SOME STUDIES ON TWO-BODY RANDOM MATRIX ENSEMBLES

Thesis submitted to

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by

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May, 2011

To my family

Declaration

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any university or institution.

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Abstract

Random matrix theory (RMT) has been established to be one of the central themes in quantum physics during the end of 20th century. This theory has emerged as a powerful statistical approach leading to paradigmatic models describing generic properties of complex systems. On the other hand, with scientific developments it was clear by mid 20th century that deterministic ideas are not valid for microscopic systems and this led to the development of a new field of research called 'quantum chaos'. One-body chaos is well understood by 90's with RMT playing a key role. More specifically, the spectral statistics predicted by RMT is a characteristic of quantum systems whose classical analogue is chaotic. However, most of the real systems are many-body in character. The classical Gaussian orthogonal (GOE), unitary (GUE) and symplectic (GSE) ensembles, introduced by Wigner and Dyson, are ensembles of multi-body interactions. In various quantum many-body systems such as nuclei, atoms, mesoscopic systems like quantum dots and small metallic grains, interacting spin systems modeling quantum computing core and BEC, the interparticle interactions are essentially two-body in nature. This together with nuclear shell-model examples led to the introduction of random matrix ensembles generated by two-body interactions in 1970-1971. These two-body ensembles are defined by representing the two-particle Hamiltonian by one of the classical ensembles and then the m ($m > 2$) particle H-matrix is generated by the Hilbert space geometry. Thus the random matrix ensemble in the two-particle spaces is embedded in the m-particle H-matrix and therefore these ensembles are generically called embedded ensembles (EEs). Simplest of these ensembles is the embedded Gaussian orthogonal ensemble of random matrices generated by two-body interactions for spinless fermion [boson] systems, denoted by EGOE(2) [BEGOE(2); here 'B' stands for bosons]. In addition to the complexity gener-

ating two-body interaction, Hamiltonians for realistic systems consist of a mean-field one-body part. Then the appropriate random matrix ensembles are EE(1+2). The spinless fermion/boson EGEs (orthogonal and unitary versions) have been explored in detail from 70's with a major revival from 1994. It is now well understood that EGEs generate paradigmatic models for many-body chaos or stochasticity exhibited by isolated finite interacting quantum systems. Besides the mean-field and the two-body character, realistic Hamiltonians also carry a variety of symmetries. In many applications of EGEs, generic properties of EGE for spinless fermions are 'assumed' to extend to symmetry subspaces. More importantly, there are several properties of real systems that require explicit inclusion of symmetries and they are defined by a variety of Lie algebras. The aim of the present thesis is to identify and systematically analyze many different physically relevant EGEs with symmetries by considering a variety of quantities and measures that are important for finite interacting quantum systems mentioned above. The embedded ensembles investigated to this end and the corresponding results are as follows.

The thesis contains nine chapters. Chapter 1 gives an introduction to the subject of two-body random matrix ensembles. Also, the known results for spinless fermion/boson EGEs are described briefly for completeness and for easy reference in the following chapters.

Finite interacting Fermi systems with a mean-field and a chaos generating two-body interaction are modeled, more realistically, by one plus two-body embedded Gaussian orthogonal ensemble of random matrices with spin degree of freedom [called EGOE(1+2)-**s**]. Numerical calculations are used to demonstrate that, as λ, the strength of the interaction (measured in the units of the average spacing of the single particle levels defining the mean-field), increases, generically there is Poisson to GOE transition in level fluctuations, Breit-Wigner to Gaussian transition in strength functions (also called local density of states) and also a duality region where information entropy will be the same in both the mean-field and interaction defined basis. Spin dependence of the transition points λ_c, λ_F and λ_d, respectively, is described using the propagator for the spectral variances and the analytical formula for the propagator is derived. We further establish that the duality region corresponds to a region of thermalization. For this purpose we have compared the single particle entropy defined by the occupancies of the single particle orbitals with thermodynamic entropy and information entropy for various λ values and they are very close to each other at $\lambda = \lambda_d$. All these results are presented in Chapter 2.

