Assignment - 7 Group - 3

Problem-1 For what values of x, the matrix

$$A = \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$$ is singular?

Solⁿ :- For matrix to be singular, $|A| = 0$

$\therefore$
$$\begin{vmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{vmatrix} = 0$$

$$\begin{vmatrix} 7-x & 2 & 2 \\ 7-x & 4-x & 1 \\ -7-x & -4 & -1-x \end{vmatrix} = 0 \quad [C_1 \to C_1 + C_2 + C_3]$$

$$\begin{vmatrix} 7-x & 2 & 2 \\ 0 & 2-x & -1 \\ 0 & -2 & 1-x \end{vmatrix} = 0 \quad \begin{bmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + R_1 \end{bmatrix}$$

on expanding by C_1, we get

$$(7-x)\begin{vmatrix} 2-x & -1 \\ -2 & 1-x \end{vmatrix} = 0$$

$$(7-x)\{\cancel{2} - 2x - x + x^2 - \cancel{2}\} = 0$$

$$(7-x)(x^2 - 3x) = 0$$

$$x(x-3)(x-7) = 0$$

$$\boxed{x = 0, 3, 7}$$

Assignment # 7

Lalit Kumar Shukla
& Rahul Yadav
Group - G-6

Ques. Express $\begin{bmatrix} 2 & 5 & -7 \\ -9 & 12 & 4 \\ 15 & -13 & 6 \end{bmatrix}$ as the sum of

→ a lower triangular matrix &
→ an upper triangular matrix with zero leading diagonal.

Sol.

$$\begin{bmatrix} 2 & 5 & -7 \\ -9 & 12 & 4 \\ 15 & -13 & 6 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 2 & 0 & 0 \\ -9 & 12 & 0 \\ 15 & -13 & 6 \end{bmatrix}}_{\text{Lower triangular matrix}} + \underbrace{\begin{bmatrix} 0 & 5 & -7 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{upper triangular matrix}}$$

ASSIGNMENT-07

G-07

Prob.3. $$\begin{bmatrix} 2 & 1 & -1 \\ 4 & -5 & 6 \\ -3 & 7 & 3 \end{bmatrix} \underline{\begin{bmatrix} 3 & 1 \\ -6 & 4 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix}}$$

first consider underlined part

$$\begin{bmatrix} 15-2 & 9+1 \\ -30-8 & -18+4 \\ -10-10 & -6+5 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 10 \\ -38 & -14 \\ -20 & -1 \end{bmatrix}$$

Now multiply $\begin{bmatrix} 2 & 1 & -1 \\ 4 & -5 & 6 \\ -3 & 7 & 3 \end{bmatrix}$ and $\begin{bmatrix} 13 & 10 \\ -38 & -14 \\ -20 & -1 \end{bmatrix}$

$$\begin{bmatrix} 26-38+20 & 7 \\ 122 & 104 \\ -365 & -131 \end{bmatrix}$$

$$\text{Ans} = \begin{bmatrix} 8 & 7 \\ 122 & 104 \\ -365 & -131 \end{bmatrix}$$

Assignment - 7

Gr - L

Newton Nath, Vikas Chand

Q.4 If $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{pmatrix}$ & I is the unit matrix of order 3, evaluate $A^2 - 3A + 9I$.

sol^n/

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{pmatrix}.$$

$$A^2 = A.A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{pmatrix}.$$

$$= \begin{pmatrix} 1-4-9 & -2-6+3 & 3+2+6 \\ 2+6+3 & -4+9-1 & 6-3-2 \\ -3+2-6 & 6+3+2 & -9-1+4 \end{pmatrix}$$

$$= \begin{pmatrix} -12 & -5 & 11 \\ 11 & 4 & 1 \\ -\cancel{8}7 & 11 & -6 \end{pmatrix}.$$

$$\therefore\ 3A = 3\begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -6 & 9 \\ 6 & 9 & -3 \\ -9 & 3 & 6 \end{pmatrix}.$$

