$
The three important parameters for the Kaon sector:

\[ \omega = \frac{A(K_S \rightarrow 2\pi)_{I=2}}{A(K_S \rightarrow 2\pi)_{I=0}} \quad |\Delta I| \neq \frac{1}{2} \]

\[ \epsilon = \frac{A(K_L \rightarrow 2\pi)_{I=0}}{A(K_S \rightarrow 2\pi)_{I=0}} \quad \text{CP} \]

\[ \epsilon' = \frac{A(K_L \rightarrow 2\pi)_{I=2} A(K_S \rightarrow 2\pi)_{I=0} - A(K_L \rightarrow 2\pi)_{I=0} A(K_S \rightarrow 2\pi)_{I=2}}{\sqrt{2}[A(K_S \rightarrow 2\pi)_{I=0}]^2} \]

Related parameters:

\[ \eta_{+-} = \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} = \epsilon + \frac{\epsilon'}{1 + \omega/\sqrt{2}} \approx \epsilon + \epsilon' \]

\[ \eta_{00} = \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)} = \epsilon - \frac{2\epsilon'}{1 - \sqrt{2}\omega} \approx \epsilon - 2\epsilon' \]
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K_{S,L} = \frac{1}{\sqrt{2(1 + \left|\epsilon_{S,L}\right|^2)}} \left( (1 + \epsilon_{S,L})K^0 \pm (1 - \epsilon_{S,L})\overline{K}^0 \right)
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\epsilon_{S,L} = \frac{i \text{Im} M_{12} - \frac{1}{2} \text{Im} \Gamma_{12} \pm \frac{1}{2} \left( M_{\overline{K}^0} - M_{K^0} - \frac{i}{2} (\Gamma_{\overline{K}^0} - \Gamma_{K^0}) \right)}{\Delta M + \frac{i}{2} \Delta \Gamma}
\]
\[
= \epsilon \pm \bar{\delta}
\]

If CPT is conserved, \(\bar{\delta} = 0\). The reverse is not true!

CPT violating parameters enter into the definition of the states and hence affect the observables.

\[
\text{Re}(\bar{\delta}) = (2.3 \pm 2.7) \times 10^{-4}, (2.51 \pm 2.25) \times 10^{-4}
\]
\[
\text{Im}(\bar{\delta}) = (0.4 \pm 2.1) \times 10^{-5}, (-1.5 \pm 1.6) \times 10^{-5}
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[KLOE 2006, KTeV 2011]
\[ K_{S,L} = \frac{1}{\sqrt{2(1 + |\epsilon_{S,L}|^2)}} \left( (1 + \epsilon_{S,L})K^0 \pm (1 - \epsilon_{S,L})\bar{K}^0 \right) \]

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[KLOE 2006, KTeV 2011]
$B^0 - \bar{B}^0$ mixing and CPT violation

Formalism is almost identical \cite{Lavoura, Ann. Phys. (1991)}

The constraints can be completely different — CPT violation may be flavour-dependent. Also, $\Delta M$ large but $\Delta \Gamma$ small.

- Lifetime difference can be significant
- CPT violation may affect direct CP-violating asymmetries, including semileptonic and dileptonic
- For semileptonic decays $B, \bar{B} \rightarrow \ell^\pm X^{\mp} f$, the time-ordering of leptonic and hadronic decays may change due to CPT violation

\cite{Datta, Paschos, Singh (PLB 2002), Balaji, Horn, Paschos (PRD 2003), Xing (PRD 1994, PLB 1999)}
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CPT violation in Mixing

Introduce CPT violation in the Hamiltonian matrix through the parameter $\delta$, can be complex:

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}},$$

Solutions:

$$\lambda = \left[ H_{11} + H_{12}\alpha \left( y + \frac{\delta}{2} \right) \right], \quad \left[ H_{22} - H_{12}\alpha \left( y + \frac{\delta}{2} \right) \right],$$

where $y = \sqrt{1 + \frac{\delta^2}{4}}$ and $\alpha = \sqrt{H_{21}/H_{12}}$. 
Eigenstates

\[ |B_H \rangle = p_1 |B^0 \rangle + q_1 |\overline{B}^0 \rangle , \]

\[ |B_L \rangle = p_2 |B^0 \rangle - q_2 |\overline{B}^0 \rangle . \]

Normalisation

\[ |p_1|^2 + |q_1|^2 = |p_2|^2 + |q_2|^2 = 1 . \]

Define

\[ \eta_1 = \frac{q_1}{p_1} = \left( y + \frac{\delta}{2} \right) \alpha ; \quad \eta_2 = \frac{q_2}{p_2} = \left( y - \frac{\delta}{2} \right) \alpha ; \quad \omega = \frac{\eta_1}{\eta_2} . \]

\( \delta \) and hence \( y \) are CPT violating. If \( |\delta| \ll 1 \), \( y \approx 1 \).
\[ \Delta a_\mu = r_1 a_\mu^1 - r_2 a_\mu^2, \quad \beta_\mu = (1, \vec{\beta}) \]

\[ \delta = -\frac{1}{2} \frac{\beta_\mu \Delta a_\mu}{\Delta M - i \Delta \Gamma / 2} \]

Numerator varies with time as \( \vec{\beta} \) rotates with \( \Delta \vec{a} \).

