043
MAJ
15357

# NUCLEAR LEVEL DENSITIES AND OTHER STATISTICAL QUANTITIES WITH INTERACTIONS IN SPECTRAL AVERAGING THEORY: SYSTEMATIC STUDIES AND APPLICATIONS

BY

DEBASISH MAJUMDAR

PHYSICAL RESEARCH LABORATORY
AHMEDABAD 380 009, INDIA

A THESIS

submitted to

The Maharaja Sayajirao University of Baroda
Baroda 390 002, India

For the degree of

Doctor of Philosophy in Physics

DECEMBER 1994

043
B15357

तस्मादसक्तः सततं कार्यं कर्म समाचर ।
असक्तो ह्याचरन्कर्म परमाप्नोति पूरुषः ।।

कर्मयोग, श्रीमद् भगवद्गीता

*"Therefore, without attachment, perform always the work that has to be done, for man attains to the highest by doing work without attachment."*

*Karmayoga, Shrimad Bhagavat Gita*

Translated by Dr. Sarvepalli Radhakrishnan

# CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

Debasish Majumdar

Debasish Majumdar

(Author)

**Certified by :**

Dr. V. Krishna Brahmam Kota

(Guide)

Physical Research Laboratotry,

Ahmedabad 380 009, India.

Prof. Vijay N. Potbhare

(Co-Guide)

The M.S. University of Baroda,

Baroda 390 002, India.

## Acknowledgements

I am greatly indebted to Dr. V.K.B. Kota for his relentless effort and meticulous guidance during the period of my research work. Without his keen interest and tenacity the thesis wouldn't have taken the present shape. I profusely thank and acknowledge him for extending me this favour.

I am grateful to Prof. V.N. Potbhare for guidance, encouragement and for many discussions, from time to time.

I am thankful to Dr. R.U. Haq for permitting me to use freely the package LVLDEN_INT which has been used for calculations throughout the thesis and for valuable inputs I received through Dr. V.K.B. Kota, Dr. K. Jain for collaboration during the initial stages of the work reported in Chapter 4 and Prof. K. Kar and Dr. S. Sarkar for supplying some of their results prior to publication.

During the course of my work I benefitted a lot from the discussions Dr. V.K.B. Kota had with Prof. J.B. French. They are helpful in many ways and most of the studies presented in the thesis have grown out of these discussions. I am also grateful to Prof. J.B. French for his keen interest in the work carried out in this thesis. I thank Profs. J.C. Parikh and S.B. Khadkikar for some valuable suggestions and comments for betterment of the thesis.

I express my gratitude to Prof. R.K. Varma, Director, Physical Research Laboratory for a grant towards computational work at Institute for Plasma Research (IPR) and Prof. Abhijit Sen for facilities at IPR. I thank Ganesh, Sudip and other students of IPR for being extremely cooperative during my visits to IPR. Thankfully I acknowledge the help I have received from Drs. D.R.

Kulkarni, B.R. Sitaram and Sai Iyer towards computational work. Thanks are due to Dr. Y.D. Devi for many useful suggestions and comments from time to time during the course of my thesis work.

This acknowledgement would never be complete if I do not mention various other facilities that I have availed and the people associated with them. To mention a few, this includes Mr. Ghanashyam Dholakia and all other staff members of PRL Computer Centre, Mrs. Urmila Ghiya, Mrs. Rohini Patil and other staff members of PRL library, Mr. D.R. Ranpura of the photography section and Mr. S.C. Bhavsar of drafting section. I remember them with deep gratitude for the support and service I have obtained from them during my thesis work.