$$9I = 9\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}.$$

$$A^2 - 3A + 9I = \begin{pmatrix} -12 & -5 & 11 \\ 11 & 4 & 11 \\ -7 & 11 & -6 \end{pmatrix} - \begin{pmatrix} 3 & -6 & 9 \\ 6 & 9 & -3 \\ -9 & 3 & 6 \end{pmatrix} + \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}.$$

$$= \begin{pmatrix} -12-3+9 & -5+6+0 & 11-9+0 \\ 11-6+0 & 4-9+9 & 11+3+0 \\ -7+9+0 & 11-3+0 & -6-6+9 \end{pmatrix} = \begin{pmatrix} -6 & 1 & 2 \\ 5 & 4 & 14 \\ 2 & 8 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} -6 & 1 & 2 \\ 5 & 4 & 14 \\ 2 & 8 & -3 \end{pmatrix} //$$

ASSIGNMENT: 7

Group: 2
Chandana
Jimia Sikdar

Q5: Factorize the matrix:

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$$

into form LU, where L is lower triangular and U is upper triangular matrix.

Solⁿ: Let,

$$\begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

$$= \begin{pmatrix} l_{11}u_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + l_{22}u_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + l_{33}u_{33} \end{pmatrix}$$

$\therefore$ $l_{11}u_{11} = 5$ $\quad l_{11}u_{12} = -2$ $\quad l_{11}u_{13} = 1$

$l_{21}u_{11} = 7$ $\quad l_{21}u_{12} + l_{22}u_{22} = 1$ $\quad l_{21}u_{13} + l_{22}u_{23} = -5$

$l_{31}u_{11} = 3$ $\quad l_{31}u_{12} + l_{32}u_{22} = 7$ $\quad l_{31}u_{13} + l_{32}u_{23} + l_{33}u_{33} = 4$

Since no. of variables are more than no. of eqⁿs, hence we need some restrictions to get the answer.

Let, $l_{11} = l_{22} = l_{33} = 1$.

$\therefore$ $u_{11} = 5$, $\quad u_{12} = -2$, $\quad u_{13} = 1$.

$l_{21} = 7/5$ and

$l_{31} = 3/5$.

Now, $l_{21}u_{12} + l_{22}u_{22} = 1$

$\therefore \frac{7}{5} \cdot (-2) + 1 \cdot u_{22} = 1 \quad \therefore -\frac{14}{5} + u_{22} = 1 \quad \therefore u_{22} = \frac{19}{5}$

Also, $l_{21} u_{13} + l_{22} u_{23} = -5$

$\therefore \frac{7}{5} \cdot u_{13} + 1 \cdot u_{23} = -5 \longrightarrow (1)$.

$\therefore \frac{7}{5} \cdot 1 + u_{23} = -5$

$\therefore u_{23} = -5 - \frac{7}{5} = \frac{-32}{5}$

Again, $l_{31} u_{12} + l_{32} u_{22} = 7$

$\therefore \frac{3}{5} \cdot (-2) + l_{32}\left(\frac{19}{5}\right) = 7$

$\Rightarrow -\frac{6}{5} + l_{32}\left(\frac{19}{5}\right) = 7$

$\therefore l_{32}\left(\frac{19}{5}\right) = 7 + \frac{6}{5} = \frac{41}{5}$

$\therefore l_{32} = 41/19$.

Also, we have: $l_{31} u_{13} + l_{32} u_{23} + l_{33} u_{33} = 4$

$\Rightarrow \frac{3}{5} \cdot 1 + \frac{41}{19} \cdot \frac{-32}{5} + 1 \cdot u_{33} = 4$

$\Rightarrow \frac{3}{5} - \frac{1312}{95} + u_{33} = 4$

$\Rightarrow \frac{57 - 1312}{95} + u_{33} = 4$

$u_{33} = 327/19$.

$$\therefore L_1 = \begin{pmatrix} 1 & 0 & 0 \\ 7/5 & 1 & 0 \\ 3/5 & 41/19 & 1 \end{pmatrix}$$

$$U_1 = \begin{pmatrix} 5 & -2 & 1 \\ 0 & 19/5 & -32/5 \\ 0 & 0 & 327/19 \end{pmatrix}$$

Cross check: $L_1 U_1 = \begin{pmatrix} 1 & 0 & 0 \\ 7/5 & 1 & 0 \\ 3/5 & 41/19 & 1 \end{pmatrix} \begin{pmatrix} 5 & -2 & 1 \\ 0 & 19/5 & -32/5 \\ 0 & 0 & 327/19 \end{pmatrix} = \begin{pmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{pmatrix} = A$

Group-4 Assignment-7

6. Q → Express $\begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix}$ as the sum of a symmetric & a skew-symmetric matrix.

