Vortices of Light and their Interaction with Matter

A thesis submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

by

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DISCIPLINE OF PHYSICS

INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR

2014 - 2015

to my family

Declaration

I declare that this written submission represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above can cause disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

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Thesis Approval

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Abstract

Optical vortices are singularities in the phase distribution of a light field. At the phase singularity, real and imaginary parts of the field vanish simultaneously and associated wavefront becomes helical. For an optical vortex of topological charge l, there are l number of helical windings in a given wavelength λ of light and it carries an orbital angular momentum of $l\hbar$ per photon. This dissertation concerns with the study of interaction of optical vortices with matter namely nonlinear optical crystal β -Barium Borate (BBO) and Bose-Einstein condensate.

A new method to determine the order of optical vortex from just the intensity distribution of a vortex has been discussed. We show that the number of dark rings in the Fourier transform (FT) of the intensity can provide us the order. To magnify the effect of FT, we have used the orthogonality of Laguerre polynomials.

We have studied the interaction of optical vortices with BBO crystal. The spatial-distribution of degenerate spontaneous parametric down-converted (SPDC) photon pairs produced by pumping type-I BBO crystal with optical vortices has been discussed. For a Gaussian pump beam, we observe a linear increase in thickness of the SPDC ring with pump size. On the other hand, the SPDC ring due to optical vortex forms two concentric bright rings with an intensity minimum in the middle. We also observe that if the beam size is lower than a particular value for a given topological charge l of the vortex, then there will be no change in full-width at half maximum of the rings formed by down-converted photons.

We have experimentally verified the quantum inspired optical entanglement for classical optical vortex beams. The extent of violation of Bell's inequality for continuous variables written in terms of the WDF increases with the increase in their topological charge. To obtain this, we have used the FT of two-point correlation function that provides us the WDF of such beams.

Quantum elliptic vortex (QEV) is generated by coupling two squeezed vacuum modes with a beam splitter (BS). The Wigner distribution function (WDF) has been used to study the properties of this quantum state. We also study how this coupling could be used to generate controlled entanglement for the application towards quantum computation and quantum information. We observe a critical point above which the increase in vorticity decreases the entanglement.

We have also studied the annihilation of vortex dipoles in Bose-Einstein condensates. To analyze this, we consider a model system where the vortexantivortex pair and gray soliton generated by annihilation of vortex dipoles are static and the system could be studied within Thomas-Fermi (TF) approximation. It is observed that the vortex dipole annihilation is critically dependent on the initial conditions for their nucleation. Noise, thermal fluctuations and dissipation destroy the superflow reflection symmetry around the vortex and antivortex pair. It is note worthy that some of our theoretical results have already been verified experimentally.

Keywords: Singular optics, Optical vortex, Spontaneous parametric downconversion, entanglement, Wigner distribution function, Bell's inequality, Vortex dipole annihilation, Bose-Einstein condensates.

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