#### Aspects of Quantum Field Theory in Phenomenology of Early Universe

#### A THESIS

#### submitted for the Award of Ph.D. degree of MOHANLAL SUKHADIA UNIVERSITY

 $in \ the$ 

Faculty of Science

by

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Under the Supervision of

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## DEPARTMENT OF PHYSICS MOHANLAL SUKHADIA UNIVERSITY UDAIPUR Year of submission: 2011

# To Maa, Bapi &

Jethubaba

#### CERTIFICATE

I feel great pleasure in certifying that the thesis entitled, "Aspects of Quantum Field Theory in Phenomenology of Early Universe" embodies a record of the results of investigations carried out by Mrs. Suratna Das under my quidance.

She has completed the following requirements as per Ph.D. regulations of the University.

(a) Course work as per the university rules.

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(c) Presented her work in the departmental committee.

(d) Published minimum of two research papers in a referred research journal.

I am satisfied with the analysis of data, interpretation of results and conclusions drawn.

I recommend the submission of thesis.

Date : May 25, 2011

Subhendra Mohanty Professor (Thesis Supervisor)

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#### DECLARATION

I, Mrs. Suratna Das, D/O Mr. Saroj Kumar Das, resident of D-108, PRL residences, Navrangpura, Ahmedabad, 380009, hereby declare that the work incorporated in the present thesis entitled, "Aspects of Quantum Field Theory in Phenomenology of Early Universe" is my own and original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma.

Date : May 25, 2011

(Suratna Das)

### Acknowledgments

This thesis would not have been possible if my parents, Shikha and Saroj Kumar Das, had not introduced me to the joys of doing science, especially physics. I convey my gratitudes for their constant support, encouragement and unconditional love.

I am indebted to my supervisor Prof. Subhendra Mohanty for his guidance and encouragement throughout the period over which this work was done. I thank him for considering me able to handle projects from quite diverse fields which results in a few publications which are not directly related to this thesis topic.

I acknowledge Dr. R. Rangarajan for numerous stimulating discussions which helped me understand the subject in a better way. I thank him and Dr. Kaushik Bhattacharya for their collaboration and constant encouragement.

I thank Prof. S. Rindani, Prof R. Amritkar, Dr. H. Misra, Dr. J Bhatt, Dr. D. Angom, Dr. S. Goswami and Dr. N. Mahanjan for taking stimulating courses on high energy physics and mathematical methods. My special gratitudes to Prof. V. Sahni of IUCAA, Pune, whose course on Cosmology, during my M.Sc. days, stimulated me enough to undertake this subject as my research topic.

My special thanks to Suman Acharyya and Anjishnu Sarkar for helping me constantly regarding computational issues and refining my poor knowledge about computers.

I am grateful to my seniors Akhilesh and Santosh and to my juniors Abhishek, Kabitri, Moumita, Siddhartha and Tanushree whose constant companionship and support always keep me recharged. Because of them leaving PRL will be like leaving my own home.

I acknowledge all my seniors especially Utpal, Suranjana, Amitava, Ritesh, Kushwaha ji, Bhavik, Manan, Harinder, Brajesh, Waliur (and his better-half Reshma), Gyan, Alok, Vishal, Lokesh, Sumanta, Naveen, Kirpa and Shuchita, all my batch-mates Anand, Ashok, Amzad, Arvind, Bhaswar, Pankaj, Naveen, Rabiul, Rohit, Sandeep, Soumya, Sreekanth, Vimal and Vineet and all my juniors, naming a few: Arko, Iman, Koushik, Srinivas, Subrata and Tapas for making my stay at PRL memorable.

I am grateful to all the PRL library, computer center and administration stuff and also to the staff members of Theoretical Physics Division of PRL for their sincere support.

At the end I will like to acknowledge my father-in-law, Late Sri Sachindranath Bhattacharjee (who was more than a friend to me), whose constant encouragement forced me to read numerous literatures in last five years and thus enabled me to look at this beautiful world from a completely new direction. Last but not the least, my special gratitudes to all my family members especially to my beloved husband, Kaushik, my dear Jethubaba, Sri Smriti Ranjan Das and my dear Grandma (Thammi), Smt. Srimati Das, for being by my side at all time.

