

# Beyond The Standard Model Physics: Grand Unification & Otherwise

A THESIS

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*There is a fine line between wrong & visionary.  
Unfortunately, one has to be a visionary to see it.*

Dr. Sheldon Cooper (*fictitious character*)  
[The Big Bang Theory: Season 3, Episode 04]

# DECLARATION

I **Mr. Ketankumar Patel**, S/o Mr. Mohanbhai Patel, resident of C/426, Vivekanand nagar society, Maninagar, Ahmedabad 380 008, hereby declare that the research work incorporated in the present thesis entitled, “**Beyond The Standard Model Physics: Grand Unification & Otherwise**” is my own work and is original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma. I have properly acknowledged the material collected from secondary sources wherever required. I solely own the responsibility for the originality of the entire content.

**Date:**

**Ketankumar Patel**  
**(Author)**

# CERTIFICATE

I feel great pleasure in certifying that the thesis entitled, “**Beyond The Standard Model Physics: Grand Unification & Otherwise**” by Mr. Ketankumar Patel under my guidance. He has completed the following requirements as per Ph.D regulations of the University.

- (a) Course work as per the university rules.
- (b) Residential requirements of the university.
- (c) Regularly submitted six monthly progress reports.
- (d) Presented his work in the departmental committee.
- (e) Published minimum of one research papers in a referred research journal.

I recommend the submission of thesis.

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Ketan M. Patel

## ABSTRACT

Grand unified theories (GUTs) which unify strong and electroweak interactions also provide a constrained and unified description of all the fermions. In particular, the renormalizable versions of GUTs based on  $SO(10)$  gauge group are considered as excellent platforms to study the peculiar patterns of quark and lepton masses and mixing angles observed in nature. In this thesis, we examine many such  $SO(10)$  models for their viability or otherwise in explaining all the fermion masses and mixing angles. The different  $SO(10)$  models, which we study in this work, can be divided in three main categories: (i) the supersymmetric  $SO(10)$  models (ii) the supersymmetric  $SO(10)$  models with flavor symmetries and (iii) the non-supersymmetric  $SO(10)$  models.

First we carry out an exhaustive analysis of supersymmetric models with minimal  $(10_H + \overline{126}_H)$  and non-minimal  $(10_H + \overline{126}_H + 120_H)$  Higgs content. Extensive numerical fits to fermion masses and mixing are carried out in each case assuming dominance of type-II or type-I seesaw mechanism. We use data corresponding to different values of  $\tan\beta$  and with or without appreciable finite supersymmetric threshold corrections. All the cases studied provide quite good fits if the type-I seesaw mechanism dominates. This is not the case in the minimal model based on several data sets and type-II seesaw mechanism. This can be traced to the absence of the  $b$ - $\tau$  unification at the GUT scale in these cases. In contrast, the type-II seesaw mechanism works uniformly well in all the non-minimal model. Required scale of the  $B - L$  breaking is identified in each case. In the case of type-I seesaw dominance in both the minimal and non-minimal models, it is observed that the  $B - L$  breaking scale inferred from neutrino masses lies closer to the GUT scale compared to the type-II seesaw mechanism.

In  $SO(10)$  models, it is not clear if the exact tribimaximal mixing (TBM) among leptons would be consistent with a precise description of the quark masses and mixing. We address this question by developing a novel formalism which allows determination of most general structures of the neutrino and charged lepton mixing matrices consistent with tribimaximal mixing. These

are then integrated into an  $SO(10)$  model within which detailed fits to fermion masses and mixing angles are given. It is shown that one can obtain excellent fits to all the fermion masses and quark mixing angles keeping tri-bimaximal leptonic mixing intact. Various perturbations to tri-bimaximal mixing which can arise in the model are considered and their impact on the predictions of the reactor mixing angle  $\theta_{13}$  is numerically discussed.

In the second part of this thesis, we study some interesting consequences of different flavor symmetries when integrated with supersymmetric  $SO(10)$  frameworks. First we integrate the  $\mu$ - $\tau$  symmetry in the non-minimal supersymmetric  $SO(10)$  model. This scenario is shown to lead to a generalized CP invariance of the mass matrices and vanishing CP violating phases if the Yukawa couplings are invariant under the  $\mu$ - $\tau$  symmetry. Small explicit breaking of the  $\mu$ - $\tau$  symmetry is then shown to provide a very good understanding of all the fermion masses and mixing. Second we propose a specific ansatz for the structure of Yukawa matrices in  $SO(10)$  models which differ from its generic expectations of hierarchical neutrinos and lead to quasi degenerate neutrino masses through the type-I seesaw mechanism. Consistency of this ansatz is demonstrated through a detailed fits to fermion masses and mixing angles all of which can be explained with reasonable accuracy in a model which uses the most general Yukawa Higgs sector of  $SO(10)$ . The proposed ansatz is shown to follow from an extended model based on the three generations of the vector like fermions and an  $O(3)$  flavour symmetry. Large neutrino mixing angles emerge as a consequence of neutrino mass degeneracy in this model. In the last section in this category, we propose a discrete symmetry  $S_4 \times Z_n$  when suitably integrated with  $SO(10)$  provides a viable framework to obtain an interesting empirical relation called Quark-Lepton Complementarity (QLC), namely  $\theta_{12}^l \sim \pi/4 - \theta_C$ . Consistency of this model is discussed through detailed analysis of fermion masses and mixing angles. The model leads to the lepton mixing matrix that is dominantly bimaximal with  $\mathcal{O}(\theta_C)$  corrections related to quark mixings which generically predicts the large reactor angle  $\theta_{13}^l \sim \theta_C/\sqrt{2}$ .

In the last part of this thesis, we carry out a detailed study of some non-

supersymmetric  $SO(10)$  models. The non-supersymmetric  $SO(10)$  models with a global  $U(1)_{PQ}$  symmetry lead to the similar Yukawa sum-rules predicted in the supersymmetric models. Detailed numerical fits to fermion masses and mixing are carried out in each case assuming dominance of type-II or type-I seesaw mechanism. It is shown that the minimal non-supersymmetric model with type-I seesaw dominance gives excellent fits. In the presence of a  $45_H$  and an intermediate scale, the model can also account for the gauge coupling unification making it potentially interesting model for the complete unification. Structure of the Yukawa coupling matrices obtained numerically in this specific case is shown to follow from a very simple  $U(1)$  symmetry and a Froggatt-Nielsen singlet. The non-minimal model with type-I seesaw also provides an excellent fit to entire fermion spectrum while the model with  $120_H + \overline{126}_H$  fails badly in this task. It is observed that type-II seesaw dominance is disfavored in all the models because of non-unification of  $b$  quark and  $\tau$  masses at the GUT scale in the absence of supersymmetry.