

Physical Research Laboratory, Ahmedabad

Mathematical and Numerical Methods
Test-I, 2012

Time: 90 Minutes

Total Marks: 50

- Instructions:
- (1) All questions are compulsory.
 - (2) The symbols have usual meanings.
 - (3) The use of un-programmable calculator is permitted.
 - (4) Support your answers with diagrams, if applicable, along with the detailed steps.
 - (5) The numbers to the right indicate marks.

- Q. 1 (a) A regular pentagon OABCD in the z-plane has two of its vertices as O(0, 0) and A(-1,0). Find out coordinates of the other vertices. (5)
- (b) If z_1, z_2, z_3 are vertices of an equilateral triangle, prove that $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$. (5)
- Q. 2 Find a suitable transformation that maps the upper half of z-plane to the set $P = \{z = x + iy : x > 0 \text{ and } y > 0\} \cup \{z = x + iy : x \leq 0 \text{ and } y > 1\}$. (5)
- Q. 3 (a) Calculate the power series expansion of $\text{Sin}^{-1} z$, about the point $z_0 = 0$. (5)
- (b) Determine the number of roots of the polynomial $f(z) = z^7 + 5 z^3 - z - 2$ in the unit disc. (5)
- Q. 4 Evaluate the integral $\int_0^\pi \log \text{Sin } x \, dx$ (10)
- Q. 5 During an experiment on a causal system, following observations were made: (15)

Time, t	0	1	2
Output, y	1	3	5

Derive a model function $y = a_1 t + a_2 (t - 1)^2$ to fit the observed data using pseudoinverse.

Also, find all the values of t when y takes a value of 2.