

DIRECTED TRANSPORT IN CHAOTIC HAMILTONIAN SYSTEMS

A THESIS

**submitted for the award of Ph.D. degree of
MOHANLAL SUKHADIA UNIVERSITY**

**in the
Faculty of Science**

**by
Harinder Pal**



**Under the Supervision of
Dr. Madabushi Srinivasan Santhanam**

Ex-Reader

Physical Research Laboratory, Ahmedabad, India

and

Co-Supervisor

Dr. Angom Dilip Kumar Singh

Associate Professor

Physical Research Laboratory, Ahmedabad, India

**DEPARTMENT OF PHYSICS
MOHANLAL SUKHADIA UNIVERSITY
UDAIPUR
2011**

To

My family

CERTIFICATE

I feel great pleasure in certifying that the thesis entitled, “**Directed Transport in Chaotic Hamiltonian Systems**” embodies a record of the results of investigations carried out by Harinder Pal under my guidance.

He has completed the following requirements as per Ph.D. regulations of the University.

- (a) Course work as per the university rules.
- (b) Residential requirements of the university.
- (c) Presented his work in the departmental committee.
- (d) Published/accepted minimum of two research papers in a referred research journal.

I am satisfied with the analysis of data, interpretation of results and conclusions drawn.

I recommend the submission of thesis.

Date :

Dr. Madabushi Srinivasan Santhanam
(Thesis Advisor)

Countersigned by
Head of the Department

DECLARATION

*I, **Harinder Pal**, S/O Mr. Harbans Lal, resident of A-2, PRL Residences, Navrangpura, Ahmedabad - 380 009, hereby declare that the work incorporated in the present thesis entitled, “**Directed Transport in Chaotic Hamiltonian Systems**” is my own and original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma.*

Date :

(Harinder Pal)

Acknowledgments

First and foremost, I offer my sincerest gratitude to my supervisor, Dr. M. S. Santhanam, who has supported me throughout my thesis with his knowledge and patience. I am grateful to him for providing me unflinching encouragement and guidance during all these years. His openness to new ideas and intellectual support for their pursuance has made research life rewarding for me. I thank him for his efforts to educate me with the concepts and tools necessary for the work in this thesis.

I owe special gratitude to my co-supervisor, Dr. Dilip Angom. I extend my warmest thanks to him for many useful discussions, encouragement and support. I am grateful to him for his kind concern regarding my academic requirements.

I thank Dr. Bimalendu Deb for giving an encouraging start to my research life by guiding me in my coursework project. I would also like to thank Prof. J. Banerji, Dr. Jitesh Bhatt, Dr. Dilip Angom, Dr. M. S. Santhanam, Prof. Shyam Lal, Prof. R Sekar, Prof. S. A. Haider, Dr. Varun Sheel and Dr. A. K. Singal for teaching various courses and making coursework an enjoyable learning experience. I extend my thanks to Prof. P. K. Panigrahi for many informal but stimulating discussions during my early days in Physical Research Laboratory. I sincerely thank Prof. Sushanta Dattagupta and Prof. Steven Tomsovic for giving me their valuable time and sharing some ideas during their visit to PRL. Special thanks are due to Dr. R. Sankaranarayanan who visited PRL during starting phase of my doctoral research and discussed his work on generalized standard map. I would also like to thank Prof. Arul Lakshminarayan for some useful discussions during International conference on "Recent Developments in Non-linear Dynamic" held in Tiruchirapalli in 2008.

I am extremely grateful to PRL for various facilities it has provided, without which this thesis would not have been possible. I would also like to thank all the staff members of library, computer center, administration, maintenance and medical dispensary for their support in making efficient use of all the facilities possible. I would like to specially thank Mrs. Nishtha AnilKumar, Mrs. Pauline Joseph, Dr.

