STUDY OF COLOUR CONFINEMENT MODEL FOR QCD

BY

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CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis of the award of any degree or diploma by any University or Institution.

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Dedicated to

My Parents

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"Circumstantial evidence is the best evidence there is, Paul. You just have to interpret it properly"

- Erle Stanley Gardner

"We seek him here, We seek him there, Those Frenchies seek him everywhere. Is he in heaven? Is he in hell?
That demmed, elusive Pimpernel?

- Baroness Orczy

ABSTRACT

The existence of colour singlet bound state of gluons called the glueball is an unique prediction of quantum chromo dynamics. Their theoretical as well as the experimental study is very crucial to the validity of this theory. Confinement of colour is an empirical result of QCD. There are confinement models for quarks to predict various properties of hadrons. Similar confinement schemes are extended for the confinement of gluons for the study of glueballs. This study aims at formulating a unified confinement basis for both quarks and gluons. Two different schemes for the confinement of coloured gluons are studied. A probable link from the basic theory of QCD to these phenomenological descriptions is obtained heuristically. The gluons in this phenomenology are considered as quasi-Maxwellian fields. This reduction from the nonlinear theory to a linear theory is the basis of all phenomenological confinement models.

In the first case a colour current confinement model (CCM) is formulated. In this formalism the nonlinear current of the gluon field is approximated to a colour super current in analogy with Ginzberg-Landau's theory of super conductivity. A particular choice of this gluon super current led to a consistent confinement scheme for the gluons in a general frame of Lorentz gauge with a secondary

gauge condition named as oscillator gauge. The two transverse modes of the confined gluons are obtained in this gauge. The gluon fields are second quantized and their energies are calculated in terms of the model parameter.

An alternate confinement scheme for the gluons is the dielectric confinement model (DCM). An inhomogeneous non-local dielectric function is obtained from the CCM by treating the CCM current as a self-induced polarization current. As the dynamical dependence is neglected the CCM dielectric function reduces to that of a simple inhomogeneous dielectric medium. Similar function is also obtained from the analogy of the Dirac Spinor equations, in the case of a confinement potential with Lorentz scalar plus vector part, with the Maxwell´s equation for a dielectric medium written in spin notations. Then a confinement model of the coloured gluons in a harmonically varying asympotically free dielectric medium is studied. In the choice of the gauge, we met with the difficulty which is similar to the bag model boundary conditions. With a restricted confinement boundary the confined gluon modes are obtained in the usual Coulomb gauge. These fields are quantized and the energies of the gluons in this model are expressed in terms of a single model parameter.

The lowest gluon modes obtained in both the schemes are characterized by $\ell = 0$, $J^{PC} = 1^{--}$ (E=gluon) and $\ell = 1$, $J^{PC} = 1^{+-}$ (M-gluon). However, their energy expressions in the two schemes are quite different. Coupling these lowest gluon modes colour singlet di-gluon and tri-gluon low-lying glueball states are constructed and their energies are calculated. The lightest glueballs are expected to have mass ranging from 1-3 GeV and have spin parities 0^{++}, 0^{-+} and 2^{++}. Experimental status of such states as the glueball candidates is discussed. Using a di-gluon glueball candidate the model parameter in both the schemes are fixed and the energies of all other glueball states are predicted. The spurious motion of the centre of the multi-gluon state is exactly taken into account in both the cases and the glueball energy states are corrected for the zeropoint motion. The resulting values for the di-gluon and tri-gluon systems are compared with the experimental candidates and similar naive bag model results.

The success of the CCM scheme for the gluons provided a harmonious confinement basis for treating both quarks and gluons together. the essential requirement here is then to obtain the confined gluon propagator. A translationally invariant ansatz has been made to develop the theory further. We obtained closed analytical expression for the relevant propagator. It is different from the usual oscillator Green's function which is not translationally invariant. We generalize this derivation for m-dimensional case. This closed analytical expression for the propagator has wider applications in the development of the bound state perturbation theory involving quarks and gluons.