EGOE(1+2)-**s** also provides a model for understanding general structures generated by pairing correlations. In the space defined by EGOE(1+2)-**s** ensemble for fermions, pairing defined by the algebra $U(2\Omega) \supset Sp(2\Omega) \supset SO(\Omega) \otimes SU_S(2)$ is identified and some of its properties are derived. Using numerical calculations it is shown that in the strong coupling limit, partial densities defined over pairing subspaces are close to Gaussian form and propagation formulas for their centroids and variances are derived. As a part of understanding pairing correlations in finite Fermi systems, we have shown that pair transfer strength sums (used in nuclear structure) as a function of excitation energy (for fixed S), a statistic for onset of chaos, follows, for low spins, the form derived for spinless fermion systems, i.e., it is close to a ratio of Gaussians. Going further, we have considered a quantity in terms of ground state energies, giving conductance peak spacings in mesoscopic systems at low temperatures, and studied its distribution over EGOE(1+2)-**s** by including both pairing and exchange interactions. This model is shown to generate bimodal to unimodal transition in the distribution of conductance peak spacings. All these results are presented in Chapter 3.

For m fermions in Ω number of single particle orbitals, each four-fold degenerate, we have introduced and analyzed in detail embedded Gaussian unitary ensemble of random matrices generated by random two-body interactions that are $SU(4)$ scalar [EGUE(2)-$SU(4)$]. Here, the $SU(4)$ algebra corresponds to the Wigner's supermultiplet $SU(4)$ symmetry in nuclei. Embedding algebra for the EGUE(2)-$SU(4)$ ensemble is $U(4\Omega) \supset U(\Omega) \otimes SU(4)$. Exploiting the Wigner-Racah algebra of the embedding algebra, analytical expression for the ensemble average of the product of any two m-particle Hamiltonian matrix elements is derived. Using this, formulas for a special class of $U(\Omega)$ irreducible representations (irreps) $\{4^r, p\}$, $p = 0, 1, 2, 3$ are derived for the ensemble averaged spectral variances and also for the covariances in energy centroids and spectral variances. On the other hand, simplifying the tabulations available for $SU(\Omega)$ Racah coefficients, numerical calculations are carried out for general $U(\Omega)$ irreps. Spectral variances clearly show, by applying the so-called Jacquod and

Stone prescription, that the EGUE(2)-$SU(4)$ ensemble generates ground state structure just as the quadratic Casimir invariant (C_2) of $SU(4)$. This is further corroborated by the calculation of the expectation values of $C_2[SU(4)]$ and the four periodicity in the ground state energies. Secondly, it is found that the covariances in energy centroids and spectral variances increase in magnitude considerably as we go from EGUE(2) for spinless fermions to EGUE(2) for fermions with spin to EGUE(2)-$SU(4)$ implying that the differences in ensemble and spectral averages grow with increasing symmetry. Also for EGUE(2)-$SU(4)$ there are, unlike for GUE, non-zero cross-correlations in energy centroids and spectral variances defined over spaces with different particle numbers and/or $U(\Omega)$ [equivalently $SU(4)$] irreps. In the dilute limit defined by $\Omega \to \infty$, $r >> 1$ and $r/\Omega \to 0$, for the $\{4^r, p\}$ irreps, we have derived analytical results for these correlations. All correlations are non-zero for finite Ω and they tend to zero as $\Omega \to \infty$. All these results are presented in Chapter 4.

One plus two-body embedded Gaussian orthogonal ensemble of random matrices with parity [EGOE(1+2)-π] generated by a random two-body interaction (modeled by GOE in two particle spaces) in the presence of a mean-field, for spinless identical fermion systems, is defined in terms of two mixing parameters and a gap between the positive ($\pi = +$) and negative ($\pi = -$) parity single particle states. Numerical calculations are used to demonstrate, using realistic values of the mixing parameters, that this ensemble generates Gaussian form (with corrections) for fixed parity state densities. The random matrix model also generates many features in parity ratios of state densities that are similar to those predicted by a method based on the Fermi-gas model for nuclei. We have also obtained a simple formula for the spectral variances defined over fixed-(m_1, m_2) spaces where m_1 is the number of fermions in the +ve parity single particle states and m_2 is the number of fermions in the −ve parity single particle states. The smoothed densities generated by the sum of fixed-(m_1, m_2) Gaussians with lowest two shape corrections describe the numerical results in many situations. The model also generates preponderance of +ve parity ground states for small values of the mixing parameters and this is a feature seen in nuclear shell-model results. All these results are presented in Chapter 5.

