Assignment - 8 Group - 3

Problem - 1

\# Solve the system of eqns. by matrix method

$$2x - 2y + z = 1 \quad \text{—(1)}$$
$$x + 2y + 2z = 2 \quad \text{—(2)}$$
$$2x + y - 2z = 7 \quad \text{—(3)}$$

Soln

These eqns can be written as

$$\begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$$

$$AX = B, \quad A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$$

or $X = A^{-1}B$, $\quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= 2(-4-2) + 2(-2-4) + 1(1-4)$$
$$= 2(-6) + 2(-6) + 1(-3)$$
$$= -12 - 12 - 3 = -27$$

Cofactors of first row are

$-6, 6, -3$

Cofactors of second row are

$-3, -6, -6$

Cofactors of third row are

$-6, -3, 6$

$$\therefore \text{adj } A = \begin{bmatrix} -6 & 6 & -3 \\ -3 & -6 & -6 \\ -6 & -3 & 6 \end{bmatrix}' = \begin{bmatrix} -6 & -3 & -6 \\ 6 & -6 & -3 \\ -3 & -6 & 6 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{-1}{27}\begin{bmatrix} -6 & -3 & -6 \\ 6 & -6 & -3 \\ -3 & -6 & 6 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} +2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 18 \\ 9 \\ -9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\therefore \boxed{x = 2, \quad y = 1, \quad z = -1}$$

Q.2. Solve the following system of eq^n's by matrix method.

$x+2y+z=4$, $2x+y=3$, $x+z=2$

Solution:- The three eq^n's are

$$x+2y+z=4$$
$$2x+y=3$$
$$x+z=2$$

In matrix form these eq^n's can be written as

$$\begin{bmatrix}1&2&1\\2&1&0\\1&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\3\\2\end{bmatrix}$$

Assume that; $A=\begin{bmatrix}1&2&1\\2&1&0\\1&0&1\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $B=\begin{bmatrix}4\\3\\2\end{bmatrix}$

~~So $AX=B$~~

~~$\Rightarrow X=A^{-1}B=\frac{(\text{adj}A)B}{|A|}$~~

$$\text{Det } A=\begin{vmatrix}1&2&1\\2&1&0\\1&0&1\end{vmatrix}=1(1-0)-2(2-0)+1(0-1)$$
$$=1-4-1=-4\neq0$$

As $|A|\neq0$ & $B\neq0$ so A^{-1} exists.

$\therefore AX=B$

$$\Rightarrow X=A^{-1}B=\frac{(\text{adj }A)\cdot B}{|A|} \quad \text{——(1)}$$

Co-factors of matrix A are

$A_{11}=(-1)^2(1-0)=1$

$A_{12}=(-1)^3(2-0)=-2$

$A_{13}=(-1)^4(0-1)=-1$

$A_{21}=(-1)^3(2-0)=-2$

$A_{22}=(-1)^4(1-1)=0$

$A_{23}=(-1)^5(0-2)=2$

$A_{31}=(-1)^4(0-1)=-1$

$A_{32}=(-1)^5(0-2)=2$

$A_{33}=(-1)^6(1-4)=-3$

So the cofactors matrix of A is $\begin{bmatrix} 1 & -2 & -1 \\ -2 & 0 & 2 \\ -1 & 2 & -3 \end{bmatrix}$

Transpose of the co-factor matrix of $A = \begin{bmatrix} 1 & -2 & -1 \\ -2 & 0 & 2 \\ -1 & 2 & -3 \end{bmatrix} = \text{adj } A$

$\therefore$ Now adj $A \cdot B$

$$= \begin{bmatrix} 1 & -2 & -1 \\ -2 & 0 & 2 \\ -1 & 2 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 6 - 2 \\ -8 + 0 + 4 \\ -4 + 6 - 6 \end{bmatrix} = \begin{bmatrix} \cancel{8} -4 \\ -4 \\ -4 \end{bmatrix}$$

~~note~~ So eq^n (1) gives ;

$$X = \frac{(\text{adj } A) \cdot B}{|A|} = \frac{1}{-4} \begin{bmatrix} \cancel{8} -4 \\ -4 \\ -4 \end{bmatrix} = \begin{bmatrix} \cancel{2} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cancel{2} 1 \\ 1 \\ 1 \end{bmatrix}$$