BaBar (0711.2713) got \( \delta \) consistent with zero (first two spectral powers) from OS dilepton events.

Belle(1203.0930):

\[ \text{Re}(\delta_d) = (-3.8 \pm 9, 9) \times 10^{-2} \]
\[ \text{Im}(\delta_d) = (1.14 \pm 0.93) \times 10^{-2} \]

Similar results in K (KTeV) and D (FOCUS) systems. Time for \( B_s \).
Consider decay to a CP eigenstate $f$.

\[ A_f = \langle f | H | B_q \rangle, \quad \bar{A}_f = \langle f | H | \bar{B}_q \rangle. \]

\[ \xi_{f_1} = \eta_1 \frac{\bar{A}_f}{A_f}, \quad \xi_{f_2} = \eta_2 \frac{\bar{A}_f}{A_f}. \]

In the SM, both are equal and $\xi_{f_1} = \xi_{f_2} = \xi_f$. For single-channel processes, $|\xi_f| = 1$.

The untagged rate $\Gamma_U[f, t] = \Gamma(B_q(t) \to f) + \Gamma(\bar{B}_q(t) \to f)$

\[ Br[f] = \frac{1}{2} \int_0^\infty dt \ \Gamma[f, t]. \]
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\[ Br[f] = \frac{1}{2} \int_0^\infty dt \Gamma[f, t]. \]
Assume $|\delta|$ to be small, make Taylor expansion up to $\delta^n$, $n \leq 2$

\[
\Gamma_U[f, t] = |A_f|^2 e^{-\Gamma_q t} \left[ (...) \cosh \left( \frac{\Delta \Gamma q t}{2} \right) + (...) \sinh \left( \frac{\Delta \Gamma q t}{2} \right) \\
+ (...) \cos (\Delta M q t) + (...) \sin (\Delta M q t) \right]
\]

Simplification:
For $B_d$ system, $\Delta \Gamma_d \ll 1$, $\cosh \to 1$, $\sinh \to 0$, easier fit to decay profile
For $|\delta| \ll 1$, keep only the linear terms. For $B_s$, keep $\Gamma_s$ too

\[
Br[f] = \frac{|A_f|^2}{2} \left[ \frac{1}{\Gamma_s} \left\{ 2 - \text{Im}(\delta)\text{Im}(\xi_f) \right\} \\
+ \frac{\Gamma_s}{(\Delta m)^2 + (\Gamma_s)^2} \text{Im}(\delta)\text{Im}(\xi_f) + \frac{\Delta \Gamma_s}{(\Gamma_s)^2} \text{Re}(\xi_f) \right].
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$$
The $B$ mesons can be tagged:

$$\Gamma_T[f, t] = \Gamma(B_q(t) \rightarrow f) - \Gamma(\bar{B}_q(t) \rightarrow f)$$

- Fit both untagged and tagged profiles
- $\text{Re}(\delta)$ from cos and sinh terms, $\text{Im}(\delta)$ from sin and cos terms

$$A_{CPT}(f, t) = \frac{\Gamma_T[f, t]}{\Gamma_U[f, t]} = \frac{\Gamma(B_q(t) \rightarrow f) - \Gamma(\bar{B}_q(t) \rightarrow f)}{\Gamma(B_q(t) \rightarrow f) + \Gamma(\bar{B}_q(t) \rightarrow f)}.$$

Goes to usual CP asymmetry $A_{CP}$ if $\delta = 0$.

No change in semileptonic CP asymmetry if only new physics is CPT violation.
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\[ A_{\text{CPT}}(f) = \frac{\int_{0}^{\infty} dt \, \Gamma_T[f, t]}{\int_{0}^{\infty} dt \, \Gamma_U[f, t]} = \frac{\int_{0}^{\infty} dt \, [\Gamma(B_q(t) \rightarrow f) - \Gamma(\bar{B}_q(t) \rightarrow f)]}{\int_{0}^{\infty} dt \, [\Gamma(B_q(t) \rightarrow f) + \Gamma(\bar{B}_q(t) \rightarrow f)]} . \]

Top to bottom: \( \text{Im}/\text{Re}(\delta) = -0.1, 0.0, 1 \)

[AK, Nandi, Patra, PRD 2010]
Consider a specific example
\[ B_s, \overline{B}_s \rightarrow D_s^\pm K^{\mp} \]

Can proceed through
\[ b \rightarrow c \bar{u}s \quad [\propto V_{cb} V_{us}^*] \quad \text{and} \]
\[ b \rightarrow u \bar{c}s \quad [\propto V_{ub} V_{cs}^* = \exp(-i\gamma)]. \]

Only tree-level in SM and \( \propto \lambda^3 \). Comparable rates.