### Abstract

In this thesis, our main focus is on applications of various Quantum Field Theoretic (QFT) treatments in analyzing early Universe phenomena primarily related to the inflationary paradigm<sup>1</sup>.

It is shown that non-perturbative QFT technique for calculating two-point correlation in flat space, known as Källén-Lehmann spectral representation, can be used to calculate the power spectrum of interacting scalar fields where the interactions are short-ranged. Decaying inflaton and composite inflaton are two such cases where our method of calculating power spectrum can be applied. Decaying inflaton suppresses the long-distance correlation while the composite inflaton yields some oscillatory features in the low l region of the TT spectrum of CMBR, which may be observed by WMAP or in the future observations with PLANCK.

We investigate whether an exotic quantum field, named the *unparticle*, can play the role of an inflaton and drive inflation. Such exotic fields yields long-range forces due to its anomalous dimension and such anomalous dimension of tensor and vector unparticle is constrained from Mercury's perihelion precession data. Signature of a scalar unparticle inflaton is the suppression of low l modes in the anisotropy spectrum in the CMBR which can be observed by WMAP or PLANCK.

Effects of pre-inflationary radiation era on the primordial non-Gaussianity is also studied using Thermal Field Theory techniques. The bispectrum contribution is enhanced by a factor of 65-90 from that of single-field slow-roll inflationary model. Thermal averaging yields trispectrum non-Gaussianity which does not depend up on the slow-roll parameters and thus can be as large as -42. Signature of such a pre-inflationary radiation era is a large trispectrum non-Gaussianity compared to the bispectrum non-Gaussianity.

<sup>&</sup>lt;sup>1</sup>keywords : Inflation, CMBR anisotropies, Power spectrum, Källén-Lehmann spectral representation, Unparticle, Primordial non-Gaussianity, Thermal Field Theory

## Contents

| Certificate |  |   |   |    |  |  |  |
|-------------|--|---|---|----|--|--|--|
| D           | Declaration<br>Acknowledgments<br>Abstract |   |   |    |  |  |  |
| A           |  |   |   |    |  |  |  |
| A           |  |   |   |    |  |  |  |
| Li          | st of                                      | Figur   | es  | x  |  |  |  |
| 1           | Intr                                       | oducti  | ion   | 1  |  |  |  |
|             | 1.1  | Macro   | cosm in the microcosm   | 1  |  |  |  |
|             | 1.2  | Quant   | um Field Theory in Inflationary paradigm                      | 3  |  |  |  |
|             | 1.3  | Notat   | ions and conventions  | 8  |  |  |  |
|             | 1.4  | Schem   | e of the thesis   | 9  |  |  |  |
| <b>2</b>    | Ger  | neral fi  | camework of Inflation and its $n$ -point functions            | 11 |  |  |  |
|             | 2.1  | Inflati   | on to the rescue of Big Bang Theory                           | 13 |  |  |  |
|             | 2.2  | 2 Inflation and the quantum origin of structure |   | 14 |  |  |  |
|             |  | 2.2.1   | Cosmological perturbation theory                              | 14 |  |  |  |
|             |  | 2.2.2   | Quantum field theory of scalar fields in de Sitter background | 17 |  |  |  |
|             |  | 2.2.3   | The Power spectrum and today's observations                   | 24 |  |  |  |
|             | 2.3  | Primo   | rdial Non-Gaussianity   | 27 |  |  |  |
|             |  | 2.3.1   | Defining Bispectrum and Trispectrum                           | 28 |  |  |  |
|             |  | 2.3.2   | Non-Gaussianity in a single-field slow-roll inflation model   | 29 |  |  |  |
|             |  | 2.3.3   | Alternative scenarios and non-Gaussianity                     | 32 |  |  |  |
|             |  | 2.3.4   | Observational bounds on primordial non-Gaussianity            | 35 |  |  |  |

