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KINETIC STABILITY OF THE BENNETT EQUILIBRIUM

BY

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TO MY SISTER

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CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

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Abstract

The present thesis is devoted to a detailed study of the linear stability properties of the Bennett equilibrium. The main emphasis is on the Kinetic aspects and non-local effects arising from the large excursion betatron orbits of the particles in the inhomogeneous magnetic field. An appropriate non-local theory is developed within the framework of the Vlasov-Maxwell equations. A matrix dispersion relation is obtained whose solutions represent the eigenfrequencies of the system against electromagnetic perturbations. In general, the dispersion relation is difficult to solve analytically except in certain simple limits. The principal approach taken in this thesis is therefore a detailed numerical solution using a variety of methods (e.g. graphical scanning, Nyquist method, Muller's method etc.). Analytical solutions are also obtained in some limits to support the numerical results as well as to delineate the physical mechanisms.

The dispersion relation is used to study three important physical problems, which are related to experiments where the Bennett equilibrium provides a realistic representation of the plasma configuration. In the low frequency electrostatic

limit, an ion-acoustic type instability is studied. This mode is driven by the relative drift between electrons and ions in a two component plasma. The ion betatron motion is found to have a stabilizing influence on the mode by raising the instability threshold and by shifting the real frequency. These results are discussed in the context of microinstability observations of Z-pinch and plasma focus experiments. For the Z-pinch the $m=1$ kink instability is another very important mode, which has been studied extensively in the past, usually within the framework of MHD theory. This instability is quite sensitive to the plasma profile. A detailed study is therefore carried out for this mode both in the MHD and the kinetic limits, using the Bennett profile. Kinetic effects are again found to have an important influence. They stabilize the mode at large k - an effect not predicted by MHD theory. The third application of the dispersion relation is made to the resistive firehose instability which is studied in the context of a non-relativistic beam propagating in a resistive background plasma. This instability which arises due to the resistive phase-lag between the plasma and the magnetic field is found to be stable at very large and very small wavenumbers. Earlier theoretical models of this mode accounted

for the phase-mixing between the particle orbits arising from the radial dependence of the betatron frequency, but ignored the wave-particle effects as well as non-local effects arising from density inhomogeneities. These effects are included in our calculation, although, phase mixing is omitted. It is found that Kinetic damping effects are comparable to the damping effect due to the phase-mixing of orbits in realistic parameter regimes. Our calculation also predicts a lower cut-off in k which is not predicted by the earlier models.

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