So $x = 1 \cancel{2}$, $y = 1$ & $z = 1$ Ans

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& Rahul yadav
Gr-6

Assignment #8

Ques. Solve the system of equations by matrix method.

$$x+y+z=6$$
$$x+2y+3z=14$$
$$x+4y+9z=36$$

Sol Using Gauss-elimination method.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 36 \end{bmatrix}$$

Subtracting ~~first~~ 2nd & 3rd row, using row 1st.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 6 \\ 8 \\ 30 \end{pmatrix}$$

Multiply eq. row 2nd with 3 & subtract 3rd row from the 2nd row.
we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 6 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 6 \\ 24 \\ 6 \end{pmatrix}$$

$$x+y+z=6$$
$$3y+6z=24$$
$$2z=6$$

Back Substitution

$$z=3$$

$y + 2z = 8$

$y = 8 - 6 = 2$

$x + y + z = 6$

$x = 6 - 5 = 1$

$x = 1 \quad , \; y = 2 \; , \; z = 3$

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Solve the eqn. by matrix method :→

Group - 8
Assignment - 8

$x+y+z=3$; $x+2y+3z=4$; $x+4y+9z=6$

We can write,

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

or, $$AX = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

where $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Now, $$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix} = 1(18-12) + 1(3-9) + 1(4-2)$$

$$= 6 - 6 + 2 = 2$$

Co-factor of $a_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 6$

$a_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -6$, $a_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 2$

$a_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -5$, $a_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 8$

$a_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -3$; $a_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1$

$a_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -2$, $a_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$
$= 1$

Co-factor matrix of

$$B = \begin{pmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{pmatrix}$$

So, the inverse of A is

$$A^{-1} = \frac{1}{|A|} B^T = \frac{1}{2} \begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix}$$

So,

$$X = A^{-1} \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

So, the solⁿ of the eqⁿ

$x = 2$, $y = 1$, and $z = 0$

Assignment-8

GROUP-10(George & Ritwik)

Question 5:

Solve the following system of equations by matrix method:

$$2x_1 - x_2 + x_3 = 4$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1 - 3x_2 - 2x_3 = 2$$

Answer:

The coefficient matrix would be

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & -3 & -2 \end{bmatrix}$$

And let’s assume that

$$B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Therefore the set of equations can be nicely put in the following form

$$AX = B$$

Where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\Rightarrow X = A^{-1}B$$

So the whole idea is to fine the inverse of the matrix A and then we are done.

$$Adj\ A = \begin{bmatrix} 1 & 3 & -4 \\ -5 & -5 & 5 \\ -2 & -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -5 & -2 \\ 3 & -5 & -1 \\ -4 & 5 & 3 \end{bmatrix}$$

$$Det\ A = |A| = -5$$

The inverse matrix would be

$$A^{-1} = \frac{Adj\ A}{|A|} = -\frac{1}{5}\begin{bmatrix} 1 & -5 & -2 \\ 3 & -5 & -1 \\ -4 & 5 & 3 \end{bmatrix}$$

$$X = A^{-1}B = -\frac{1}{5}\begin{bmatrix} 1 & -5 & -2 \\ 3 & -5 & -1 \\ -4 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

By comparing two sides we can always get the solution

$$x_1 = 1$$

$$x_2 = -1$$

$$x_3 = 1$$

ASSIGNMENT -08
G-07

Prob-08, We have to determine rank of given 4x4 matrix

Given matrix is

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

Applying Row transformation
We can reduce the matrix to Echelon form

In Echelon form this matrix is

$$\begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

clearly this matrix is of rank3
i.e., it's three rows are linearly independent

Group - 11 Assignment - 8 Venkatesh Chinni
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(7) Find Inverse of matrix $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ by Gauss-Jordan method

Sol

Definition:- Let 'A' be an invertible $n \times n$ matrix. Suppose that a sequence of elementary row operations reduces 'A' to Identity matrix. Then the same sequence of elementary row operations when applied to identity matrix yields A^{-1}.