It is impossible to name all of my friends and colleagues at PRL hostel and PDF quarters in this limited space whose constant moral support and lively company have been a source of joy, pleasure and refreshment in many ups and downs during my stay at Ahmedabad. Himadri Chakrabarty and his family, S. Ramachandran, M.S. Santhanam, Devashis Banerjee, G. Srinivasan, V. Sivakumaran, Sam Ragland, Tarun Pant, D. Pallam Raju, V. Prahlad, W. Watson, P. Poulose and Biju Thomas to name a few. Thanks are due to Mr. T.P. Muralidharan for some secretarial help. My experiences, stretching beyond the curricular boundaries at PRL, still remain green in my memory. Every time, I derive more and more pleasure and feel rejuvenated as I reveal those with a new dose of vision and imagination. I always try to enshrine them from not being scattered through the winds of time.

Words are perhaps inadequate to express my overwhelming gratitude to my parents back home in Calcutta. They have extended their full moral support

during this long period of my thesis work. I am grateful to my other family members and relatives for providing me constant inspiration through their letters or otherwise.

I conclude by extending my thanks to the innumerable tax payers of my country for supporting me indirectly.

# Abstract

The subject of statistical nuclear spectroscopy, that started with Bethe's statistical mechanical level density formula , Wigner's treatment of spectral fluctuations using matrix ensembles and French's embedded ensembles and Gaussian densities, continues to be an important field of research as it has large variety of applications in nuclear structure and nuclear astrophysics. Statistical spectroscopy divides into two parts; one is spectral averages which give smoothed (with respect to energy) forms of observables and the other being spectral fluctuations around these smoothed forms; the later are ignored for most purposes. The smoothed forms of spectroscopic observables follow from the action of Central Limit Theorems (CLT) in nuclear shell model spaces. Recent progress in statistical spectroscopy is in deriving and applying the smoothed forms in indefinitely large spaces with *interactions* by using unitary group decompositions (of hamiltonians and the spectroscopic spaces), CLT's locally, and convolutions - the resulting theory is the Spectral Averaging Theory in Large Shell Model Spaces (SAT-LSS). The aim of the present thesis is to study (and test) some aspects of SAT-LSS for state, expectation value and strength densities in large spaces and apply them to calculate nuclear level densities and $\beta$-decay rates. The problems solved to this end are described below along with corresponding results.

In SAT-LSS, (i) using a significant unitary group decomposition under which a one plus two-body nuclear hamiltonian decomposes into orthogonal effective one-body part $\mathbf{h}$ and irreducible two-body part $\mathbf{V}$ ($H \rightarrow \mathbf{h} + \mathbf{V}$); (ii) applying the CLT's locally and (iii) decomposing the shell model space

into distant non-interacting $S$ - subspaces (for light nuclei $S$ denotes $\hbar\omega$ excitation), the state densities take a convolution form, $I^H = \sum_S I^{\mathrm{h},S} \otimes \rho_{\mathcal{G}}^{\mathrm{V},S}[E]$ where $I^{\mathrm{h}}$ is renormalized non-interacting particle (NIP) state density and $\rho_{\mathcal{G}}$ is a normalized spreading Gaussian. Similar convolution forms are derived for spin-cutoff and occupancy densities. Firstly, systematic studies of two important aspects of the level density theory are carried out in the thesis and they are: (a) a part of the interaction that produces non-NIP like shifts of the single particle energies, that is neglected in the theory, is conclusively demonstrated (using norms of operators and considering large number of effective interactions) to be indeed small all across the periodic table; (b) moment methods for constructing locally smoothed forms for NIP state, spin-cutoff and occupancy densities which are one of the convolution factors in the IP theory, are comprehensively tested for the three densities and also for their $S$ - decompositions in an example of 12 protons and 12 neutrons in a large shell model space consisting of 16 orbits ($s$, $p$, $ds$, $fp$, $sdg$, $1h_{11/2}$) with $0 - 4$ $\hbar\omega$ excitations; moment methods give an excellent representation of exact results.