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Figure 5.1 : Asymptotic Behaviour of the Confined Gluon Propagator

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GENERAL INTRODUCTION

In recent years, the understanding of the most fundamental interactions among elementary particles has undergone substantial changes in its experimental as well as theoretical aspects. As it stands today, the leptons (e, μ, τ, ν_e, ν_μ, ν_τ) and quarks (u,d,s,c,b,t) are the most fundamental constituents of matter. The hadrons, i.e. the baryons and mesons are composites[1-3] and they are made up of three quarks and quark-antiquark pairs respectively. The electromagnetic, weak and strong interactions can be understood by knowing the exact dynamics of these leptons and quarks. Leptons carry integral electric charges, i.e. 0 or + e, where as the quarks carry fractional electric charges, i.e. 1/3 or 2/3 e (see Table 1.1) [4]. Leptons undergo electromagnetic and weak interactions while quarks undergo strong interactions also. But unlike the leptons, quarks are not found free in nature. The experimental evidences are that they are permanently confined objects[5,6]. This leads to the confinement theory [7-9], which will be discussed in subsequent chapters.

The difficulty in the simple quark model for baryons [1,10] led to the introduction of a new degree of freedom for quarks. This is called the colour degree of freedom of the quarks [7,8]. Accordingly by the Pauli's principle, to construct a completely antisymmetric wavefunction for the baryons, each quark must exist in three colours. The theory describing the dynamics of this colour is called the Quantum Chromo Dynamics (QCD) [11,12].

Like the photon in Quantum Electrodynamics (QED) gluons are the gauge bosons in QCD [9]. But QCD is a non-abelian gauge theory [11,12] unlike QED which is abelian. The salient features of QCD are: (1) Colour-colour interactions are very weak at very high momentum transfers or at very short distances (asymptotic freedom) [13], and (2) these interactions grow strong at low momentum transfer or at very large distances (infra red slavery) [14]. The latter presumably is giving rise to confinement [15]. While the QCD (Yang-Mills) theory is shown to exhibit asymptotic freedom, the confinement is more of an empirical necessity, i.e., only colour singlet states are seen free. The gluons which are the quanta of the colour field carry colour charges and they interact among themselves. One of the evidences for QCD is then the existence of glueballs which are the colour singlet bound states of multigluons [16]. Thus the study of glueballs and their experimental confirmation is very crucial to the validity of quantum chromodynamics. Since these coloured gluons are many in number (SU3-colour octet) the coupled nonlinear equations obeyed by them are too complex to solve. Hence to understand the nature of glueball states, the strong interactions and the properties of hadrons from their microscopic structure, one has to go for phenomenological models incorporating confinement. This is the spirit in which various models like bag models [17-19], potential models [20-22], etc., for confinement are developed to study various aspects of hadrons and its properties. One of the very successful models is the relativistic confinement potential model for quarks using Lorentz scalar plus vector harmonic oscillator potential (RHM) [23], for the predictions of the diverse aspects of hadronic properties. Successes of this simple model (RHM) motivated us to look for a similar confinement model for the QCD colour fields (i.e., gluons) for the prediction of the glueball states and thus to formulate a unified harmonic confinement basis for both quarks and gluons.

The gauge theory of fundamental interactions of elementary particles is reviewed in the first chapter. QED is described to illustrate the abelian gauge theory and the QCD is described to illustrate the nonabelian gauge theory. The general properties of QCD and some of the phenomenological colour confinement model for QCD are discussed in chapter two.

New confinement models for the coloured gluons motivated from the RHM for quarks [23] are proposed in chapter three. A Colour Confinement Model (CCM) and a Dielectric Confinement Model (DCM) for the gluons satisfying the 'quasi-Maxwellian' type of field equations are developed.

We construct in the fourth chapter the colour singlet multi-gluon glueball states using the confined gluon modes in both CCM and DCM. We discuss, in this chapter, the experimental status of such exotic states and their identification as glueball candidates. We calculate the low-lying di-gluon and tri-gluon glueball energies by fitting some of the experimental candidates for glueballs. The spurious motion of the centre of a multi-gluon state is taken into account as in the case of RHM and the results are presented in this chapter.

Having developed a harmonic theory for the confinement of gluons for the study of glueballs, in chapter five, we calculate the confined gluon propagator corresponding to the CCM gluons for the future application of the unified confinement theory of quarks and gluons. We are able to obtain an analytical closed expression for the m-dimensional harmonic oscillator propagator in a translationally invariant ansatz.

In the last chapter the conclusions and future applications of this study are discussed.