Given $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ $\Rightarrow \cancel{AI}$ $\Rightarrow \cancel{A^{-1}AI}$ $\Rightarrow \cancel{IA^{-1}}$

(1) $\simeq \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

(2) $\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & -3 & -2 & 0 & 1 \end{array}\right]$

$R_1 \to R_1 - R_2$, $R_3 \to R_3 - R_2$

(3) $\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -4 & -1 & -1 & 1 \end{array}\right]$

$R_3 \to R_3 / -4$

(4) $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/4 & 1/4 & -1/4 \end{array}\right]$

$R_2 \to R_2 - R_3$

(5) $\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & -\frac{5}{4} & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{array}\right]$

$R_1 \to R_1 - R_3$

(6) $\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/4 & -5/4 & 1/4 \\ 0 & 1 & 0 & -5/4 & 3/4 & 1/4 \\ 0 & 0 & 1 & 1/4 & 1/4 & -1/4 \end{array}\right]$

By definition of Gauss-Jordan method the inverse of given matrix is

$$\begin{bmatrix} 7/4 & -5/4 & 1/4 \\ -5/4 & 3/4 & 1/4 \\ 1/4 & 1/4 & -1/4 \end{bmatrix}$$

Assignment - 8

Gr - 1

Newton Nath & Vikas Chand

(8) Use Gauss-Jordan reduction method to find the inverse of

$$\begin{pmatrix} 7 & 6 & 2 \\ -1 & 2 & 4 \\ 3 & 6 & 8 \end{pmatrix}.$$

Soln/

$$A = \begin{pmatrix} 7 & 6 & 2 \\ -1 & 2 & 4 \\ 3 & 6 & 8 \end{pmatrix} = \left(\begin{array}{ccc|ccc} 7 & 6 & 2 & 1 & 0 & 0 \\ -1 & 2 & 4 & 0 & 1 & 0 \\ 3 & 6 & 8 & 0 & 0 & 1 \end{array}\right).$$

$$= \begin{pmatrix} 1 & 6/7 & 2/7 & 1/7 & 0 & 0 \\ 0 & 4 & 20/3 & 0 & 1 & 1/3 \\ 3 & 6 & 8 & 0 & 0 & 1 \end{pmatrix} \quad \begin{array}{l} R_1 \to \frac{1}{7} R_1 \\ R_2 \to R_2 + \frac{1}{3} R_3 \end{array}$$

$$= \begin{pmatrix} 1 & 6/7 & 2/7 & 1/7 & 0 & 0 \\ 0 & 4 & 20/3 & 0 & 1 & 1/3 \\ 0 & 24/7 & 50/7 & -3/7 & 0 & 1 \end{pmatrix} \quad R_3 \to R_3 - 3R_1$$

$$= \begin{pmatrix} 1 & 6/7 & 2/7 & 1/7 & 0 & 0 \\ 0 & 4 & 20/3 & 0 & 1 & 1/3 \\ 0 & 1 & 50/24 & -3/24 & 0 & 7/24 \end{pmatrix} \quad R_3 \to \frac{7}{24} R_3$$

$$= \begin{pmatrix} 1 & 6/7 & 2/7 & 1/7 & 0 & 0 \\ 0 & 4 & 20/3 & 0 & 1 & 1/3 \\ 0 & 0 & 5/12 & -1/8 & -1/4 & 5/24 \end{pmatrix} \quad R_3 \to R_3 - \frac{1}{4} R_2$$

$$= \begin{pmatrix} 1 & 6/7 & 2/7 & 1/7 & 0 & 0 \\ 0 & 1 & 5/3 & 0 & 1/4 & 1/12 \\ 0 & 0 & 1 & -3/10 & -3/5 & 1/2 \end{pmatrix} \quad \begin{array}{l} R_2 \to \frac{1}{4} R_2 \\ R_3 \to \frac{12}{5} R_3 \end{array}$$

$$= \left(\begin{array}{cccccc} 1 & 6/7 & 2/7 & 1/7 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & 5/4 & -3/4 \\ 0 & 0 & 1 & -3/10 & -3/5 & 1/2 \end{array}\right) \quad R_2 \to R_2 - 5/3\, R_3$$