|                  | 2.4  | Summary   | 36   |  |
|------------------|--|---|--|--|
| 3                | Källén-Lehmann representation of QFT and interacting inflaton  |   |  |  |
|                  | 3.1  | Introduction  | 38   |  |
|                  | 3.2  | Power spectrum of interacting scalar field - general case   | 41   |  |
|                  | 3.3  | Inflaton with a decay width   | 43   |  |
|                  | 3.4  | Inflaton as Composite Particle  | 45   |  |
|                  | 3.5  | Conclusion  | 48   |  |
| 4                | An   | exotic quantum field : Unparticle as inflaton   | 50   |  |
|                  | 4.1  | Introduction  | 50   |  |
|                  | 4.2  | Constraining unparticle anomalous dimension from long range forces  | 52   |  |
|                  |  | 4.2.1 Ungravity from tensor unparticles   | 53   |  |
|                  |  | 4.2.2 Long range force from vector unparticles  | 58   |  |
|                  | 4.3  | Unparticle as inflaton  | 60   |  |
|                  | 4.4  | Conclusions   | 62   |  |
| 5                | Thermal field theory and enhanced non-Gaussianity  |   |  |  |
|                  | 5.1  | Introduction  | 64   |  |
|                  | 5.2  | Thermal average of inflaton newer spectra   |  |  |
|                  | 0.2  | Thermal average of militation power spectra   | 65   |  |
|                  | 5.3  | Non-Gaussianity in bispectrum from thermal distribution of inflaton   | 65<br>70   |  |
|                  | 5.2<br>5.3<br>5.4  | Non-Gaussianity in trispectrum from thermal distribution of inflaton<br>Non-Gaussianity in trispectrum from thermal distribution of inflaton  | 65<br>70<br>74   |  |
|                  | <ol> <li>5.3</li> <li>5.4</li> <li>5.5</li> </ol>  | Non-Gaussianity in bispectrum from thermal distribution of inflaton<br>Non-Gaussianity in trispectrum from thermal distribution of inflaton<br>Conclusion   | <ul><li>65</li><li>70</li><li>74</li><li>76</li></ul>  |  |
| 6                | <ul><li>5.3</li><li>5.4</li><li>5.5</li><li>Con</li></ul>  | Non-Gaussianity in bispectrum from thermal distribution of inflaton<br>Non-Gaussianity in trispectrum from thermal distribution of inflaton<br>Conclusion   | <ul> <li>65</li> <li>70</li> <li>74</li> <li>76</li> <li>78</li> </ul>   |  |
| 6<br>A           | 5.3<br>5.4<br>5.5<br>Con<br>Bas  | Non-Gaussianity in bispectrum from thermal distribution of inflaton         Non-Gaussianity in trispectrum from thermal distribution of inflaton         Conclusion         clusion         ics of FLRW Cosmology   | <ul> <li>65</li> <li>70</li> <li>74</li> <li>76</li> <li>78</li> <li>82</li> </ul>   |  |
| 6<br>A<br>B      | <ul> <li>5.3</li> <li>5.4</li> <li>5.5</li> <li>Con</li> <li>Bas</li> <li>Dyr</li> </ul>               | Non-Gaussianity in bispectrum from thermal distribution of inflaton<br>Non-Gaussianity in trispectrum from thermal distribution of inflaton<br>Conclusion   | <ul> <li>65</li> <li>70</li> <li>74</li> <li>76</li> <li>78</li> <li>82</li> <li>85</li> </ul>   |  |
| 6<br>A<br>B      | <ul> <li>5.3</li> <li>5.4</li> <li>5.5</li> <li>Con</li> <li>Bass</li> <li>Dyr</li> <li>B.1</li> </ul> | Non-Gaussianity in bispectrum from thermal distribution of inflaton         Non-Gaussianity in trispectrum from thermal distribution of inflaton         Conclusion   | <ul> <li>65</li> <li>70</li> <li>74</li> <li>76</li> <li>78</li> <li>82</li> <li>85</li> <li>86</li> </ul>                                     |  |
| 6<br>A<br>B<br>C | 5.3<br>5.4<br>5.5<br>Cor<br>Bas<br>Dyr<br>B.1<br>Per   | Non-Gaussianity in bispectrum from thermal distribution of inflaton         Non-Gaussianity in trispectrum from thermal distribution of inflaton         Conclusion       Conclusion         Iclusion         ics of FLRW Cosmology         namics of single-field slow-roll inflationary model         Conditions for slow-rolling of the inflaton         turbations during Inflation   | <ul> <li>65</li> <li>70</li> <li>74</li> <li>76</li> <li>78</li> <li>82</li> <li>85</li> <li>86</li> <li>88</li> </ul>                         |  |
| 6<br>A<br>B<br>C | 5.3<br>5.4<br>5.5<br>Cor<br>Bas<br>Dyr<br>B.1<br>Per<br>C.1  | Non-Gaussianity in bispectrum from thermal distribution of inflaton         Non-Gaussianity in trispectrum from thermal distribution of inflaton         Conclusion         conclusion         ics of FLRW Cosmology         namics of single-field slow-roll inflationary model         Conditions for slow-rolling of the inflaton         turbations during Inflation         Perturbed Einstein field equations   | <ul> <li>65</li> <li>70</li> <li>74</li> <li>76</li> <li>78</li> <li>82</li> <li>85</li> <li>86</li> <li>88</li> <li>89</li> </ul>             |  |
| 6<br>A<br>B<br>C | 5.3<br>5.4<br>5.5<br>Cor<br>Bas<br>Dyr<br>B.1<br>Per<br>C.1<br>C.2                                     | Non-Gaussianity in bispectrum from thermal distribution of inflaton         Non-Gaussianity in trispectrum from thermal distribution of inflaton         Conclusion         Conclusion         ics of FLRW Cosmology         namics of single-field slow-roll inflationary model         Conditions for slow-rolling of the inflaton         turbations during Inflation         Perturbed Einstein field equations         Perturbed Klein-Gordon Equation | <ul> <li>65</li> <li>70</li> <li>74</li> <li>76</li> <li>78</li> <li>82</li> <li>85</li> <li>86</li> <li>88</li> <li>89</li> <li>94</li> </ul> |  |

