# Vortices of Light and their Interaction with Matter 

## A thesis submitted in partial fulfillment of the requirements for the degree of

## Doctor of Philosophy

by

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DISCIPLINE OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR

## to <br> my family

## Declaration

I declare that this written submission represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above can cause disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

## Thesis Approval

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Place: PRL, Ahmedabad

## Acknowledgements

This thesis is the result of a very nice and revealing research time during 2009-14 at Physical Research Laboratory (PRL), Ahmedabad. It gives me a great pleasure to acknowledge all those who supported me directly or indirectly and made this thesis possible.

First and foremost, I wish to express my sincere thanks to my supervisor, Prof. R. P. Singh, for giving me the opportunity and the resources to conduct my doctoral research program at PRL. I owe my deep sense of gratitude to him. I am also very thankful to him for constant guidance that he has provided throughout my research endeavor. He helped in my study to large extent from basics of optics to image analysis as well as from independent thinking to scientific writing. His persistent support, far-sighted guidance and broad scientific experience made my research work possible. It is a great privilege for me to get a companionship with him. I consider myself fortunate to have him as my supervisor.

I thank Prof. J. N. Goswami, Director; Mr. Y. M. Trivedi and Wg. Cdr. (Retd.) Vibhas Gupta, Registrar; Prof. A. Joshipura, Prof. U. Sarkar, Prof. A. K. Singhvi and Prof. S. Rindani, former Deans of PRL; Prof. Prof. P. Janardhan, present Dean; Prof. S. Rindani, former area chairman and Prof. S. Mohanty current area chairman, for providing necessary facilities to carry out research work. I thank Dr. Goutam K. Samanta and Prof. Jitesh R. Bhatt for reviewing the work and motivating constantly during these five years.

I am highly obliged to Prof. J. Banerji for his suggestions and critical comments which helped me to improve scientific outcomes. The fruitful discussions with him always helped me to improve the quality of the scientific results and manuscripts. A big thank you for Dr. Goutam K. Samanta who has helped in keeping the motivation for perfectness inside me. Discussions on fundamentals and experimental techniques with Dr. Samanta was always enjoyable and fruitful.

I thank Prof. J. S. Ray, Prof. Angom D. Singh, Dr. D. Banerjee, Dr. N. Mahajan, Prof. R. Rangarajan, Mr. T. Sarvaiya, Prof. H. Mishra and Prof. K. P. Subramanian for teaching the courses during my course work at PRL which turned out to be very useful in tenure of research work. In this petabyte era of computational capabilities, data handling and computations have been a very crucial but tedious task. In this regard, I thank all the staff members of PRL computer center for providing excellent computational and internet facilities. Especially, I must acknowledge the help received from Jigar Raval, Samuel Johnson, Hitendra Mishra, Alok Srivastava and Tejas Sarvaiya from computer center. In addition, I express my appreciation for the co-operation and help extended by the library and
its staff members in general and Mrs. Nistha Anilkumar and Pragya Pandey in particular.

I take this opportunity to thank my seniors, Ashok Kumar and Pravin Vaity, who introduced me to the world of Optics and paved a smooth way for further research in this area. Sincere discussion with them and sometimes "not" receiving ready made answers helped me a lot in understanding the subject. The work culture maintained by Ashok and Pravin in the lab was highly motivational. I am also thankful to Sunita Kedia from whom I have learnt "not busy" attitude. I have learnt a lot from Virendra Jaiswal and Jitendra Bhatt, whose occasional visits to lab were very informative.

I am thankful to all my colleagues, Gangi, Aadhi, Chithra, Apurv, Vinayak, Jabir, Ali and Vinayak in Lab for being friendly and co-operative during this research journey. Our academic and non-academic discussions have made my stay enjoyable and eventful in lab. I have learned a lot about different fields in which my colleagues are working. In one way, I have learnt about different fields, and not just one in the last five years of my life. I would like to thank the visitors which include summer and engineer trainees, particularly to Aabhaas, Shaival, Kinchit, Binal, Chirag, Mehul, Gaurav, Megh and others.

My special thanks and gratitude goes to my batchmates Devagnik, Fazlul, Kabitri and Tanushree. Their presence has made me feel that I am not alone and someone is there to care for me during odd days. Special thanks to Fazlul who has helped me in some of my experiments. He has also helped me in numerical techniques and computer related issues.

I would like to thank Prof. Sudhir K. Jain, Director; Prof. Amit Prashant, Dean; Prof. B. Datta, associate Dean; and the academic section of Indian Institute of Technology, Gandhinagar for the support and help. Special thanks to Mr. Piyush for his prompt response and help during registration.