$$= \left(\begin{array}{cccccc} 1 & 6/7 & 0 & 8/35 & 6/35 & -1/7 \\ 0 & 1 & 0 & 1/2 & 5/4 & -3/4 \\ 0 & 0 & 1 & -3/10 & -3/5 & 1/2 \end{array}\right) \quad R_1 \to R_1 - \frac{2}{7} R_3$$

$$= \left(\begin{array}{cccccc} 1 & 0 & 0 & -1/5 & -9/10 & 1/2 \\ 0 & 1 & 0 & 1/2 & 5/4 & -3/4 \\ 0 & 0 & 1 & -3/10 & -3/5 & 1/2 \end{array}\right) \quad R_1 \to R_1 - \frac{6}{7} R_2.$$

$\therefore$ The reqd. A^{-1} is $\begin{pmatrix} -1/5 & -9/10 & 1/2 \\ 1/2 & 5/4 & -3/4 \\ -3/10 & -3/5 & 1/2 \end{pmatrix}$ //

Assignment 8

Group 5: Apurv & Sanjay

Question

9. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Also find two non-singular matrices P and Q such that PAQ = $\mathbb{I}$, where $\mathbb{I}$ is the unit matrix and verify that $A^{-1} = QP$.

Solution

The combined matrix [A I] is given by

$$\begin{bmatrix} 3 & -3 & 4 & 1 & 0 & 0 \\ 2 & -3 & 4 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

By further row operations,

$$\sim R_1 \mapsto R_1 - R_2 \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 2 & -3 & 4 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix} \sim R_2 \mapsto 2R_2 - R_1 \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & -3 & 4 & -2 & 3 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim R_3 \mapsto R_2 - 3R_3 \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & -3 & 4 & -2 & 3 & 0 \\ 0 & 0 & 1 & -2 & 3 & -3 \end{bmatrix} \sim R_2 \mapsto R_2 - 4R_3; R_2 \mapsto \frac{R_2}{-3} \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -4 \\ 0 & 0 & 1 & -2 & 3 & -3 \end{bmatrix}$$

So finally $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

Now second part of the question is to find Q and P such that QAP = $\mathbb{I}$. The most straight forward answer is to choose either P or Q to be A^{-1} and to choose the other as $\mathbb{I}$.So one of the choice will be

$$P = A^{-1} \qquad \text{and} \qquad Q = \mathbb{I} \tag{0.0.1}$$

Then

$$PAQ = A^{-1}A\mathbb{I} = \mathbb{I} \times \mathbb{I} = \mathbb{I} \tag{0.0.2}$$

It also satisfies the condition

$$PQ = A^{-1}\mathbb{I} = A^{-1} \tag{0.0.3}$$

We also note that there are infinitely many choice for P and Q which satisfies the condition in the question.

ASSIGNMENT: 8

Group 2:
Chandana
Jimia Sikdar.

Q.10. Examine whether the following eq^n's are consistent & solve them, if they are consistent.

$$x + y + z = 6$$
$$2x + y + 3z = 13$$
$$5x + 2y + z = 12$$
$$2x - 3y - 2z = -10$$

Sol^n:- Writing the above eq^n's in Augmented Matrix form, we get:

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & 1 & 3 & 13 \\ 5 & 2 & 1 & 12 \\ 2 & -3 & -2 & -10 \end{bmatrix}$$

Carrying out Row operation, we get:

$$\begin{matrix} R_2 - 2R_1 \longrightarrow \\ R_3 - 5R_1 \longrightarrow \\ R_4 - 2R_1 \longrightarrow \end{matrix} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & -3 & -4 & -18 \\ 0 & -5 & -4 & -22 \end{bmatrix}$$

$$R_4 - 5R_2 \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & -3 & -4 & -18 \\ 0 & 0 & -9 & -27 \end{bmatrix}$$

$$\frac{1}{9} R_4 \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & -3 & -4 & -18 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

$$R_2 \times 3 \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 3 & 3 \\ 0 & -3 & -4 & -18 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

$R_3 - R_2 \longrightarrow \begin{vmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 3 & 3 \\ 0 & 0 & -7 & -21 \\ 0 & 0 & -1 & -3 \end{vmatrix}$

$\frac{1}{7} R_3 \longrightarrow \begin{vmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 3 & 3 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -1 & -3 \end{vmatrix}$

$R_4 - R_3 \longrightarrow \begin{vmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 3 & 3 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{vmatrix}$

$\therefore$ This is a consistent ~~sot~~ eq^n.