Sheetal Patel, Dr. Bhushit, Mrs. Parul Parikh, Miss Jayshree, Mr. Keyur Shah, Mr. Gangadharia, Mr. B. M. Joshi, Mr. Ghanshyam Patel, Mr. Ranganathan, Miss Pragya, and Mr. N. P. M. Nair for always being happy to help. I acknowledge the extensive use of High Performance Computing cluster facility for my thesis work. I am thankful to Mr. G. G. Dholakia, Mr. D. V. Subhedar, Mr. Jigar Raval, Mr. Hitendra, Mr. Tejas, Mr. Alok and Mr. Mrugesh Gajjar for their prompt actions to my computation related requests. I would like to extend my warmest regards for Mr. Manjunatha and Mr. Anish of CDAC, Pune for their help in using various packages on HPC. Their zeal to deal with HPC related issues made my life easy. I am also grateful to IISER, Pune for its hospitality during my visits for thesis related work. I also thank the staff of MLSU, Udaipur for their kind co-operation and prompt services.

I would like to thank all my friends in PRL for their affectionate company and best wishes, as well as express my apology that I could not mention names of all of them because they are just too many. Vimal and Suman have been good “school-mates”. Their company during the schools and conferences we have attended together made those learning periods more enjoyable. I would like to thank Suman also for helping me out whenever I had some software or operating system related issues. I also thank my senior Dr. Rajneesh Atre for his help and guidance.

I express my deepest thanks to my parents for their untiring efforts and encouragement throughout my studies. I also thank my sisters, Sunita and Sandeep, for their loving support. I express my sincere gratitude to my elder sister Sunita for her guidance during my schooling and also for encouraging me for higher education. I am deeply thankful to my wife Manan whose understanding, patience and unconditional support has kept my spirits up through thick and thin.

Finally, I feel great pleasure to express my gratitude to all others who have directly or indirectly contributed to this thesis.

ABSTRACT

We study the dynamics and directed transport in a class of chaotic Hamiltonian systems. The system we consider is a δ -kicked particle in the presence of (i) double-barrier potential and (ii) periodic lattice of square-well potentials. In contrast to the well studied kicked rotor, the kicked system, in the presence of two variants of square-well potentials, studied in this thesis does not obey the Kolmogorov-Arnold-Moser (KAM) theorem. Due to this, invariant curves are absent and instead the phase space displays intricate chains of islands and fully connected chaotic layer even for very small kick strength. However, a special feature of the system reported in this thesis is that, inspite of being a non-KAM system, dynamics is KAM-like in some regions of phase space. We study the effect of interplay between of non-KAM and KAM-like phase space dynamics on dynamical properties of the system. We report a number of novel and interesting dynamical features like (a) the classically induced suppression of energy growth, (b) non-equilibrium steady state and (c) momentum filtering effect. We also report results for the quantum analogues of these dynamical features.

To study the directed transport properties of the system, we study evolution of a set of initial states. We study the effect of spatio-temporal symmetries on net current of a set of states. We observe that the system shows ratchet effect, *i.e.*, directed current in absence of net bias, upon breaking certain spatio-temporal symmetries. We explain how the non-KAM nature of the system imparts some useful characteristics to it as ratchet model. Throughout this work, we also analyse the quantum dynamics of the system, mainly in the semiclassical regime, and study the consequences of quantum effects. We also show that the system can act as a quantum ratchet.

Contents

Certificate	ii
Declaration	iii
Acknowledgments	iv
Abstract	vi
List of Figures	x
1 Introduction	1
1.1 The Ratchets and Their History	2
1.2 Deterministic Ratchets	4
1.3 Kicked Particle System	5
1.4 Quantum Ratchets	10
1.5 Motivation	11
2 Kicked Particle in a Double-barrier Structure: The Phase Space Dy-	
namics	14
2.1 The System	15

