

Binary Mixtures of Ultracold Quantum Gases in Optical Lattices

A thesis submitted in partial fulfilment of
the requirements for the degree of

Doctor of Philosophy

by

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DISCIPLINE OF PHYSICS

INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR

2016

to

my family

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It is certified that the work contained in the thesis titled "**Binary Mixtures of Ultracold Quantum Gases in Optical Lattices**" by Mr. Kuldeep Suthar (Roll No. 11330018), has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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Abstract

The stationary state, and dynamics of the ultracold bosons in optical lattices at zero temperature are well described by the discrete nonlinear Schrödinger equation (DNLSE). This equation is valid in the tight-binding limit, and used for the superfluid (SF) phase of bosons or Bose-Einstein condensate (BEC) in optical lattices. The recent experimental realizations of the binary mixtures of ultracold bosons in optical lattices provide the motivation to study the effects of finite temperatures in the lattice system. We report the development of the coupled DNLSEs, and the Hartree-Fock-Bogoliubov formalism with the Popov (HFB-Popov) approximation for the two-component BECs (TBECs) or binary condensate mixtures of dilute atomic gases in optical lattices. This method is ideal to study the ground state density profiles, and the evolution of the low-lying quasiparticle modes at zero as well as finite temperatures. The thesis can be broadly divided into three parts. The first two parts are results of the zero temperature calculations, which examine the quasiparticle excitation spectra of the TBECs in quasi-1D and quasi-2D optical lattices. The third part deals with the finite temperature results, and pertains to the investigation of the finite temperature effects on the quasiparticle mode evolution of the TBECs. The spontaneous symmetry breaking of $U(1)$ global gauge symmetry results into two Nambu-Goldstone (NG) modes corresponding to each of the species in quasi-1D TBECs. However, at phase separation an extra NG mode emerges with *sandwich* type density profile in the immiscible phase. We investigate the role of quantum fluctuations on the quasiparticle mode evolution for quasi-1D TBECs. In the presence of the fluctuations, an extra NG mode which appears at phase separation gets hardened, and a symmetry broken *side-by-side* density profile appears in the immiscible phase. Furthermore, we examine the ground state geometry, and the quasiparticle spectra of quasi-2D TBECs. We observe that the TBECs acquire the side-by-side geometry when it is tuned from miscible to the immiscible phase. The energies of the quasiparticle modes are softened as the system is tuned towards the phase separation, and harden after phase separation. In the miscible domain the quasiparticle modes are degenerate, and this degeneracy is lifted after the phase separation. Furthermore, in the miscible domain, the quasiparticles have well-defined azimuthal quantum numbers, and hence shows a clear structure in the dispersion curve. On the other hand,

the dispersion curve of the immiscible phase does not have a discernible trend due to the presence of the mode mixing. We also report the enhancement in the miscibility of the condensates of quasi-2D TBEC in the presence of the thermal fluctuations.

Keywords: Bose-Einstein condensation, Multicomponent condensates, Optical lattice, Bose-Hubbard model, Phase separation, Quantum fluctuations.

Abbreviations

BEC	Bose-Einstein condensate
BCS	Bardeen-Cooper-Schrieffer
SF	Superfluid
MI	Mott insulator
ODLRO	Off-diagonal long-range order
TBEC	Two-component Bose-Einstein condensate
BZ	Brillouin zone
BH	Bose-Hubbard
DMRG	Density matrix renormalization group
QMC	Quantum Monte Carlo
DNLSE	Discrete nonlinear Schrödinger equation
BdG	Bogoliubov-de Gennes
HFB	Hartree-Fock-Bogoliubov
GPE	Gross-Pitaevskii equation
TBA	Tight-binding approximation
SSB	Spontaneous symmetry breaking
NG	Nambu-Goldstone
RTI	Rayleigh-Taylor instability
IC	Iteration cycle

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List of Publications

Publications in Journals

1. **K. Suthar**, Arko Roy, and D. Angom

Acoustic radiation from vortex-barrier interaction in atomic Bose-Einstein condensate,

J. Phys. B **47**, 135301 (2014). 9pp, arXiv:1312.7811.

2. **K. Suthar**, Arko Roy, and D. Angom

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4. **K. Suthar** and D. Angom

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Conference Paper

1. S. Gautam, **K. Suthar**, and D. Angom

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