Ans:- We know any square matrix A can be expressed as the sum of a symmetric & skew symmetric matrix.

i.e. $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = B + C$ (say)

where $\frac{1}{2}(A + A^T) = B$ = symmetric matrix

$\frac{1}{2}(A - A^T) = C$ = skew symmetric matrix

Assume $A = \begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 0 & 1 & 4 \\ 5 & 1 & 5 \\ -3 & 1 & 9 \end{bmatrix}$

Now $B = \frac{1}{2}(A + A^T) = \frac{1}{2}\left\{\begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 4 \\ 5 & 1 & 5 \\ -3 & 1 & 9 \end{bmatrix}\right\}$

$= \frac{1}{2}\begin{bmatrix} 0 & 6 & 1 \\ 6 & 2 & 6 \\ 1 & 6 & 18 \end{bmatrix} = \begin{bmatrix} 0 & 3 & \frac{1}{2} \\ 3 & 1 & 3 \\ \frac{1}{2} & 3 & 9 \end{bmatrix}$

& $C = \frac{1}{2}(A - A^T) = \frac{1}{2}\left\{\begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 4 \\ 5 & 1 & 5 \\ -3 & 1 & 9 \end{bmatrix}\right\}$

$= \frac{1}{2}\begin{bmatrix} 0 & 4 & -7 \\ -4 & 0 & -4 \\ 7 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & -\frac{7}{2} \\ -2 & 0 & -2 \\ \frac{7}{2} & 2 & 0 \end{bmatrix}$

$\therefore$ Here $\begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 3 & \frac{1}{2} \\ 3 & 1 & 3 \\ \frac{1}{2} & 3 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -\frac{7}{2} \\ -2 & 0 & -2 \\ \frac{7}{2} & 2 & 0 \end{bmatrix}$

Ans

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Assignment - 7

Group 10. (Birin & Ritwik).

Q.7. Find the inverse of the matrix $\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 0 \\ 1 & 4 & 1 \end{bmatrix}$

Soln:-

$$A^{-1} = \frac{1}{|A|} \text{ adj } A.$$

$$|A| = 1(0) + 3(2) + 2(8)$$
$$= \underline{\underline{22}}.$$

$$\text{Cofactor matrix of } A = \begin{bmatrix} 0 & -2 & 8 \\ 11 & -1 & -7 \\ 0 & -4 & 6 \end{bmatrix}$$

adj A = Transpose of cofactor matrix of A.

$$\Rightarrow \text{adj } A = \begin{bmatrix} 0 & 11 & 0 \\ -2 & -1 & -4 \\ 8 & -7 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A.$$

$$A^{-1} = \frac{1}{22}\begin{bmatrix} 0 & 11 & 0 \\ -2 & -1 & -4 \\ 8 & -7 & 6 \end{bmatrix}.$$

Gr - 11

Assignment - 7

Venkatesh chinni
Durgaprasad

⑧ Show that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan\theta/_2 \\ \tan\theta/_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta/_2 \\ -\tan\theta/_2 & 1 \end{bmatrix}^{-1}$

Sol

suppose $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Take $\begin{bmatrix} 1 & \tan\theta/_2 \\ -\tan\theta/_2 & 1 \end{bmatrix}^{-1} = \frac{1}{\sec^2\theta/_2}\begin{bmatrix} 1 & -\tan\theta/_2 \\ \tan\theta/_2 & 1 \end{bmatrix} \rightarrow (1)$

R.H.S

$\begin{bmatrix} 1 & -\tan\theta/_2 \\ \tan\theta/_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta/_2 \\ -\tan\theta/_2 & 1 \end{bmatrix}^{-1}$

