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A STUDY OF SOME NONLINEAR INTERACTIONS  
OF KINETIC ALFVEN WAVES

BY

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TO MY PARENTS

CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

Chitra Kar

Certified:

A handwritten signature in black ink, appearing to read 'Abhijit Sen', with a horizontal line drawn underneath the signature.

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## Abstract

The present thesis is devoted to the study of some nonlinear aspects of kinetic Alfvén waves. It is motivated by the importance of kinetic Alfvén waves in supplementary heating of tokamak plasmas. In particular, near the mode conversion layer, the wave has an enhanced amplitude and can thus interact nonlinearly with other normal modes of the plasma. Two such interactions have been chosen for detailed investigation in this thesis - namely, the nonlinear excitation of tearing modes and that of drift modes.

The basic non-linear process is the parametric decay of the kinetic Alfvén wave into another Alfvén wave and a low frequency wave (the tearing or the drift mode). Several aspects of this interaction are studied - contributions from resonant (side band coupling) terms, non-resonant (ponderomotive) terms, nonlinear equilibrium drifts as well as phase mixing effects. The low frequency modes considered include resistive  $m=1$  and  $m=2$  tearing modes, collisionless tearing modes, kinetic drift modes and drift temperature modes. For the drift modes the effect of background inhomogeneity is also taken into account.

The calculations are based on both fluid and kinetic descriptions of the plasma. The method of solution is mainly analytical - relying on variational and matched asymptotic techniques. Some numerical support to the analytical results is also provided. It is found that the growth rates of the nonlinearly excited low frequency modes are quite large for realistic tokamak parameters. They can be comparable or even exceed growth rates of other nonlinear processes proposed earlier [1] for heating purposes. Since drift waves play an important role in plasma transport and can significantly influence plasma confinement, their nonlinear excitation can have serious implications for the Alfvén wave heating schemes.

A brief discussion on this aspect is made in light of some of the preliminary experimental evidence of such low frequency activity in tokamak experiments.

1. Hasegawa, A. and Chen, L., Phys. Fluids 19 1924 (1976)

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