2.2	The Classical Map	18
2.3	Phase Space Dynamics	22
2.4	KAM-like Behavior: Role of Symmetries	29
2.5	Quantum Dynamics	30
3	Kicked Particle in a Double-barrier Structure: Dynamical Features	35
3.1	Classically Induced Suppression of Energy Growth	36
3.1.1	Mechanism of Escape from DBS	36
3.1.2	Saturation of Energy Growth	37
3.1.3	Behavior for Large Kick Strengths	43
3.2	Momentum Squeezing	45
3.3	Pumping Action	51
3.4	Non-equilibrium Steady State	52
4	Kicked Particle in a Lattice of Finite Wells: The Classical Ratchet	58
4.1	The System	59
4.2	The Phase Space Unit Cell	60
4.3	The Classical Map	62
4.4	Phase Space Features	66
4.5	The Classical Ratchet Effect	72
4.5.1	The Lattice of Double Square Wells	73
4.5.2	The Effect of Spatio Temporal Symmetries	74
4.5.3	Effect of barrier width	78
5	Kicked Particle in a Lattice of Finite Wells: The Quantum Ratchet	81
5.1	The Unperturbed System	81
5.2	The Kicked System	87
5.3	The Quantum ‘Phase Space’	94
5.4	Quantum Ratchet Current	100

6	Summary and Future Directions	105
6.1	Summary	105
6.2	Future Directions	108
A	Effect of Barrier-width on Refraction	110
B	KAM-like Behavior: Effect of (R, ϕ)	112
	Bibliography	114
	List of Publications	121

List of Figures

1.1	Phase space of standard map for different values of kick strength, ϵ . The kick strengths are (a) 0.15, (b) 1.0, (c) 4.5, and (d) 10. Gradual transition from regular to chaotic dynamics occur with increasing ϵ	8
2.1	Grey line shows the periodic lattice of identical equally spaced finite wells. Dotted lines superimposed on it show two kinds of periodic units. Solid black line shows the double barrier structure.	15
2.2	Schematic of the stationary part of the potential: The double-barrier structure	16
2.3	Stroboscopic Poincare section (black) for $R = 0.95, \epsilon = 0.15, V_0 = 0.5, \phi = 0$ and $b = 0.5$. All the continuous curves (in color) marked C_1 to C_6 are for the corresponding standard map with kick strength 0.15. The black box at position $x = \pm x_w$ indicates the width b of the barrier. The solid circles (in red) show a trajectory starting from A_1 until it exits the potential well at A_9 . The time ordered sequence of the trajectory is A_1 to A_2 , reflection at $-x_w$, A_3 to A_4 , reflection at x_w , A_5 to A_6 , cross the boundary at x_w , A_7 to A_8 , cross the boundary at $x_w + b$, exit the potential at A_9 . See text for details. Open red circles show trajectory of a particle with initial state at B_1 evolving to B_2	22

- 2.4 Figure shows schematically the effect of refraction at the barrier. Grey boxes represent the barrier region in phase space. The three phase points a, b and c represent the states of particle evolving on $C(\mu_5)$ starting from a in absence of barriers. Red arrows connects the phase point c , the evolving state would reach in absence of barrier, with the point d or e it actually reaches in presence of barrier after evolving between the kicks. 25
- 2.5 Stroboscopic plot for $b = 10^{-5}, R = 0.7, \phi = 0, \epsilon = 0.15, V_0 = 0.5$. Dashed line (in red) represents the boundary of region \mathcal{M} . The scatter of points between $C_+(\mu_c)$ and $C_+(\mu_{th})$ on right and between $C_-(\mu_c)$ and $C_-(\mu_{th})$ on left side of the DBS represent the particles escaping out of the well (whose initial states were in \mathcal{M}). 26
- 2.6 Stroboscopic plot for $R = 0.5$. All the other parameters are same as in Fig. 2.5. Dashed line (in red) represents the boundary of region \mathcal{M} . The scatter of points between $C_+(\mu_c)$ and $C_+(\mu_{th})$ on right and between $C_-(\mu_c)$ and $C_-(\mu_{th})$ on left side of the DBS represent the particles escaping out of the well (whose initial states were in \mathcal{M}). 26
- 2.7 Stroboscopic Poincare section for the Hamiltonian in Eq. (2.1) showing the region $x \in (-x_l, x_r), p \in (-p_c, p_c)$ for $b = 0, \epsilon = 0.15, V_0 = 0.5$. The other parameters are (a) $R = 0.95, \phi = 0$ (b) $R = 1.0, \phi = 0$, (c) $R = 1.05, \phi = 0$, (d) $R = 0.45, \phi = \pi/2$, (e) $R = 0.5, \phi = \pi/2$ and (f) $R = 0.55, \phi = \pi/2$ 31

