DIRECTED TRANSPORT IN CHAOTIC HAMILTONIAN SYSTEMS

A THESIS

submitted for the award of Ph.D. degree of

MOHANLAL SUKHADIA UNIVERSITY

in the

Faculty of Science

by

Harinder Pal

Under the Supervision of

Dr. Madabushi Srinivasan Santhanam

Ex-Reader

Physical Research Laboratory, Ahmedabad, India

and

Co-Supervisor

Dr. Angom Dilip Kumar Singh

Associate Professor

Physical Research Laboratory, Ahmedabad, India

DEPARTMENT OF PHYSICS
MOHANLAL SUKHADIA UNIVERSITY
UDAIPUR
2011

To

My family

CERTIFICATE

I feel great pleasure in certifying that the thesis entitled, **"Directed Transport in Chaotic Hamiltonian Systems"** embodies a record of the results of investigations carried out by Harinder Pal under my guidance.

He has completed the following requirements as per Ph.D. regulations of the University.

(a) Course work as per the university rules.

(b) Residential requirements of the university.

(c) Presented his work in the departmental committee.

(d) Published/accepted minimum of two research papers in a referred research journal.

I am satisfied with the analysis of data, interpretation of results and conclusions drawn.

I recommend the submission of thesis.

Date :

Dr. Madabushi Srinivasan Santhanam
(Thesis Advisor)

Countersigned by
Head of the Department

DECLARATION

*I, **Harinder Pal**, S/O Mr. Harbans Lal, resident of A-2, PRL Residences, Navrangpura, Ahmedabad - 380 009, hereby declare that the work incorporated in the present thesis entitled, **"Directed Transport in Chaotic Hamiltonian Systems"** is my own and original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma.*

Date :

(Harinder Pal)

Acknowledgments

First and foremost, I offer my sincerest gratitude to my supervisor, Dr. M. S. Santhanam, who has supported me throughout my thesis with his knowledge and patience. I am grateful to him for providing me unflinching encouragement and guidance during all these years. His openness to new ideas and intellectual support for their pursuance has made research life rewarding for me. I thank him for his efforts to educate me with the concepts and tools necessary for the work in this thesis.

I owe special gratitude to my co-supervisor, Dr. Dilip Angom. I extend my warmest thanks to him for many useful discussions, encouragement and support. I am grateful to him for his kind concern regarding my academic requirements.

I thank Dr. Bimalendu Deb for giving an encouraging start to my research life by guiding me in my coursework project. I would also like to thank Prof. J. Banerji, Dr. Jitesh Bhatt, Dr. Dilip Angom, Dr. M. S. Santhanam, Prof. Shyam Lal, Prof. R Sekar, Prof. S. A. Haider, Dr. Varun Sheel and Dr. A. K. Singal for teaching various courses and making coursework an enjoyable learning experience. I extend my thanks to Prof. P. K. Panigrahi for many informal but stimulating discussions during my early days in Physical Research Laboratory. I sincerely thank Prof. Sushanta Dattagupta and Prof. Steven Tomsovic for giving me their valuable time and sharing some ideas during their visit to PRL. Special thanks are due to Dr. R. Sankaranarayanan who visited PRL during starting phase of my doctoral research and discussed his work on generalized standard map. I would also like to thank Prof. Arul Lakshminarayan for some useful discussions during International conference on "Recent Developments in Non-linear Dynamic" held in Tiruchirapalli in 2008.

I am extremely grateful to PRL for various facilities it has provided, without which this thesis would not have been possible. I would also like to thank all the staff members of library, computer center, administration, maintenance and medical dispensary for their support in making efficient use of all the facilities possible. I would like to specially thank Mrs. Nishtha AnilKumar, Mrs. Pauline Joseph, Dr.

Sheetal Patel, Dr. Bhushit, Mrs. Parul Parikh, Miss Jayshree, Mr. Keyur Shah, Mr. Gangadharia, Mr. B. M. Joshi, Mr. Ghanshyam Patel, Mr. Ranganathan, Miss Pragya, and Mr. N. P. M. Nair for always being happy to help. I acknowledge the extensive use of High Performance Computing cluster facility for my thesis work. I am thankful to Mr. G. G. Dholakia, Mr. D. V. Subhedar, Mr. Jigar Raval, Mr. Hitendra, Mr. Tejas, Mr. Alok and Mr. Mrugesh Gajjar for their prompt actions to my computation related requests. I would like to extend my warmest regards for Mr. Manjunatha and Mr. Anish of CDAC, Pune for their help in using various packages on HPC. Their zeal to deal with HPC related issues made my life easy. I am also grateful to IISER, Pune for its hospitality during my visits for thesis related work. I also thank the staff of MLSU, Udaipur for their kind co-operation and prompt services.

I would like to thank all my friends in PRL for their affectionate company and best wishes, as well as express my apology that I could not mention names of all of them because they are just too many. Vimal and Suman have been good “school-mates”. Their company during the schools and conferences we have attended together made those learning periods more enjoyable. I would like to thank Suman also for helping me out whenever I had some software or operating system related issues. I also thank my senior Dr. Rajneesh Atre for his help and guidance.

I express my deepest thanks to my parents for their untiring efforts and encouragement throughout my studies. I also thank my sisters, Sunita and Sandeep, for their loving support. I express my sincere gratitude to my elder sister Sunita for her guidance during my schooling and also for encouraging me for higher education. I am deeply thankful to my wife Manan whose understanding, patience and unconditional support has kept my spirits up through thick and thin.

Finally, I feel great pleasure to express my gratitude to all others who have directly or indirectly contributed to this thesis.

ABSTRACT

We study the dynamics and directed transport in a class of chaotic Hamiltonian systems. The system we consider is a $\delta-$kicked particle in the presence of (i) double-barrier potential and (ii) periodic lattice of square-well potentials. In contrast to the well studied kicked rotor, the kicked system, in the presence of two variants of square-well potentials, studied in this thesis does not obey the Kolmogorov-Arnold-Moser (KAM) theorem. Due to this, invariant curves are absent and instead the phase space displays intricate chains of islands and fully connected chaotic layer even for very small kick strength. However, a special feature of the system reported in this thesis is that, inspite of being a non-KAM system, dynamics is KAM-like in some regions of phase space. We study the effect of interplay between of non-KAM and KAM-like phase space dynamics on dynamical properties of the system. We report a number of novel and interesting dynamical features like (a) the classically induced suppression of energy growth, (b) non-equilibrium steady state and (c) momentum filtering effect. We also report results for the quantum analogues of these dynamical features.

To study the directed transport properties of the system, we study evolution of a set of initial states. We study the effect of spatio-temporal symmetries on net current of a set of states. We observe that the system shows ratchet effect, *i.e.*, directed current in absence of net bias, upon breaking certain spatio-temporal symmetries. We explain how the non-KAM nature of the system imparts some useful characteristics to it as ratchet model. Throughout this work, we also analyse the quantum dynamics of the system, mainly in the semiclassical regime, and study the consequences of quantum effects. We also show that the system can act as a quantum ratchet.

Contents

List of Figures