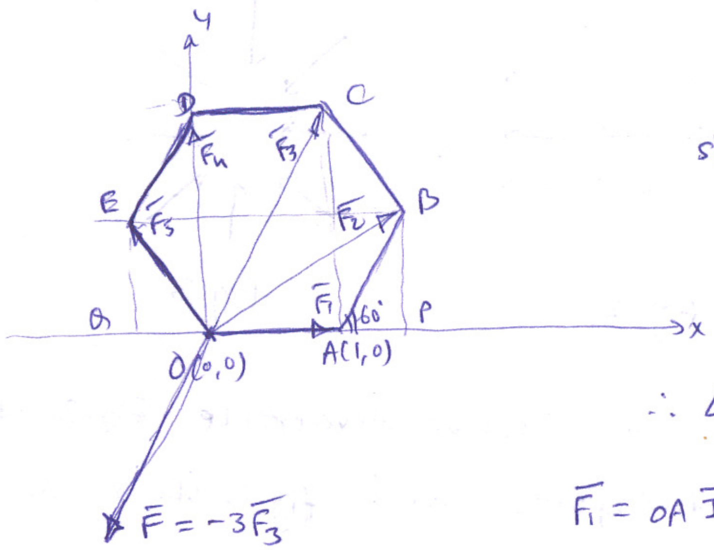
MNM Test-2 Solution

Ans. 1(a)



For regular hexagon OABCDE shown in the figure,

$OA = AB = 1$ and

internal angle is $\frac{(n-2)\times 180}{n} = 120°$

$\therefore \angle BAP = 60°$

$\bar{F_1} = OA\,\bar{I} = \bar{I}$ —— (1)

$BP = AB\sin 60° = 0.87$, $AP = AB\cos 60° = 0.5$

$\bar{F_2} = OP\,\bar{I} + BP\,\bar{J} = 1.5\,\bar{I} + 0.87\,\bar{J}$ —— (2)

$AC = 2BP = 2(0.87) = 1.74$, $F_3 = OA\,\bar{I} + AC\,\bar{J} = \bar{I} + 1.74\,\bar{J}$ —— (3)

$\bar{F_4} = OD\,\bar{J} = AC\,\bar{J} = 1.74\,\bar{J}$ —— (4)

$\bar{F_5} = -OQ\,\bar{I} + EQ\,\bar{J} = -AP\,\bar{I} + BP\,\bar{J} = -0.5\,\bar{I} + 0.87\,\bar{J}$ —— (5)

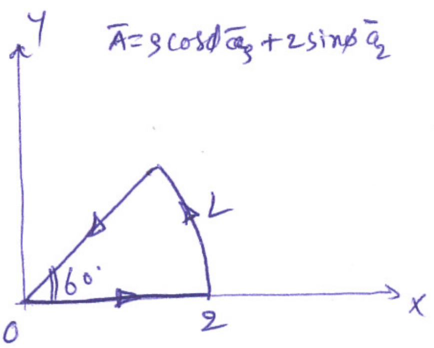
Resultant force $\bar{R} = \bar{F_1} + \bar{F_2} + \bar{F_3} + \bar{F_4} + \bar{F_5} = 3\bar{I} + 5.22\bar{J}$ (From (1), (2), (3), (4) and (5))

$\bar{R} = 3\bar{F_3}$

Balancing force $\bar{F} = -\bar{R} = -3\bar{F_3} = -3\bar{I} - 5.22\bar{J}$

Ans. 1(b)

$\bar{A} = \rho\cos\phi\,\bar{a}_\rho + 2\sin\phi\,\bar{a}_z$



$$\oint \bar{A}\cdot d\bar{L} = \int_{\rho=0}^{\rho=2} \bar{A}\cdot d\rho\,\bar{a}_\rho + \int_{\phi=0°}^{\phi=60°} \bar{A}\cdot \rho\, d\phi\,\bar{a}_\phi + \int_{\rho=2}^{\rho=0} \bar{A}\cdot d\rho\,\bar{a}_\rho$$

$$= \int_{\rho=0}^{\rho=2} \rho\cos\phi\, d\rho + \int_{\phi=0°}^{\phi=60°} 0\, d\phi + \int_{\rho=2}^{\rho=0} \rho\cos\phi\, d\rho$$

$$= \int_{\rho=0}^{\rho=2} \rho\cos 0°\, d\rho + \int_{\rho=2}^{\rho=0} \rho\cos 60°\, d\rho$$

$$= \int_{\rho=0}^{\rho=2} \rho\, d\rho + 0.5\int_{\rho=2}^{\rho=0} \rho\, d\rho$$

$$= \left[\frac{\rho^2}{2}\right]_0^2 + 0.5\left[\frac{\rho^2}{2}\right]_2^0 = 2 + 0.5(-2) = 2 - 1 = 1$$

Ans. 2 (a)

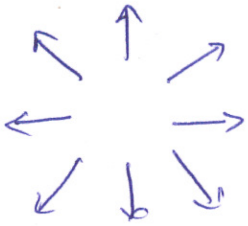


Fig. 1

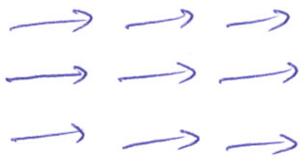


Fig. 2

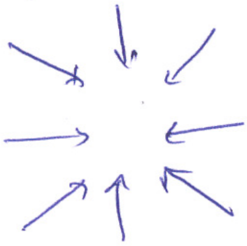


Fig. 3

Figures 1, 2 and 3 show the vector fields with positive divergence, zero divergence and negative divergence, respectively.