- 2.8 (Top) Husimi distribution for evolved wave packet. Initial wave function corresponds to $Q(x_0, p_0, n)$ sharply localized inside chaotic region around $(0, 0)$. In Grey scale version, grossly the darker areas represent the region with larger value of Husimi distribution function (for figure at the bottom as well). It shows that the Husimi function decays very steeply outside $[x_{-w}, x_w]$ and acquires negligible values compared to those for region inside $[x_{-w}, x_w]$. We have taken $\hbar_s = 0.0025$, $R = 0.85$, $b = 0.2$, $\epsilon = 0.15$, $V_0 = 0.5$, $\phi = 0$. (Bottom) Enlarged and better resolved view of inset from figure on the top shows path followed by probability density outside the barrier region. 34
- 3.1 Stroboscopic section for $R = 0.5$, $\epsilon = 0.3$, $b = 0.2$, $V_0 = 0.5$ and $\phi = 0$. Lower and upper limits on momenta of escaped particles are represented by p_{min} and p_{max} , respectively. The width of momentum band in which escaped particles lie is given denoted with Δp . The momentum span of trajectories followed by escaped particles at a fixed value of x is represented by Δp_x . Grey strip around $p = 0$ is the region in which all the initial states were distributed uniformly. 38
- 3.2 Distribution of states in momentum space at (a) $n = 0$, (b) $n = 50$, (c) $n = 100$ corresponding to parameters and initial set of states used in Fig. 3.1. 39
- 3.3 Classical (black) and quantum (red) momentum distribution at different times corresponding to parameters and initial states used in Fig. 3.1. For quantum simulation $\hbar_s = 0.0025$. These nearly identical distributions indicate that the evolved distribution has very well converged to a steady state. 41

- 3.4 Classical (solid line) and quantum (dashed line) $\langle E \rangle$ as function of time n . Numerically estimated value of $\langle E \rangle_s$ for classical system is shown through horizontal line. Parameters are same as those for Fig. 3.3. The triangles in the x -axis are the times for which momentum distribution is drawn if Fig. 3.3 41
- 3.5 (Top) $\langle E \rangle$ vs time n for (a) full chaos between the barriers ($R = 0.5$) and (b) mixed phase space ($R = 0.8$). (Bottom) Number of particles remained inside the well region N_n vs time n for (a) full chaos and (b) mixed phase space between the barriers. Other parameters are: $\epsilon = 0.2, b = 0.2, V_0 = 0.5$ and $\phi = 0$ 42
- 3.6 $\langle E \rangle$ vs n for (brown) $\epsilon = 0.1$, (red) $\epsilon = 0.2$ and (green) $\epsilon = 0.3$. Other parameters are: $R = 0.5, b = 0.2, V_0 = 0.5$ and $\phi = 0$ 43
- 3.7 (Black line) Theoretical $\langle E \rangle$ vs n for standard map for $\epsilon = 5$. (Red circles) Numerically calculated $\langle E \rangle$ vs n for system defined in Eq.(2.6) for same value R . Other parameters for second case (red circle): $R = 0.9, b = 0.2, V_0 = 0.5$ and $\phi = 0$ 44
- 3.8 Classical (solid black line) and quantum (dashed line) momentum distributions at $n = 700$ for $R = 0.5, b = 0.2, \epsilon = 0.1, V_0 = 0.5$ and $\phi = 0$. For quantum simulation $\hbar_s = 0.0025$. Initial momentum distribution is uniform as shown by rectangular blue curve. 45
- 3.9 Evolved momentum distribution at $n = 5000$ for parameters and initial state same as used in Fig. 3.8. Zero density between two peaks indicate that all the particles have escaped from DBS. 46