$\Rightarrow \begin{bmatrix} 1 & -\tan\theta/_2 \\ \tan\theta/_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan\theta/_2 \\ \tan\theta/_2 & 1 \end{bmatrix} \frac{1}{\sec^2\theta/_2}$ [$\because$ By ①]

$\Rightarrow \cos^2\theta/_2 \begin{bmatrix} 1 & -\tan\theta/_2 \\ \tan\theta/_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan\theta/_2 \\ \tan\theta/_2 & 1 \end{bmatrix}$

$\Rightarrow \cos^2\theta/_2 \begin{bmatrix} 1-\tan^2\theta/_2 & -2\tan\theta/_2 \\ 2\tan\theta/_2 & 1-\tan^2\theta/_2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos^2\theta/_2 - \sin^2\theta/_2 & -2\sin\theta/_2\cos\theta/_2 \\ 2\sin\theta/_2\cos\theta/_2 & \cos^2\theta/_2 - \sin^2\theta/_2 \end{bmatrix}$ [$\because \cos 2\theta = \cos^2\theta - \sin^2\theta$, $\sin 2\theta = 2\sin\theta \sin\theta$]

$\Rightarrow \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = L.H.S$

Hence proved

Group-8 assignment-7

(9)

If $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$, verify that

$(AB)^T = B^T A^T$ and $(AB)^{-1} = B^{-1} A^{-1}$.

Ans:- $A^T = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 0 & 5 \\ -1 & 2 & 0 \end{pmatrix}$ and $B^T = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix}$

Now, $B^T A^T = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 3 & 4 \\ 2 & 0 & 5 \\ -1 & 2 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 5 & 3 & 14 \\ 1 & 2 & 5 \\ -3 & 6 & 0 \end{pmatrix} \quad \text{— (1)}$$

$AB = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{pmatrix} \quad \text{— (2)}$$

So, $(AB)^T = \begin{pmatrix} 5 & 3 & 14 \\ 1 & 2 & 5 \\ -3 & 6 & 0 \end{pmatrix} = B^T A^T$ (proved)

$|A| = -9.$

Co-factor of A :-

$a_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 2 \\ 5 & 0 \end{vmatrix} = -10$

$a_{12} = (-1)^{1+2} \begin{vmatrix} 3 & 2 \\ 4 & 0 \end{vmatrix} = 8$

$a_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 0 \\ 4 & 5 \end{vmatrix} = 15$

$a_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -1 \\ 5 & 0 \end{vmatrix} = -5$

$a_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ 4 & 0 \end{vmatrix} = 4$

$a_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = 3$

$a_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = 4$

$a_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = -5$

$a_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} = -6$

co-factor matrix of $A = \begin{pmatrix} -10 & 8 & 15 \\ -5 & 4 & 3 \\ 4 & -5 & -6 \end{pmatrix}$

So, $A^{-1} = -\frac{1}{9}\begin{pmatrix} -10 & -5 & 4 \\ 8 & 4 & -5 \\ 15 & 3 & -6 \end{pmatrix}$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

$B_{11} = (-1)^{1+1}(3-0) = 3$, $B_{23} = (-1)(1-0) = -1$

$B_{12} = -1(6-0) = -6$, $B_{31} = 0$

$B_{13} = (1)(2-0) = 2$, $B_{32} = 0$

$B_{21} = 0$, $B_{33} = 1.1 = 1$

$B_{22} = (1)(3-0) = 3$

$|B| = 3$

$$B^{-1} = \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ -6 & 3 & 0 \\ 2 & -1 & 1 \end{bmatrix}$$

$$(AB) = \begin{pmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{pmatrix}$$

