Study of Synchronization in Coupled Dynamical Systems

A THESIS

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by

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Year of Submission: 2012

To

My Family

DECLARATION

I ***Mr. Suman Acharyya****, S/o Mr. Debiprasad Acharyya, resident of C-9, PRL residences, Navrangpura, Ahmedabad 380009, hereby declare that the work incorporated in the present thesis entitled,* ***"Study of synchronization in coupled dynamical systems"*** *is my own and original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma.*

Date :

(Suman Acharyya)

CERTIFICATE

I feel great pleasure in certifying that the thesis entitled, **"Study of synchronization in coupled dynamical systems"** embodies a record of the results of investigations carried out by Mr. Suman Acharyya under my guidance.

He has completed the following requirements as per Ph.D. regulations of the University.

(a) Course work as per the university rules.

(b) Residential requirements of the university.

(c) Presented his work in the departmental committee.

(d) Published minimum of two research papers in a referred research journal.

I am satisfied with the analysis of data, interpretation of results and conclusions drawn.

I recommend the submission of thesis.

Date :

R. E. Amritkar
Professor
(Thesis Supervisor)

Countersigned by
Head of the Department

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– Suman Acharyya

Abstract

The work of this thesis can be divided into two parts. In the first part we study synchronization of coupled nonidentical dynamical systems and in the later part we analyze the desynchronization bifurcation of coupled dynamical systems.

When two or more identical systems are coupled then synchronization comes out as equality of the state variables of the coupled systems, which is known as complete (or identical) synchronization (CS). The conditions for stability of complete synchronization are well analyzed by the Master Stability Function (MSF). For coupled nonidentical systems it is not possible to get complete synchronization, instead one can find out a functional relationship between the state variables of the coupled systems which is known as generalized synchronization (GS). In this thesis we develop a theory to construct an approximate MSF for determining stability of GS for coupled nonidentical systems. Next, by using the stability criteria provided by the MSF we construct synchronized optimized network by rewiring the links of a given network. In the optimized network the nodes which have extreme values (maximum or minimum depending on the nature of MSF) of parameter mismatch are chosen as hubs and the pair of nodes having larger parameter mismatch are chosen to create links.

In the second part of this thesis we study desynchronization bifurcation of coupled dynamical systems. In some coupled dynamical systems one can find an interval of coupling strength where the synchronized state is stable. When the coupling strength is increased beyond this stable region, the synchronized state becomes unstable and the coupled systems undergo desynchronization bifurcation. We analyze this desynchronization bifurcation in coupled chaotic systems and we observe that this desynchronization bifurcation is pitchfork bifurcation of transverse manifold. We propose an integrable model which shows similar desynchronization bifurcation. In this context we propose Systems' Transverse Lyapunov Exponents (STLE) for determining the stability of individual systems in a network.

Keywords: Coupled Systems, Synchronization, Master Stability Function, Optimization, Desynchronization Bifurcation.

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