- 3.10 (Left) Stroboscopic section for $R = 0.95, b = 0.2, \epsilon = 0.1, V_0 = 0.5$ and $\phi = 0$. The phase space between the barriers is mixed phase space. However, the chaotic region is empty because after long evolution all the chaotic particles have escaped. Two sets of initial states (i) uniformly distributed in brown box (ii) uniformly distributed on purple line, are used to get evolved momentum distribution at $n = 50000$. The section corresponds to second set. (Right) Evolved momentum distribution at $n = 50000$ for first set of initial states is shown in green and for second set it is shown in red. 47
- 3.11 Evolved momentum distribution for $\epsilon = 0.05$ (blue), $\epsilon = 0.1$ (green) and $\epsilon = 0.2$ (red). Other parameters are: $R = 0.5, b = 0.2, V_0 = 0.5$ and $\phi = 0$ 48
- 3.12 Evolved momentum distribution at $n = 1000$ for $V_0 = 0.5$ (red), $V_0 = 1.25$ (green). Other parameters are: $R = 0.5, b = 0.2, \epsilon = 0.1$ and $\phi = 0$. The threshold momenta corresponding to $V_0 = 0.5$ and $V_0 = 1.25$ are marked as p_{c1} and p_{c2} , respectively. 49
- 3.13 Evolved momentum distribution at time n scaled with p_c for two values of V_0 corresponding to $p_c = 1$ (red) and $p_c = 1 + 4\pi$ (green). Since the time taken for sufficient number of particles to escape from the DBS, so that their distribution can analyzed, is longer for larger p_c , we take $n = 5000$ for $p_c = 1 + 4\pi$ and $n = 1000$ for $p_c = 1$. Other parameters are: $R = 0.5, b = 0.2, \epsilon = 0.1$ and $\phi = 0$ 50
- 3.14 Green curves shows evolved distribution of $\langle p \rangle_{\Delta_n}$ (refer to text for its definition) at $n = 5000$ for parameters corresponding to Fig. 3.3 (left) and at $n = 50000$ for parameters corresponding to Fig. 3.10 (right). Red curves show corresponding evolved momentum distributions. 51
- 3.15 Initial (brown) and evolved (black) momentum distribution for $R = 1.0, b = 0.2, \epsilon = 0.1, V_0 = 0.5$ and $\phi = 0.5$. Asymmetric momentum distribution indicates net transport. 53

3.16	Stroboscopic section for $R = 1.0, b = 0.2, \epsilon = 0.1, V_0 = 0.5$ and $\phi = 0.5$. For non-zero ϕ the phase space is also asymmetric.	53
3.17	Phase space distribution of unescaped particles at (a) $n = 0$, (b) $n = 20$, (c) $n = 80$, (d) $n = 150$ for $R = 0.5, b = 10^{-5}, \epsilon = 0.15, V_0 = 0.5$ and $\phi = 0.0$. Nearly identical distributions in (c) and (d) indicate the non-equilibrium steady state.	54
3.18	Nonequilibrium steady state in the system in Hamiltonian 2.6. The solid lines are the classical results and the symbols correspond to quantum results. The mean energy for the particles held in between the double-barrier structure $\langle E \rangle_{in}$ saturates to different constants for different values of ϵ . The other parameters are $R = 0.5, b = 10^{-5}, V_0 = 0.5$ and $\phi = 0.0$ and for quantum simulations $\hbar_s = 0.0025$. The solid symbol (triangle up) marks the time scale τ_r at which the system relaxes to the steady state. . .	56
3.19	Classical steady-state momentum distribution for $\epsilon = 0.25$ at $n = 100$ solid and $n = 200$ (dashed red). The other parameters are $R = 0.5, b = 10^{-5}, V_0 = 0.5$ and $\phi = 0.0$	56
4.1	Schematic of one dimensional lattice of finite square wells.	59
4.2	Periodic series of δ -kicks. Positions of vertical line represent the times at which kicks act. $\nabla^b(0)$ and $\nabla^a(0)$ represent the durations of free evolutions before and after the kick for $\tau = 0$. $\nabla^b(\tau_1)$ and $\nabla^a(\tau_1)$ represent the durations of free evolutions before and after the kick for $\tau = \tau_1$	64
4.3	Periodic cycle of δ -kicks. Each cycle contains two kicks of unequal strength. Positions of vertical line represent the times at which kicks act and their heights represent their strengths. The sequences of the kicking and free evolution parts are shown for $\tau = 0$ and $\tau = \tau_1$	66
4.4	Stroboscopic section corresponding to $\tau = 0$, i.e. $S(0.0)$, for $b = 0.2, \epsilon = 0.15, \phi = 0$ and $V_0 = 0.5$. System exhibits mixed dynamics in all parts of phase space.	67