$AB_{11} = (1)(0-30) = -30$

$AB_{12} = (-1)(0-84) = +84$

$AB_{13} = (1)(15-28) = -13$

$AB_{21} = (-1)(0+15) = -15$, $AB_{31} = (1)(6+6) = 12$

$AB_{22} = (1)(0-(-42)) = 42$, $AB_{32} = (-1)(30+9) = -39$

$AB_{23} = (-1)(25-14) = -11$, $AB_{33} = (1)(10-3) = 7$

$AB^{-1} = ($

$$|AB| = 5(0-30) - 1(0-84) + (-3)(15-28)$$
$$= -150 + 84 + 39$$
$$= -27$$

$$(AB)^{-1} = \frac{-1}{27}\begin{vmatrix} -30 & -15 & 12 \\ +84 & 42 & -39 \\ -13 & -11 & 7 \end{vmatrix}$$

$$B^{-1}A^{-1} = \frac{-1}{27}\begin{pmatrix} 3 & 0 & 0 \\ -6 & 3 & 0 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} -10 & -5 & 4 \\ 8 & 4 & -5 \\ 15 & 3 & -6 \end{pmatrix}$$

$$= \frac{-1}{27}\begin{bmatrix} -30 & -15 & 12 \\ +84 & 42 & -39 \\ -13 & -11 & 7 \end{bmatrix}$$

$$= (AB)^{-1} \quad \text{(proved)}$$

Date : 24.08.2012 Assignment -7. Group-9

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10 If $D = \mathrm{diag}\,[d_1, d_2, \ldots, d_n]$; $d_1, d_2, \ldots, d_n \neq 0$; prove that

$D^{-1} = \mathrm{diag}\,[d_1^{-1}, d_2^{-1}, \ldots, d_n^{-1}]$.

$\Rightarrow$

$$D = \begin{pmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ 0 & \vdots & \cdots & & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & d_n \end{pmatrix}$$

$\therefore$ determinant of D is : $\Delta = d_1 \cdot d_2 \cdots d_n$

$$= \prod_{i=1}^{n} d_i$$

$$\text{adjoint } D = \begin{pmatrix} d_2 \cdots d_n & 0 & 0 & \cdots & 0 \\ 0 & d_3 \cdots d_n d_1 & 0 & \cdots & 0 \\ 0 & 0 & d_4 \cdots d_n d_1 d_2 & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & d_{n-1} \cdots d_1 \end{pmatrix}^T$$

$$= \begin{pmatrix} \prod_{i=2}^{n} d_i & 0 & \cdots & 0 \\ 0 & \prod_{\substack{i=1 \\ i\neq 2}}^{n} d_i & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \prod_{i=1}^{n-1} d_i \end{pmatrix}$$

So, $$D^{-1} = \frac{\text{adjoint } D}{\Delta} = \prod_{i=1}^{n} \begin{pmatrix} \prod_{i=2}^{n} d_i & 0 & \cdots & 0 \\ 0 & \prod_{\substack{i=1 \\ i\neq 2}}^{n} d_i & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & \prod_{i=1}^{n-1} d_i \end{pmatrix}$$

$$D^{-1} = \begin{pmatrix} 1/d_1 & 0 & 0 & \cdots & 0 \\ 0 & 1/d_2 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & & \\ 0 & 0 & \cdots & \cdots & 1/d_n \end{pmatrix}$$

$\therefore D^{-1} = \operatorname{diag}[d_1^{-1}, d_2^{-1}, \ldots, d_n^{-1}]$. (proved)

Assignment 7

Group 5: Apurv & Sanjay

Question

11. If A and B are two $n \times n$ non- singular matrix, show that $adj(AB) = adj(B)adj(A)$

Solution

Given: A and B are non singular

$\Rightarrow A^{-1}$ and B^{-1} exists, and is given by

$$A^{-1} = \frac{adj(A)}{|A|} \tag{0.0.1}$$

Where $|A|$ is the determinant of A, we note that it is a scalar, so taking it to the left side.

$$|A|A^{-1} = adj(A) \tag{0.0.2}$$

Similarly $|B|B^{-1} = adj(B)$, going a step further,

$$\begin{aligned}
adj(AB) &= |AB|(AB)^{-1} \\
&= |A||B| \;\; B^{-1}A^{-1} && \because |AB| = |A||B| \;\; and \;\; (AB)^{-1} = B^{-1}A^{-1} \\
&= |B|B^{-1} \;\; |A|A^{-1} && \because \text{Determinant is a scalar, order can be interchanged} \\
&= adj(B) \;\; adj(A) && \because \text{From Eq.0.0.2} \quad Q.E.D
\end{aligned}$$