\[ \text{Br}(B_s \rightarrow D_s K) = (1.90 \pm 0.23) \times 10^{-4} \quad \text{[LHCb 2012]} \]
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[\text{LHCb 2012}]
\[ \Gamma(B_s(t) \rightarrow f) - \Gamma(B_s^*(t) \rightarrow f) = \left[ P_1 \sinh(\Delta \Gamma_s t/2) + Q_1 \cosh(\Delta \Gamma_s t/2) + R_1 \cos(\Delta M_s t) + S_1 \sin(\Delta M_s t) \right] \times e^{-\Gamma_s t} |A_f|^2, \]

Tagged

\[ \Gamma(B_s(t) \rightarrow f) + \Gamma(B_s^*(t) \rightarrow f) = \left[ P_2 \sinh(\Delta \Gamma_s t/2) + Q_2 \cosh(\Delta \Gamma_s t/2) + R_2 \cos(\Delta M_s t) + S_2 \sin(\Delta M_s t) \right] \times e^{-\Gamma_s t} |A_f|^2, \]

Untagged

Absence of CPT violation means \( P_1 = Q_1 = R_2 = S_2 = 0 \)
Similar observables $\bar{P}_1 - \bar{S}_2$ for $B_s \to \bar{f}(= D_s^- K^+)$

\[
\frac{R_1 + \bar{R}_1}{P_2 + \bar{P}_2} = \frac{\text{Re}(\delta)}{2}, \quad \frac{Q_2 - \bar{Q}_2}{S_1 - \bar{S}_1} = \frac{\text{Im}(\delta)}{2}.
\]

Hadronic uncertainties and BSM effects in mixing cancel out in the ratio!

One can refine the analysis. LHCb with 200 fb$^{-1}$ can reach up to $\text{Re}(\delta) \sim 0.1$

[AK, Nandi, Patra, Soni, PRD 2013]
CP violation can be present in decay only and not mixing. Parametrize by some complex parameter \( y_f \).

\[
A(B_s \to D_s^+ K^-) = T_1 e^{i\gamma} (1 - y_f) \\
A(B_s \to D_s^- K^+) = T_2 (1 + y_f^*) \\
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\]

\[
A_{br}^{CPT} = \frac{\langle Br(B_s \to D_s^+ K^-) \rangle - \langle Br(B_s \to D_s^- K^+) \rangle}{\langle Br(B_s \to D_s^+ K^-) \rangle + \langle Br(B_s \to D_s^- K^+) \rangle}
\]

\[
= -2 \frac{\text{Re}(y_f)}{1 + |y_f|^2} \approx -2 \text{Re}(y_f)
\]

LHCb at 200 fb\(^{-1}\): \( \text{Re}(y_f) \approx 0.003 \)
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A(\overline{B}_s \rightarrow D_s^- K^+) &= T_1 e^{-i\gamma} (1 + y_f^*)
\end{align*}
\]

\[
A^{CPT}_{br} = \frac{\langle Br(B_s \rightarrow D_s^+ K^-) \rangle - \langle Br(B_s \rightarrow D_s^- K^+) \rangle}{\langle Br(B_s \rightarrow D_s^+ K^-) \rangle + \langle Br(B_s \rightarrow D_s^- K^+) \rangle} \\
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LHCb at 200 fb$^{-1}$: $\text{Re}(y_f) \sim 0.003$
Triple-product asymmetries

- Consider $B \rightarrow V_1 V_2$

  $B(p) \rightarrow V_1(k_1, \varepsilon_1) + V_2(k_2, \varepsilon_2)$

- Construct $\alpha \equiv \vec{k}_1 \cdot (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$

- The asymmetry

  $\frac{\Gamma(\alpha > 0) - \Gamma(\alpha < 0)}{\Gamma(\alpha > 0) + \Gamma(\alpha < 0)}$

  is odd under the time-reversal operator $T$. If CPT holds, this is a signal of CP violation.

- TP asymmetries should be observables in other systems too.
A lot of TPs are zero in SM but nonzero in BSM with a second amplitude. TPs can be nonzero even if the strong phase difference is zero.

One can also relate the s,p,d wave amplitudes with the so-called transversity amplitudes $A_0$, $A_{||}$, $A_{\perp}$.

Final state decay distributions probe the interference terms of these amplitudes — probe for $T$ violation.

Some asymmetries are zero in SM and CPT conserving BSM but become nonzero in SM + CPTV.

[AK and Patra 2013]
Conclusions

- CPT is supposed to be a good symmetry in any local Lorentz-invariant QFT.
- However, CPT may be violated if LI is broken. LI can be broken by string interactions, noncommutative coordinates, strong gravity ....
- CPTV needs LV, the reverse is not true.
- Various particle physics tests for CPTV: neutrinos, \((g - 2)_{e,\mu}\), etc.
- Strong bounds in K and \(B_d\) systems.
- Time to look in \(B_s\), LHC can uncover such signals. We should stay tuned.

Thank you. Bon appetit.
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\textit{Thank you. Bon appetit.}