For a first systematic analysis of experimental data using the convolution forms for state and spin-cutoff densities, in a given region of the periodic table, $fp$-shell nuclei are chosen as the example. Nuclei in this region of the periodic table have acquired new significance recently as some of them play an important role in nuclear astrophysics problems, double $\beta$ - decay, problem of quenching of $GT$ sum rule strengths etc. The calculations, using surface delta interaction with strength $G$, are carried out in large spaces by including configurations upto $2\hbar\omega$ excitations and by employing eight spherical orbits ($ds$, $fp$, $1g_{9/2}$) for the five nuclei $^{55}Mn$, $^{56}Fe$, $^{59}Co$, $^{60}Co$ and $^{60}Ni$ and by extending for obvious reasons, to ten spherical orbits ($ds$, $fp$, $1g_{9/2}$, $2d_{5/2}$,

$1g_{7/2}$) for the three $A > 60$ nuclei $^{62}Ni$, $^{63}Cu$ and $^{65}Cu$. In general the convolution form for the densities is seen to provide an excellent representation of the observed total level densities and spin-cutoff factors. The calculations determine a magnitude parameter of the interaction - the strength $G$ which is found to be $\sim 20/A\ MeV$ for all the nuclei, is compatible with the values deduced from low-energy spectroscopy.

In SAT-LSS the bivariate strength density $\mathbf{I}^{H}_{\mathcal{O}}(E_i, E_f)$ for a transition operator $\mathcal{O}$ takes a convolution form, $\mathbf{I}^{H}_{\mathcal{O}}(E_i, E_f) = \mathbf{I}^{\mathbf{h}}_{\mathcal{O}} \otimes \rho^{\mathbf{V}}_{BIV-\mathcal{G};\mathcal{O}}[E_i, E_f]$. Leaving aside the spreading bivariate Gaussian $\rho^{\mathbf{V}}_{BIV-\mathcal{G};\mathcal{O}}$, one needs good methods for constructing the NIP density $\mathbf{I}^{\mathbf{h}}_{\mathcal{O}}$ in large spaces. For one-body transition operators, a complete formalism for constructing the exact NIP strength densities using spherical orbits is worked out. For rapid construction of the bivariate NIP densities and also to carry out $S$-decomposition, trace propagation formulas for bivariate moments $M_{rs}$ with $r + s \leq 2$ are derived for unitary orbit partial densities which determine the bivariate Gaussian representation for the partial densities. This formalism is tested in a large space numerical example with $GT(\beta^-)$ transition operator. Trace propagation formulas for $M_{rs}$ with $r + s \leq 4$ are also derived in $m$-particle scalar spaces, which are useful for many purposes.

A method to calculate $\beta$-decay rates at finite temperature is developed by writing the expression for the rates explicitly in terms of bivariate $GT$ strength densities ($\mathbf{I}^{H}_{\mathcal{O}(GT)}$) and state densities of parent nucleus besides having the usual phase space factors. For constructing $\mathbf{I}^{H}_{\mathcal{O}(GT)}$, the method developed in this thesis for constructing NIP strength densities is applied together with a plausible

procedure for generating the $GT$ bivariate spreading Gaussians due to interactions. Recently there is considerable interest in evaluating $\beta$-decay rates for $A > 60$ neutron excess $fp$-shell nuclei as these rates play a very important role in determining the structure of the core of massive supernova stars and hence on their subsequent evolution towards gravitational collapse and supernova explosion phases. Therefore as an example, using our method, $\beta$-decay rates for $^{61,62}Fe$ and $^{62-64}Co$ isotopes are calculated at presupernova matter densities $\rho = 10^7 - 10^8$ gm/cc, temperatures $T = (3-5) \times 10^9\ ^oK$ and for various values of the electron fraction $Y_e$. As a biproduct, the calculations produced a value (via observed $\beta$-decay half lifes) for the bivariate correlation coefficient $\bar{\zeta}$ between the hamiltonian and the $GT$ operator to be $\bar{\zeta} \sim 0.67$, and thus new information about the nuclear hamiltonians is deduced. In addition an expression for the $GT$ non-energy weighted summed strengths is also deduced.

