

STUDIES OF NONLINEAR WAVES IN PLASMAS

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THE DEGREE OF**

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BY

N. NAGESHA RAO

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PHYSICAL RESEARCH LABORATORY
AHMEDABAD**

TO
MY UNCLE

SARODE NARAYANA RAO

C E R T I F I C A T E

I hereby declare that the work presented in this Thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

N. Nagesha Rao

N. NAGESHA RAO
(Author)

Certified by :

RK Varma

RAM K. VARMA
(Professor-in-charge)

Ahmedabad
JULY 10, 1981

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ABSTRACT OF THE THESIS

We investigate, in this Thesis, some problems pertaining to the existence and propagation of nonlinear ion acoustic waves and nonlinear amplitude modulated, high frequency Langmuir waves and the associated low frequency ion waves. In the case of nonlinear ion acoustic waves, we first investigate the physical implications of the different sets of stretched co-ordinates employed in the Reductive Perturbation Method for obtaining the relevant evolution equation, namely, the Korteweg-de Vries equation. It is found that the two sets of co-ordinates yield different Korteweg-de Vries equations in the (x,t) co-ordinate system and that the initial value problems associated with these equations are different. The experimental implications of these differences have been discussed. We investigate, next, the effect of plasma inhomogeneities on the propagation characteristics of the nonlinear ion acoustic waves using the Reductive Perturbation Analysis. In the presence of spatial gradients in the ion density and ion temperature, these waves are found to be governed by a modified Korteweg-de Vries equation. Soliton solution of this equation shows that as the nonlinear ion acoustic waves propagate along the ion temperature (or density) gradient, their amplitudes are reduced. A linear analysis of the problem of ion acoustic wave propagation in inhomogeneous plasmas has also been carried out.

For the problem of nonlinear, amplitude modulated Langmuir waves we develop a theory valid in the entire range of the soliton Mach number, namely, $0 < M < 1$. A set of governing equations for the

stationary propagation of the high frequency Langmuir waves and the associated ion waves has been derived by taking into account the full ion nonlinearity and complete departures from the charge neutrality for the low frequency ion waves. A method is then developed to solve these coupled, nonlinear equations. This method is capable of taking into account any arbitrary degree of ion nonlinearity, consistent with the nonlinearity retained in the Langmuir field amplitude. A class of double-hump Langmuir solitons having non-zero Langmuir field intensity at the centre is found for intermediate values of the Mach number in the range $0 < M < 1$. These solutions are found to provide a smooth transition from single-hump Langmuir solitons to the double-hump Langmuir solitons having zero Langmuir field intensity at the centre. The regions of parameter values for the existence of different types of Langmuir soliton solutions are explicitly obtained. The existence and structure of these solutions have been studied by means of the Sagdeev Potential Analysis. The theory developed here yields, under suitable limiting conditions, various Langmuir soliton solutions discussed earlier by other authors. Finally, a conjecture is made about the existence of many-hump Langmuir solitons for higher order nonlinearities in the low frequency ion potential.

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