Let, $\begin{vmatrix} 1 & 1 & 1 & \cancel{6} \\ 0 & -3 & 3 & \cancel{3} \\ 0 & 0 & -1 & \cancel{-3} \\ 0 & 0 & 0 & \cancel{0} \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 6 \\ 3 \\ -3 \\ 0 \end{vmatrix}$

$\therefore$

$x + y + z = 6 \longrightarrow$ (i)

$-3y + 3z = 3 \longrightarrow$ (ii)

$-z = -3 \longrightarrow$ (iii).

Solving the 3 eq^{n}s, we get

$z = 3$

Putting value of z in eq^n (ii) $\Rightarrow$ $-3y + 9 = 3$

$\therefore -3y = -6$

$y = 2$

(i) $\Rightarrow$ $x + 2 + 3 = 6$

$\Rightarrow$ $x = 1$

$\therefore \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$.

ASSIGNMENT 5

GROUP-2
CHANDANA
JINIA

Show that $\tan^{-1} z = \frac{i}{2} \log \frac{i+z}{i-z}$

let $\tan^{-1} z = x$

$$z = \tan x$$

$$\therefore \quad z = \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} \times \frac{1}{i}$$

$$zi = \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}}$$

$$\frac{z}{-i} = \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}}$$

$$\frac{z}{i} = \frac{e^{-ix} - e^{+ix}}{e^{ix} + e^{-ix}}$$

$$\frac{i+z}{i-z} = \frac{e^{-ix} + e^{ix} + e^{-ix} - e^{ix}}{e^{-ix} + e^{ix} - e^{-ix} + e^{ix}}$$

$$\frac{i+z}{i-z} = \frac{e^{-ix}}{e^{ix}}$$

$$\frac{i+z}{i-z} = e^{-2ix}$$

$$x = \frac{1}{2(-i)} \log\left(\frac{i+z}{i-z}\right)$$

$$\tan^{-1} z = \frac{i}{2} \log\left(\frac{i+z}{i-z}\right)$$

— x —

Assignment no. 7.

Group 2
Chandana
Jinia Sikdar

Q5: Factorize the matrix $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ into the form LU, where L is lower triangular and U is upper triangular matrix.

Ans: Carrying out row operations to make the matrix in row-echelon form, we get:

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$$

$$R_2 - R_3 \longrightarrow \begin{bmatrix} 5 & -2 & 1 \\ 4 & -6 & -9 \\ 3 & 7 & 4 \end{bmatrix}$$

$$R_2 - R_1 \longrightarrow \begin{bmatrix} 5 & -2 & 1 \\ -1 & -4 & -10 \\ 3 & 7 & 4 \end{bmatrix}$$

$$R_3 + 3R_2 \longrightarrow \begin{bmatrix} 5 & -2 & 1 \\ -1 & -4 & -10 \\ 0 & -5 & -26 \end{bmatrix}$$

$$5R_2 + R_1 \longrightarrow \frac{1}{5}\begin{bmatrix} 5 & -2 & 1 \\ 0 & -22 & -49 \\ 0 & -5 & -26 \end{bmatrix}$$

$$R_3 \times \frac{-22}{5} \longrightarrow \frac{1}{5} \times \frac{-5}{22}\begin{bmatrix} 5 & -2 & 1 \\ 0 & -22 & -49 \\ 0 & 22 & \frac{26 \times 22}{5} \end{bmatrix}$$

$$R_3 + R_2 \longrightarrow \begin{bmatrix} 5 & -2 & 1 \\ 0 & -22 & -49 \\ 0 & 0 & 327/5 \end{bmatrix}$$

To get L, we write the elementary matrices and their inverses for each of the operation above.