To summarize, SAT-LSS is explored by carrying out first systematic analysis of $fp$-shell level density data using the convolution forms for the state and spin-cutoff densities, developed further by deriving an essentially complete theory for constructing NIP strength densities produced by one-body transition operators, and applied to neutron excess $fp$-shell nuclei relevant for presupernova evolution calculations in nuclear astrophysics by developing a method to calculate $\beta$-decay rates at finite temperature in terms of state and bivariate strength densities. The results of the present thesis together with the earlier investigations, show that the convolution forms for state, expectation value and strength densities in large spaces as given by SAT-LSS can be used profitably to study a variety of nuclear physics and nuclear astrophysics problems.

# Contents


**Abstract** vii

**Chapters**

**1 Introduction** 1

**2 Some Mathematical Aspects of Spectral Averaging Theory** 22

2.1 Preview . . . . . 22

2.2 Statistical distributions . . . . . 23

2.2.1 Univariate distributions . . . . . 23

2.2.2 Bivariate distributions . . . . . 26

2.3 State, expectation value and strength densities: CLT results . 31

2.3.1 State densities . . . . . 31

2.3.2 Expectation values and expectation value densities . . 32

2.3.3 Strength densities . . . . . 34

2.4 Partitioning : spherical and unitary configurations . . . . . 36

2.5 Unitary decomposition of operators . . . . . 40

2.5.1 $U(N)$ group . . . . . 40

2.5.2 Spherical configuration group $U(N_\alpha) \oplus U(N_\beta) \oplus$ ..... . . 43

2.5.3 Unitary configuration group $U(N_{\boldsymbol{\alpha}}) \oplus U(N_{\boldsymbol{\beta}}) \oplus$ ..... . . 44

2.6 Trace propagation . . . . . 46

2.6.1 $U(N)$ group averages . . . . . 46

2.6.2 Spherical configuration averages . . . . . 51

2.6.3 Unitary configuration averages . . . . . 52

2.7 Summary . . . . . 55

**3 Tests of Some Aspects of Spectral Averaging Theory in Large Shell Model Spaces for State, Spin-Cutoff and Occupancy Densities** **56**

3.1 Introduction . . . . . 56

3.1.1 $S$ - decomposition . . . . . 57

3.1.2 Convolution form for state densities . . . . . 61

3.1.3 Convolution forms for spin-cutoff and occupancy densities 67

3.1.4 Moment methods and convolution forms for NIP state, spin-cutoff and occupancy densities . . . . . 68

3.1.5 Summary of convolution results and conventions for pn systems . . . . . 71

3.2 Test of the smallness of $V^{[0][2]}$ part of various interactions . . . 73

3.3 Tests of moment methods for NIP state, spin-cutoff and occupancy densities . . . . . 91

3.4 Summary . . . . . 103

**4 Analysis of Level Density Data for $fp$-Shell Nuclei: Systematic Calculations Using SAT-LSS** **105**

4.1 Preview . . . . . 105

4.2 Selection of nuclei, choice of orbits and the hamiltonian and the procedure for calculations . . . . . 107

4.2.1 Choice of $fp$-shell nuclei . . . . . 107

4.2.2 Reference energy . . . . . 108

4.2.3 Choice of the single particle orbits and the two-body interaction . . . . . 110

4.2.4 Unitary orbits for spreading variances and NIP calculations . . . . . 112

4.2.5 Ratio of $J^2$-density variance and state density variance 116

4.2.6 Procedure for calculations . . . . . 118

4.3 8-orbit results for $^{55}Mn$, $^{56}Fe$, $^{59}Co$, $^{60}Co$, $^{60}Ni$ and $^{63}Cu$ . . . 121

4.3.1 Nucleus $^{55}Mn$ . . . . . 121

4.3.2 Nucleus $^{56}Fe$ . . . . . 128

4.3.3 Nucleus $^{59}Co$ . . . . . 131