| D   | Esse          | entials of de Sitter spacetime  | 96  |  |  |
|---|---------------|---|-----|--|--|
|   | D.1           | Horizon in de Sitter spacetime  | 96  |  |  |
|   | D.2           | Quasi de Sitter spacetime   | 97  |  |  |
| E Uniqueness of mode functions in Minkowski spacetime |               |   | 98  |  |  |
|   | E.1           | Bogolyubov transformation   | 98  |  |  |
|   | E.2           | Preferred mode functions in Minkowski space                                   | 99  |  |  |
| $\mathbf{F}$  | Thr           | ee-point and connected four-point function of $\mathcal{R}_{NL}$              | 101 |  |  |
|   | F.1           | Calculating three-point correlation function of $\mathcal{R}_{NL}$            | 101 |  |  |
|   | F.2           | Calculating connected four-point correlation function of $\mathcal{R}_{NL}$   | 103 |  |  |
| G   | $\mathbf{KL}$ | representation in Minkowski space   | 106 |  |  |
|   | G.1           | Feynman propagator for interacting scalar field $\ldots \ldots \ldots \ldots$ | 107 |  |  |
|   | G.2           | Wightman function for interacting scalar field                                | 108 |  |  |
| List of Publication 117                               |               |   |     |  |  |

## List of Figures

| 2.1 | The 7-year temperature (TT) power spectrum from WMAP. $\ldots$                              | 27 |
|-----|---|----|
| 3.1 | $\mathcal{P}_{\mathcal{R}}(k)$ vs. k plot for decaying scalar inflaton                      | 44 |
| 3.2 | The TT angular spectrum for the inflaton with a decay width                                 | 45 |
| 3.3 | $\mathcal{P}_{\mathcal{R}}(k)$ vs k plot for composite inflaton                             | 47 |
| 3.4 | The TT angular spectrum for the inflaton as a composite particle                            | 48 |
|     |   |    |
| 4.1 | $\log\left(\frac{M_u}{\text{GeV}}\right)$ vs. $d_u$ plot for tensor unparticles             | 57 |
| 4.2 | $-\log(\lambda)$ vs. $d_u$ plot for vector unparticles $\ldots \ldots \ldots \ldots \ldots$ | 59 |
| 4.3 | The TT angular spectrum for the Unparticle-inflaton $\ldots$                                | 61 |
|     |   |    |
| 5.1 | ${\rm TT}$ spectrum for a inflationary scenario with prior radiation era $~$ .              | 68 |
| 5.2 | ${\rm TT}$ spectrum for a inflationary scenario with prior radiation era $~$ .              | 69 |
| 5.3 | $\frac{f_{NL}^{\rm th}}{f_{NL}}$ vs. $\beta k$ plot   | 73 |
| 5.4 | $	au_{NL}^{\mathrm{th}}$ vs. $\beta k$ plot   | 76 |