Pairing in Degenerate Fermi Gases

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Under the Supervision of

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Certificate

I feel great pleasure in certifying that the thesis entitled "Pairing in Degenerate Fermi Gases" embodies a record of the results of investigations carried out by Silotri Salman Ahmed under my guidance. I am satisfied with the analysis of data, interpretation of results and conclusions drawn.

He has completed the residential requirement as per rules.

I recommend the submission of thesis.

Date:

January 28, 2010

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Declaration

I hereby declare that the work incorporated in the present thesis entitled "Pairing in Degenerate Fermi Gases" is my own work and is original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma.

Silotri Salman Ahmed Ziya Ahmed

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Abstract

The advances in cooling and trapping of atoms present the unique opportunity to study exotic many body phases which were previously elusive in conventional condensed matter systems. In these systems, the inter atomic interaction can be tuned via *Feshbach resonances* and the population of each atomic species can be controlled. In this thesis, we study Cooper pairing in two component degenerate atomic Fermi gases. The superfluid systems with matched Fermi surfaces are well described by celebrated Bardeen-Cooper-Schrieffer (BCS) theory. We discuss, in this framework, the crossover from Bardeen-Cooper-Schrieffer (BCS) limit of weakly bound Cooper pairs of fermionic atoms to the Bose-Einstein condensate (BEC) of diatomic molecules as the strength of the interaction is varied. In presence of mismatched Fermi surfaces, however, the system is proposed to admit variety of exotic superfluid phases.

This mismatch can arise due to population imbalance or the mass difference between the two trapped components or both. We, in particular, study the *breached pairing* phase which is potential candidate as a ground state for such imbalanced systems. In this state, excess unpaired fermions occupy the negative quasi-particle energies thereby minimizing the thermodynamic potential. Moreover, it exhibits gapless modes and is also termed as gapless superfluidity.

We consider a variational ground state for the system of nonrelativistic

fermions with a four fermion point interaction to model the phase structure of the ultracold atomic Fermi mixture with equal and unequal population and the mass. We find that *breached pairing* phase with one Fermi surface which admits only one gapless mode, is the stable phase. This rules out the proposal that mass asymmetry between the pairing components can lead to breached pairing with two Fermi surfaces also referred to as *interior gap* state. We also present the temperature effects on these systems within mean field approximation. The temperature effects are taken into account by thermal Bogoliubov transformation. We then extend the formalism from homogeneous system to trapped systems where present day experiments are carried out.

We study equal mass population imbalanced two-component atomic Fermi gas with unequal trap frequencies ($\omega_{\uparrow} \neq \omega_{\downarrow}$) at zero temperature using the local density approximation (LDA). We consider the strongly attracting Bose-Einstein condensation (BEC) limit where polarized (gapless) superfluid, breached pairing phase with one Fermi surface (BP1), is stable. The system exhibits shell structure: unpolarized superfluid \rightarrow gapless superfluid (BP1) \rightarrow normal state. Compared to the trap symmetric case, when the majority component is tightly confined the gapless superfluid shell grows in size leading to reduced threshold polarization to form a polarized (gapless) superfluid core. In contrast, when the minority component is tightly confined, we find that the superfluid phase is dominated by the unpolarized superfluid phase with the gapless phase forming a narrow shell. The shell radii for various phases as a function of polarization at different values of trap asymmetry are presented and the features are explained using the phase diagram.

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