Ans. 2 (b)

$\bar{P} = x^2 y z \, \bar{a}_x + x z \, \bar{a}_z$

$\bar{\nabla} \cdot \bar{P} = \frac{\partial}{\partial x}(P_x) + \frac{\partial}{\partial y}(P_y) + \frac{\partial}{\partial z}(P_z) = \frac{\partial}{\partial x}(x^2 y z) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(xz)$

$= 2xyz + x$

The point is given in the cylindrical system as $(1, 1, 1)$

$\rho = 1$, $\phi = 1$ in radian, $z = 1$

$x = \rho \cos\phi = 1 \cos 1 = 0.54$

$y = \rho \sin\phi = 1 \sin 1 = 0.84$

$\bar{\nabla} \cdot \bar{P} = 2(0.54)(0.84)(1) + 0.54 = 1.4472$

Ans. 3

$$\frac{d^2y}{dx^2} - y = 0 \quad \text{——— (1)}$$

Assume that the series solution of (1) is

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots + a_n x^n \quad \text{——— (2)}$$

$$\frac{dy}{dx} = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots + n a_n x^{n-1}$$

$$\frac{d^2y}{dx^2} = 2a_2 + 6a_3 x + 12 a_4 x^2 + \cdots + n(n-1) a_n x^{n-2} \quad \text{——— (3)}$$

From (1), (2) and (3)

$$\left(2a_2 + 6a_3 x + 12 a_4 x^2 + \cdots + n(n-1) a_n x^{n-2}\right)$$

$$- \left(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots + a_n x^n\right) = 0$$

Equating all coefficients to zero

$2a_2 - a_0 = 0 \qquad \therefore a_2 = \frac{a_0}{2} = \frac{a_0}{2!}$

$6a_3 - a_1 = 0 \qquad \therefore a_3 = \frac{a_1}{6} = \frac{a_1}{3!}$

$12a_4 - a_2 = 0 \qquad \therefore a_4 = \frac{a_2}{12} = \frac{a_0}{12 \cdot 2} = \frac{a_0}{4!}$

$(n+2)(n+1)a_{n+2} - a_n = 0 \qquad \therefore a_{n+2} = \frac{a_n}{(n+2)(n+1)}$

For even n, $a_n = \frac{a_0}{n!}$ and for odd n, $a_n = \frac{a_1}{n!}$

$\therefore y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \text{---} + a_n x^n$

$\therefore y = a_0\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots\right) + a_1\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right)$

Ans. 4 $\quad \left(\frac{dy}{dx}\right)^2 (2-3y)^2 = 4(1-y)$

$\therefore (y')^2 (2-3y)^2 = 4(1-y) \quad \text{—— (1)}$

p-discriminant $\quad$ Diff. (1) w.r.t y', $\quad 2y'(2-3y)^2 = 0$

$\therefore y'(2-3y)^2 = 0 \qquad \therefore (y')^2 (2-3y)^4 = 0 \quad \text{—— (2)}$

From (1) and (2) $\quad \frac{4(1-y)}{(2-3y)^2}(2-3y)^4 = 0$

$\therefore (1-y)(2-3y)^2 = 0 \quad \text{—— (3)}$

general solution $\quad$ From (1), $\quad \left(\frac{dy}{dx}\right)^2 = \frac{4(1-y)}{(2-3y)^2}$

$\therefore \frac{dy}{dx} = \pm \frac{2\sqrt{1-y}}{(2-3y)}$

$\therefore \frac{(2-3y)}{2\sqrt{1-y}}\, dy = \pm\, dx$

Integrating, $\int \frac{2-3y}{2\sqrt{1-y}}\,dy = \pm \int dx + C$

For $1-y=t$ in left side integral, $dy = -dt$

$2-3y = 2-3(1-t) = 3t-1$

$$\int \frac{3t-1}{2\sqrt{t}}(-dt) = \pm x + c$$

$$\int \frac{1-3t}{2\sqrt{t}}\,dt = \pm x + c$$

$$\therefore \int \frac{1}{2}t^{-1/2} - \frac{3}{2}t^{1/2}\,dt = \pm x + c$$

$$\therefore \frac{1}{2}\left(\frac{t^{1/2}}{1/2}\right) - \frac{3}{2}\left(\frac{t^{3/2}}{3/2}\right) = \pm x + c$$

$$\therefore t^{1/2} - t^{3/2} = \pm x + c$$

$$\therefore \sqrt{t}\,(1-t) = \pm x + c$$

$$\therefore \sqrt{1-y}\,(y) = \pm x + c$$

$$\therefore (1-y)\,y^2 = (x+c)^2 \quad \text{—— (4)}$$

The general Solution is $(1-y)y^2 = (x+c)^2$

c-discriminant Diff. (4) w.r.t c

$0 = 2(x+c) \qquad \therefore x+c=0$

From (4) $(1-y)y^2 = 0$ —— (5)

$\psi_p = (1-y)(2-3y)^2 = 0$

$\psi_c = (1-y)\,y^2 = 0$

The equation of envelope is $1-y=0$ $\therefore \boxed{y=1}$ is a singular solution

The equation of Tac locus is $(2-3y)^2=0$

$\therefore \boxed{y=\frac{2}{3}}$

The equation of Node locus is $y^2=0$ $\therefore \boxed{y=0}$

Ans. 5 $\sqrt{\frac{\partial z}{\partial x}} + \sqrt{\frac{\partial z}{\partial y}} = 1$

$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ $\therefore \sqrt{p} + \sqrt{q} = 1$ —— ①

The general solution of ① is $z = ax + by + c$ where a, b and c are constants.

$\therefore p = a$, $q = b$

$\therefore \sqrt{a} + \sqrt{b} = 1$ $\therefore \sqrt{b} = 1 - \sqrt{a}$

$\therefore b = (1 - \sqrt{a})^2$

$\therefore$ $z = ax + (1 - \sqrt{a})^2 y + c$ is the solution.