This acknowledgement section remains incomplete if there is no mention of my wonderful seniors cum friends who were always there to support and help at my stay in hostel. My sportsman spirit became alive after I saw them playing volley ball, badminton and table tennis. My seniors Arun, Amrendra, Sunil, Siddharth, Koushik, Chinmay, Sushant and Yogita have helped to keep my loneliness away from me. I am also thankful to Ashwini, Arvind Singh, Pankaj, Vineet, Neeraj, Prashant, Bhaswar, Sandeep, Vimal, Ketan, Sudhanwa, Suman, Amzad, Srikant, Satinder, Arvind Saxena for making my stay comfortable. In addition, I express my sincere thanks to my dear juniors Avdhesh, Damu, Lekshmy, Midhun, Anjali, Arko, Bhavya, Dillip, Gaurav, both Girish, Gulab, Monojit, Naveen, Priyanka, both Tanmoy, Yashpal, Dinesh, Upendra, Wageesh, Arun, Gaurava,

Abhaya, Anirban, Guru, Ikshu, Kuldeep, Manu, Shraddha, Alok, Sanjay, Bivin, Deepak, Dipti, Jinia, Lalit, Rahul, Ashim, Chandana, Newton, Pankaj, Venky, Chandan, Hemant, Kuldeep, Venkata, Navpreet, Prahlad, Satish, Rukmani, Rupa, Yasir, Komal, Manish, Vaidehi, Sneha, Upasana, Lijo and several others who made my stay in PRL pleasant and having a wonderful experience.

This thesis is incomplete if I do not acknowledge "Google". It has always made my life easier like a friend. Google has not put in such a situation that I have searched something and there was zero result(s). I also thank open-source packages like XUbuntu, Firefox, Zotero, Texmaker, PDFtk and others which had made my work very simple.

I am deeply indebted to my school teachers, specially Ms. Lipika Paul for excellent teaching and encouragement. A special thanks goes to my long time friends Gautam and Tribedi for their constant support and encouragements. Their constant push and motivational chats have helped me to recover from odd times.

Last, but not the least, I would be nowhere around here without the love and support of my parents. Words cannot express my gratitude for my parents, cousins and my family members. Finally, I must also acknowledge the love, support and blessings of friends, teachers, relatives and all known and unknown persons whom I met with or interacted with in life.


#### Abstract

Optical vortices are singularities in the phase distribution of a light field. At the phase singularity, real and imaginary parts of the field vanish simultaneously and associated wavefront becomes helical. For an optical vortex of topological charge $l$, there are $l$ number of helical windings in a given wavelength $\lambda$ of light and it carries an orbital angular momentum of $l \hbar$ per photon. This dissertation concerns with the study of interaction of optical vortices with matter namely nonlinear optical crystal $\beta$-Barium Borate (BBO) and Bose-Einstein condensate.

A new method to determine the order of optical vortex from just the intensity distribution of a vortex has been discussed. We show that the number of dark rings in the Fourier transform (FT) of the intensity can provide us the order. To magnify the effect of FT, we have used the orthogonality of Laguerre polynomials.

We have studied the interaction of optical vortices with BBO crystal. The spatial-distribution of degenerate spontaneous parametric down-converted (SPDC) photon pairs produced by pumping type-I BBO crystal with optical vortices has been discussed. For a Gaussian pump beam, we observe a linear increase in thickness of the SPDC ring with pump size. On the other hand, the SPDC ring due to optical vortex forms two concentric bright rings with an intensity minimum in the middle. We also observe that if the beam size is lower than a particular value for a given topological charge $l$ of the vortex, then there will be no change in full-width at half maximum of the rings formed by down-converted photons.

We have experimentally verified the quantum inspired optical entanglement for classical optical vortex beams. The extent of violation of Bell's inequality for continuous variables written in terms of the WDF increases with the increase in their topological charge. To obtain this, we have used the FT of two-point correlation function that provides us the WDF of such beams.

Quantum elliptic vortex (QEV) is generated by coupling two squeezed vacuum modes with a beam splitter (BS). The Wigner distribution function (WDF) has been used to study the properties of this quantum state. We also study how this coupling could be used to generate controlled entanglement for the application towards quantum computation and quantum information. We observe a critical


point above which the increase in vorticity decreases the entanglement.
We have also studied the annihilation of vortex dipoles in Bose-Einstein condensates. To analyze this, we consider a model system where the vortexantivortex pair and gray soliton generated by annihilation of vortex dipoles are static and the system could be studied within Thomas-Fermi (TF) approximation. It is observed that the vortex dipole annihilation is critically dependent on the initial conditions for their nucleation. Noise, thermal fluctuations and dissipation destroy the superflow reflection symmetry around the vortex and antivortex pair. It is note worthy that some of our theoretical results have already been verified experimentally.

Keywords: Singular optics, Optical vortex, Spontaneous parametric downconversion, entanglement, Wigner distribution function, Bell's inequality, Vortex dipole annihilation, Bose-Einstein condensates.

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