STUDIES ON NONCOMMUTATIVE FIELD THEORIES

A THESIS
SUBMITTED IN PARTIAL FULFILMENT
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

BY

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TO MY PARENTS

Declaration

I hereby declare that the material presented in this thesis is, the result of investigations carried out by me in the School of Physics, University of Hyderabad, under the supervision of Dr. Prasanta K. Panigrahi (currently at the Physical Research Laboratory, Ahmedabad-380 009).

The results reported in the thesis are new to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

In keeping with the general practice of reporting scientific observations due acknowledgement has been made whenever the work described is based on the findings of other investigators.

T. SHREECHARAN

Place: Hyderabad

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Certificate

This is to certify that the work embodied in this thesis entitled "Studies on noncommutative field theories", has been carried out by Mr. T. Shreecharan, under my supervision and the same has not been submitted in whole or part for a degree in any university.

Dr. PRASANTA K. PANIGRAHI

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