For m number of bosons, carrying spin ($\mathbf{s} = \frac{1}{2}$) degree of freedom, in Ω number of single particle orbitals, each doubly degenerate, we have introduced and analyzed embedded Gaussian orthogonal ensemble of random matrices generated by random two-body interactions that are spin (S) scalar [BEGOE(2)-**s**]. The ensemble BEGOE(2)-**s** is intermediate to the BEGOE(2) for spinless bosons and for bosons with spin $\mathbf{s} = 1$ which is relevant for spinor BEC. Embedding algebra for the BEGOE(2)-**s** ensemble and also for BEGOE(1+2)-**s** that includes the mean-field one-body part is $U(2\Omega) \supset U(\Omega) \otimes SU(2)$ with $SU(2)$ generating spin. A method for constructing the ensembles in fixed-(m,S) spaces has been developed. Numerical calculations show that the fixed-(m,S) density of states is close to Gaussian and generically there is Poisson to GOE transition in level fluctuations as the interaction strength (measured in the units of the average spacing of the single particle levels defining the mean-field) is increased. The interaction strength needed for the onset of the transition is found to decrease with increasing S. Propagation formulas for the fixed-(m,S) space energy centroids and ensemble averaged spectral variances are derived. Using these, covariances in energy centroids and spectral variances are analyzed. Variance propagator clearly shows that the BEGOE(2)-**s** ensemble generates ground states with spin $S = S_{max}$. This is further corroborated by analyzing the structure of the ground states in the presence of the exchange interaction $\hat{S}^2$ in BEGOE(1+2)-**s**. Natural spin ordering (S_{max}, $S_{max}-1$, $S_{max}-2$, ..., 0 or $\frac{1}{2}$) is also observed with random interactions. Going beyond these, we have also introduced pairing symmetry in the space defined by BEGOE(2)-**s**. Expectation values of the pairing Hamiltonian show that random interactions exhibit pairing correlations in the ground state region. All these results are presented in Chapter 6.

Parameters defining many of the important spectral distributions (valid in the chaotic region), generated by EGEs, involve traces of product of four two-body operators. For example, these higher order traces are required for calculating nuclear structure matrix elements for $\beta\beta$ decay and also for establishing Gaussian density of states generated by various embedded ensembles. Extending the binary correlation approximation method for two different operators and for traces over two-orbit configurations, we have derived formulas, valid in the dilute limit, for the skewness and excess parameters for EGOE(1+2)-π ensemble. In addition, we have derived a formula for the traces defining the correlation coefficient of the bivariate transition strength distribution generated by the two-body transition operator appropriate for calculating 0ν-$\beta\beta$ decay nuclear transition matrix elements and also for other higher order traces required for justifying the bivariate Gaussian form for the strength distribution. With applications in the subject of regular structures generated by random interactions, we have also derived expressions for the coefficients in the expansions to order $[J(J+1)]^2$ for the energy centroids $E_c(m,J)$ and spectral variances $\sigma^2(m,J)$ generated by EGOE(2)-J ensemble members for the single-j situation. These expansion coefficients also involve traces of four two-body operators. All these results are presented in Chapter 7.

In Chapter 8, to establish random matrix structure of nuclear shell model Hamiltonian matrices, we have presented a comprehensive analysis of the structure of Hamiltonian matrices based on visualization of the matrices in three dimensions as well as in terms of measures for GOE, banded and embedded random matrix ensembles. We have considered two nuclear shell-model examples, ^{22}Na with $J^\pi T = 2^+0$ and ^{24}Mg with $J^\pi T = 0^+0$ and, for comparison we have also considered SmI atomic example with $J^\pi = 4^+$. It is clearly established that the matrices are neither GOE nor banded. For the EGOE [strictly speaking, EGOE(2)-JT or EGOE(2)-J] structure we have examined the correlations between diagonal elements and eigenvalues, fluctuations in the basis states variances and structure of the two-body part of the Hamiltonian in the eigenvalue basis. Unlike the atomic example, nuclear examples show that the nuclear shell-model Hamiltonians can be well represented by EGOE.

Finally, Chapter 9 of the thesis gives conclusions and future outlook. To summarize, we have obtained large number of new results for embedded ensembles and in particular for EGOE(1+2)-**s**, EGUE(2)-$SU(4)$, EGOE(1+2)-π and BEGOE(1+2)-**s**, with EGUE(2)-$SU(4)$ introduced for the first time in this thesis. Moreover, some results are presented for EGOE(2)-J and for the first time BEGOE(1+2)-**s** has been explored in detail in this thesis. In addition, formulas are derived, by extending the binary correlation approximation method, for higher order traces for embedded ensembles with $U(N) \supset U(N_1) \oplus U(N_2)$ embedding and some of these are needed for new applications of statistical nuclear spectroscopy. Results of the present thesis establish that embedded Gaussian ensembles can be used gainfully to study a variety of problems in many-body quantum physics and this includes quantum information science and the thermodynamics of isolated finite interacting quantum systems.

Contents