- 4.5 Evolving set of initial states shown on Stroboscopic sections $S(0.0)$ for parameters same as for Fig. 4.4 at (a) $n = 0$, (b) $n = 50000$, (c) $n = 100000$, (d) $n = 200000$. For this all the initial states were taken on a small region around $(0,0)$ as shown in (a). 69
- 4.6 Due to continual spread in phase space as shown in Fig. 4.5, the mean energy $\langle E \rangle$ of all the particles keeps increasing. Parameters are same as in Figs. 4.4 and 4.5. 69
- 4.7 Stroboscopic section $S(0.0)$, for $b = 10^{-5}$, $\epsilon = 0.15$, $\phi = 0.0$ and $V_0 = 0.5$. It shows that the phase space is regular if $b \rightarrow 0$, as well as $\phi = 0$. . . 70
- 4.8 Stroboscopic section $S(0.0)$, for $b = 10^{-5}$, $\epsilon = 0.15$, $\phi = 0.5$ and $V_0 = 0.5$. It shows that the phase space comprises mixed dynamics region trapped between two regular regions for $b \rightarrow 0$ 71
- 4.9 Due to trapping of mixed phase space region between the regular regions (see Fig. 4.7), the growth of energy gets arrested after some finite time of evolution. Parameters for this figure are same those for Fig. 4.7 71
- 4.10 Schematic of lattice of double square wells. Barrier width b is taken to be negligible. Two consecutive finite square wells define one periodic unit. The length d of a periodic unit of this new stationary potential is twice the wavelength λ of kicking field. 74
- 4.11 Stroboscopic section $S(0.0)$, for $b = 10^{-5}$, $\epsilon = 0.15$, $\phi = 0$ and $V_0 = 0.5$. The corresponding stationary potential is shown in Fig. 4.10. 74
- 4.12 $\langle p \rangle$ vs n for ensemble of initial states all lying in chaotic layer for spatially symmetric (red) and spatially asymmetric (green) system. Temporal symmetry is maintained in both the cases. $\langle p \rangle$ is calculated on $S(0.5)$, ie. corresponding to $\tau = 0.5$. Parameters are $b = 10^{-5}$, $V_0 = 0.5$, $\epsilon = 0.2$, $\phi = 0.5$ 75

