Physical Research Laboratory, Ahmedabad

Mathematical and Numerical Methods
Test-III, 2012

Time: 90 Minutes Total Marks: 50

Instructions: (1) All questions are compulsory.
(2) The symbols have usual meanings.
(3) The numbers to the right indicate marks.
(4) The use of un-programmable calculator is permitted.
(5) Support your answers with diagrams, if applicable, along with the detailed steps.

Q. 1 (a) Find the polynomial of degree three which takes the values prescribed below **(8)**

x_k	0	1	2	4
y_k	1	1	2	5

(b) For k = -1, 0, 1; verify that **(5)**

$$y_k = y_0 + \binom{k}{1} \delta y_{-1/2} + \binom{k+1}{2} \delta^2 y_0$$

Q. 2 Prove that **(10)**

$$[x_0, x_1, x_2] = \frac{\begin{vmatrix} 1 & x_0 & y_0 \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{vmatrix}}{\begin{vmatrix} 1 & x_0 & {x_0}^2 \\ 1 & x_1 & {x_1}^2 \\ 1 & x_2 & {x_2}^2 \end{vmatrix}}$$

Q. 3 Find the root of the following equation correct to two decimal places by Horner's method **(9)**

$$f(x) = 2x^3 - 6x^2 + 2x - 1$$

Q. 4 Starting with (0, 0), apply Jacobi's method to the equations **(10)**

$$x - 5y = -4 \quad (1)$$
$$7x - y = 6 \quad (2)$$

up to three iterations. Now, interchange the equations (1) and (2) and apply Jacobi's method to the new set of equations up to three iterations. What are your observations?

Q. 5 Calculate the value of $\int_0^{\pi/2} \sin x \ dx$ by Simpson's one-third rule, using 11 ordinates. Use minimum three places after the decimal point in the calculations. Give the result without rounding. **(8)**

Hint 1. $\int_{x_0}^{x_0+n\,h} f(x)\ dx = \frac{h}{3}\left[(y_0 + y_n) + 4\,(y_1 + y_3 + \cdots + y_{n-1}) + 2\,(y_2 + y_4 + \ldots + y_{n-2})\right]$

2. $\binom{-k}{i} = \frac{(-k)\,(-k-1)\ldots(-k-i+1)}{i!}$