Let us have a unit matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$E_1 = R_2 - R_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore \quad E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_2 = R_2 - R_1 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore \quad E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_3 = R_3 + 3R_2 \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ -3 & 3 & -2 \end{bmatrix} \quad \therefore \quad E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 3 & 1 \end{bmatrix}$$

$$E_4 = 5R_2 + R_1 \longrightarrow \frac{1}{5}\begin{bmatrix} 1 & 0 & 0 \\ -4 & 5 & -5 \\ -3 & 3 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -4/5 & 1 & -1 \\ -3 & 3 & -2 \end{bmatrix} \quad \therefore \quad E_4^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 7/5 & -2 & 1 \\ 3/5 & 3 & 1 \end{bmatrix}$$

$$E_5 = R_3\left(\frac{-22}{5}\right) \Rightarrow \frac{-5}{22}\begin{bmatrix} 1 & 0 & 0 \\ -4/5 & 1 & -1 \\ -\frac{66}{5} & +\frac{66}{5} & -\frac{44}{5} \end{bmatrix} \quad \therefore \quad E_5^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 7/5 & -2 & 5/22 \\ 3/5 & -3 & 5/22 \end{bmatrix}$$

$$E_6 = R_3 + R_2 \longrightarrow \begin{array}{c} \\ \frac{-5}{22} \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 \\ -4/5 & 1 & -1 \\ -15 & 67/5 & -54/5 \end{bmatrix}$$

$$E_6^{-1} = \frac{-22}{13} \begin{bmatrix} 13/5 & 0 & 0 \\ 159/25 & -54/5 & 1 \\ 107/25 & 67/5 & 1 \end{bmatrix}$$

We have,

$$U = \begin{bmatrix} 5 & -2 & 1 \\ 0 & -22 & -49 \\ 0 & 0 & 327/5 \end{bmatrix}$$

$$L = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} E_5^{-1} E_6^{-1}.$$

$$\therefore L = \frac{-22}{13} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 7/5 & -2 & 1 \\ 3/5 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 7/5 & -2 & 5/22 \\ 3/5 & -3 & 5/22 \end{bmatrix} \begin{bmatrix} 13/5 & 0 & 0 \\ 159/25 & -54/5 & 1 \\ 107/25 & 67/5 & 1 \end{bmatrix}$$

$$= \frac{-22}{13} \begin{bmatrix} 13/5 & 0 & 0 \\ \frac{-27632}{275} & \frac{15002}{55} & \frac{-233}{11} \\ \frac{-16163}{550} & \frac{659}{5} & \frac{-127}{5} \end{bmatrix}$$

$$= \begin{bmatrix} -22/5 & 0 & 0 \\ \frac{55264}{325} & \frac{-2308}{5} & \frac{466}{13} \\ \ldots & \ldots & \ldots \end{bmatrix}$$

Factores are :

$$U = \begin{bmatrix} 5 & -2 & 1 \\ 0 & -22 & -49 \\ 0 & 0 & 327/5 \end{bmatrix}$$

$$L = \begin{bmatrix} -22/5 & 0 & 0 \\ 55264/325 & \frac{-2308}{5} & \frac{466}{13} \\ \frac{32326}{650} & \frac{-14498}{65} & \frac{2794}{65} \end{bmatrix}$$

—x—

Date: 24.08.2012 Assignment - 8 Group - 9

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11. Solve completely the following system of equations:

$$x + y - 2z + 3w = 0$$
$$x - 2y + z - w = 0$$
$$4x + y - 5z + 8w = 0$$
$$5x - 7y + 2z - w = 0$$

$\Rightarrow$ Let $A = \begin{pmatrix} 1 & 1 & -2 & 3 \\ 1 & -2 & 1 & -1 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{pmatrix}$

$X = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$ $\quad O = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

So, the matrix equation is:

$$AX = O$$

Determinant of the eqn is:

$$\Delta = 1[-2(5-16) - (-1+56) - 1(2-35)] - 1[1(5-16) - (-4-40) - 1(8+25)] - 2[1(-1+56) + 2(-4-40) - 1(-28-5)] - 3[1(2-35) + 2(8+25) + 1(-28-5)]$$

$= 0.$

So, as the determinant is 0, so the equations given are inconsistent. Ans