- 4.13 $\langle p \rangle$ vs n for ensemble of initial states all lying in chaotic layer for spatially asymmetric but T-symmetric case corresponding to different values of τ . Saturated values of $\langle p \rangle$ are equal and opposite for equal and opposite values of τ . Parameters are $b = 10^{-5}$, $V_0 = 0.5$, $\epsilon = 0.15$, $\phi = 0.5$ 77
- 4.14 $\langle p \rangle$ vs n for ensemble of initial states all lying in chaotic layer for broken spatial and temporal symmetry corresponding to different values of τ . Saturated values of $\langle p \rangle$ are not equal and opposite for equal and opposite values of τ . Parameters are $b = 10^{-5}$, $V_0 = 0.5$, $\epsilon_1 = 0.08$, $\epsilon_2 = 0.16$, $\phi = 0.5$. 77
- 4.15 $\langle p \rangle$ vs τ for time-symmetric (left) and time-asymmetric (right) case. Spatial symmetry is broken in both the cases. Clearly, for time-symmetric case $\langle p \rangle$ values are symmetrically distributed about zero leading to net current zero. This symmetry in $\langle p \rangle$ distribution along τ does not hold when temporal symmetry is broken. Parameters for the two cases are same as those for Figs. 4.13 and 4.14, respectively. 78
- 4.16 $\langle p \rangle$ vs n at $\tau = 0.5$ for different values of b . Other parameters are $V_0 = 0.5$, $\epsilon = 0.15$, $\phi = 0.5$ 79
- 4.17 $\langle p \rangle$ vs n at $\tau = 0.5$ for two different sets initial states (discussed in text) in two different colors. The graph at the top corresponds to $b = 10^{-5}$ and the one at bottom corresponds to $b = 0.4$. Other parameters are $V_0 = 0.5$, $\epsilon = 0.15$, $\phi = 0.5$. The value of $\langle p \rangle$ at a given n is independent of the set of initial states used for $b = 10^{-5}$, but is different for different sets of initial states for $b = 0.4$ 80
- 5.1 Figure shows the three regions (I, II and III) of periodic unit of $V_{sq}(x)$. . . 83
- 5.2 (Left) Energy levels of eigen states of unperturbed system, calculated for $b = 0.1\pi$, $V_0 = 0.5$, $\hbar_s = 0.0067$ and $N = 1$. It shows that energy levels are densely distributed for $\hbar_s \ll 1$. (Right) Energy levels of eigen states of unperturbed system, calculated for $b = 0.6\pi$, $V_0 = 0.5$, $\hbar_s = 1.1$ and $N = 64$. For large values of \hbar_s , the energies of basis states are distributed in broad well separated bands. 87

- 5.3 The probability density $|V_j^l|^2$ for some Floquet states vs energy of the basis states. The parameters are $\hbar_s = 0.0067, \phi = 0.5, V_0 = 0.5, \epsilon = 0.15, R = 1.9$ 93
- 5.4 The horizontal axis shows the index (state number) assigned to Floquet states. The bottom and top points connected with a given vertical line represent the minimum energy E_{min} and maximum energy E_{max} at which the corresponding Floquet state $|\Psi_i\rangle$ has probability above the cut-off value 0.0005. Only those states which have minimum one percent probability with $E < V_0$ are shown. E_t represents the energy at which the basis is truncated. 94
- 5.5 Husimi distribution for some of the Floquet states. Parameters: $\epsilon = 0.15, b = 0.1\pi, V_0 = 0.5, \hbar = 0.0067$ and $\phi = 0.5$ 95
- 5.6 Husimi distribution for some of the Floquet states. The parameters are $\epsilon = 0.15, V_0 = 0.5, \hbar = 0.0067$, (a,b) $b = 0.3\pi, \phi = 0.5$, (c,d) $b = 0.1\pi, \phi = 1.0$, (d,e) $b = 0.1\pi, \phi = 0$ 97
- 5.7 Husimi distribution for some of the Floquet states. Parameters: $\epsilon = 0.15, b = 0.00001\pi, V_0 = 0.5, \hbar = 0.0067$ 99
- 5.8 Husimi distribution for some of the Floquet states. Parameters: $\epsilon = 0.15, b = 0.0001\pi, V_0 = 0.5, \hbar = 0.0067$ 100
- 5.9 $\langle j \rangle$ separately for Floquet states carrying positive current and states carrying negative current for time symmetric system. (Right) Mean current of all chosen (see text) Floquet states at different times. Parameters: $\epsilon = 0.15, b = 0.1\pi, V_0 = 0.5, \hbar = 0.0067$ 101
- 5.10 (Left) $\langle j \rangle$ separately for Floquet states carrying positive current and states carrying negative current for time asymmetric system. (Right) Mean current of all chosen (see text) Floquet states at different times. Parameters: $\epsilon_1 = 0.8, \epsilon_2 = 0.16, b = 0.1\pi, V_0 = 0.5, \hbar = 0.0067$ 102

- 5.11 τ integrated current of eigen states for which at least five percent density lies below V_0 . Parameters: $\epsilon = 0.1$, $b = 0.1\pi$, $V_0 = 0.5$, $\hbar = 0.0067$. The vertical red line mark the mean I of the distribution. The green curve shows the distribution of I values for time symmetric case. The distribution in green is not normalised. It is scaled down by factor of 20. 103