Study of Dynamical Processes on Complex Networks

A THESIS

submitted for the award of Ph.D. degree of

MOHANLAL SUKHADIA UNIVERSITY

in the

Faculty of Science

by

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Under the Supervision of

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DEPARTMENT OF PHYSICS
MOHANLAL SUKHADIA UNIVERSITY
UDAIPUR
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All our knowledge brings us nearer to our ignorance...

–T. S. Eliot

Dedicated to,

My Family

DECLARATION

I **Mr. Vimal Kishore**, *S/o Mr. Mannu Lal Saxena, resident of Room no. 204, Thaltej Hostel, Bodakdev, Ahmedabad, hereby declare that the work incorporated in the present thesis entitled,* **"Study of Dynamical Processes on Complex Networks"** *is my own and original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma.*

Date :

(Vimal Kishore)

CERTIFICATE

I feel great pleasure in certifying that the thesis entitled, **"Study of Dynamical Processes on Complex Networks"** embodies a record of the results of investigations carried out by Mr. Vimal Kishore under my guidance.

He has completed the following requirements as per Ph.D. regulations of the University.

(a) Course work as per the university rules.

(b) Residential requirements of the university.

(c) Presented his work in the departmental committee.

(d) Published minimum of two research papers in a referred research journal.

I am satisfied with the analysis of data, interpretation of results and conclusions drawn.

I recommend the submission of thesis.

Date :

Madabushi Srinivasan Santhanam

Ex-Reader

(Thesis Supervisor)

Countersigned by

Head of the Department

Acknowledgements

I get by with a little help from my friends.

–John Lennon, Paul McCartney

This thesis, an extreme event in my life, is an outcome of my random walk on the path leading to PhD. It took almost six years for me to travel this road which has always been little less traveled. The journey of was full of many felicitous moments because I had the privilege to work with valuable colleagues, to enjoy the friendship of many and the love of the ones most close to me. I can not mention them all, neither I can acknowledge them enough, but all I can say surely is that my true acknowledgements are ampler than these few pages.

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List of Publications

- Extreme Events on Complex Networks,
  Vimal Kishore, M. S. Santhanam and R. E. Amritkar,
  Phys. Rev. Lett **106**, 188701 (2011).
  arXiv:1102.1789 [cond-mat.stat-mech]

- Extreme Events and Event-size Fluctuations in Biased Random Walks on Networks,
  Vimal Kishore, M. S. Santhanam and R. E. Amritkar
  Phys. Rev. E **85**, 056120 (2012).
  arXiv:1112.2112 [cond-mat.stat-mech]

Abstract

Networks are everywhere and most of the the networks support a number of transport processes. Fluctuations in traffic flow constitute one of the main factor affecting the dynamics of these systems. Fluctuations above a certain threshold can be labeled as extreme events. This definition of extreme events as exceedances above a prescribed quantile is not necessarily related to the constraints imposed by the capacity of the node. It arises from the natural fluctuations in the traffic passing through a node. The transport model that we have adopted is the random walk on complex network. Thus, in this thesis we place our results in the context of both the random walks and extreme events in a network setting.

In the case of a simple random walk, we show that the small degree nodes of a network are more likely to encounter extreme events than the hubs. The result remains unchanged even with the use of shortest path strategy on networks. We also obtain the extreme event probability in the case of topologically biased random walk and show that biasing the traffic towards hubs can increase the risk of bottlenecks on networks.

Using the above notion of extreme events, we study the nature of failure of a network by removing nodes which experience an extreme event and redistributing the walkers on the remaining or active nodes. We find that in an all-to-all network, cascade failures cause the sudden collapse of the network.

This thesis, as a whole, is an attempt to understand extreme events occurring on the nodes due to flow on networks. It discusses the importance of lower degree nodes in scale free network and presents a different but natural mechanism for the complete failure of a network. The work done in the thesis can help in designing the networks which will be better prepared to meet the expected extreme events. However, extreme events discussed here being due to inherent fluctuations will nevertheless take place and can not be avoided.

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