

Physical Research Laboratory, Ahmedabad

Mathematical and Numerical Methods
Test-II, 2012

Time: 90 Minutes

Total Marks: 50

- Instructions:
- (1) All questions are compulsory.
 - (2) The bold letters represent vectors.
 - (3) The symbols have usual meanings.
 - (4) The numbers to the right indicate marks.
 - (5) The use of un-programmable calculator is permitted.
 - (6) Support your answers with diagrams, if applicable, along with the detailed steps.

- Q. 1 (a) Five forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_4$ and \mathbf{F}_5 with magnitudes equal to the distances between a vertex O and other vertices of a regular hexagon $OABCDE$ as shown in Figure 1 are pulling the vertex O . Find out and show in the figure, a force \mathbf{F} acting on the point O in such a way that the regular hexagon is not distorted. (7)

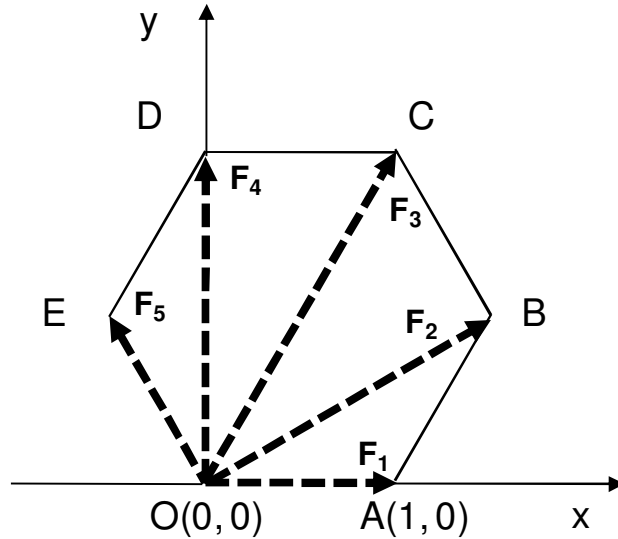


Figure 1

- (b) Calculate the circulation of $\mathbf{A} = \rho \cos \phi \mathbf{a}_\rho + z \sin \phi \mathbf{a}_z$ around the edge L of the wedge defined by $0 \leq \rho \leq 2, 0 \leq \phi \leq 60^\circ, z = 0$ as shown in Figure 2. (3)

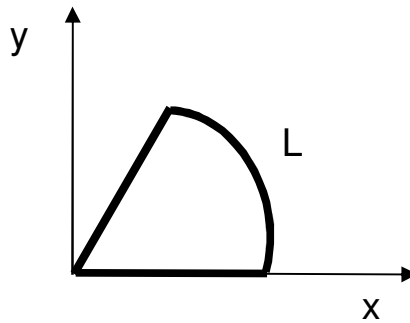


Figure 2

Q. 2 (a) Draw the vector fields whose (1) divergence is positive, (2) zero and (3) negative in a plane. **(9)**

(b) Evaluate the divergence of a field $\mathbf{P} = x^2 y z \mathbf{a}_x + x z \mathbf{a}_z$ at a point which is given in the cylindrical system as (1, 1, 1). **(6)**

Q. 3 Solve the following equation in series, starting with an assumption that the first term of the series is a constant term. **(10)**

$$\frac{d^2 y}{dx^2} - y = 0$$

Q. 4 Find the general solution, singular solution, Tac locus and Node locus of the following equation. **(10)**

$$\left(\frac{dy}{dx}\right)^2 (2 - 3y)^2 = 4(1 - y)$$

Q. 5 Solve the following equation **(5)**

$$\sqrt{\frac{\partial z}{\partial x}} + \sqrt{\frac{\partial z}{\partial y}} = 1$$