Assignment - 3

Qr - 1

Newton Nath & Vikas Chand.

(1) Prove that →

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

Sol^n/ From De Moivre's Theorem we know

$$(\cos n\theta + i\sin n\theta) = (\cos\theta + i\sin\theta)^n$$

Considering $n = 6$, we have.

$$(\cos 6\theta + i\sin 6\theta) = (\cos\theta + i\sin\theta)^6 \rightarrow (1)$$

Now, expanding eq^n (1) Binomially →.

$$(\cos 6\theta + i\sin 6\theta) = \cos^6\theta + {}^6C_1\cos^5\theta\,(i\sin\theta)$$
$$+ {}^6C_2\cos^4\theta\,(i\sin\theta)^2 + {}^6C_3\cos^3\theta\,(i\sin\theta)^3$$
$$+ {}^6C_4\cos^2\theta\,(i\sin\theta)^4 + {}^6C_5\cos\theta\,(i\sin\theta)^5 + {}^6C_6\,(i\sin\theta)^6$$

Now separating real & imaginary parts →.

$$\cos 6\theta + i\sin 6\theta = (\cos^6\theta - {}^6C_2\cos^4\theta\sin^2\theta + {}^6C_4\cos^2\theta$$
$$\sin^4\theta - \sin^6\theta) + i\,({}^6C_1\cos^5\theta\sin\theta - {}^6C_3\cos^3\theta\sin^3\theta$$
$$+ {}^6C_5\cos\theta\sin^5\theta)$$

Now comparing real & imaginary part.

$$\cos 6\theta = \cos^6\theta - {}^6C_2 \cos^4\theta \sin^2\theta + {}^6C_4 \cos^2\theta \sin^4\theta$$
$$- \sin^6\theta.$$

Since we want only $\cos 6\theta \longrightarrow$.

L.H.S

$$\cos 6\theta = \cos^6\theta - \frac{6.5}{2}\cos^4\theta(1-\cos^2\theta) + \frac{6.5}{2}\cos^2\theta(1-\cos^2\theta)^2$$
$$-(1-\cos^2\theta)^3$$

$$= \cos^6\theta - 15\cos^4\theta + 15\cos^6\theta + 15\cos^2\theta(1+\cos^4\theta - 2\cos^2\theta)$$
$$-(1-\cos^6\theta - 3\cos^2\theta + 3\cos^4\theta)$$

$$= \cos^6\theta - 15\cos^4\theta + 15\cos^6\theta + 15\cos^2\theta + 15\cos^6\theta$$

$$-30\cos^4\theta - 1 + \cos^6\theta + 3\cos^2\theta - 3\cos^4\theta$$

$$= 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$

R.H.S

Proved

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ASSIGNMENT : 4

GROUP 2
Chandana
Jinia Sikdar

PROBLEM 2:

~~Solve~~ Prove that

$$\frac{\sin 6\theta}{\cos\theta} = 32\sin^5\theta - 32\sin^3\theta + 6\sin\theta$$

Solution:

Using Expansion Series on LHS, we get:

$$\sin 6\theta = {}^6C_1 (\cos\theta)^5 \sin\theta - {}^6C_3 (\cos\theta)^3 \sin^3\theta + {}^6C_5 \cos\theta \sin^5\theta$$

$$\sin 6\theta = 6\cos^5\theta \sin\theta - \frac{6\times5\times4}{3\times2\times1}\cos^3\theta \sin^3\theta + 6\cos\theta\sin^5\theta$$

$$\sin 6\theta = 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta$$

dividing both the sides by $\cos\theta$, we get:

$$\frac{\sin 6\theta}{\cos\theta} = \frac{6\cos^5\theta\sin\theta}{\cos\theta} - \frac{20\cos^3\theta\sin^3\theta}{\cos\theta} + \frac{6\cos\theta\sin^5\theta}{\cos\theta}$$

$$\frac{\sin 6\theta}{\cos\theta} = 6\cos^4\theta\sin\theta - 20\cos^2\theta\sin^3\theta + 6\sin^5\theta \quad \text{—— (1)}$$

substituting $\cos^2\theta = 1 - \sin^2\theta$ in eqn (1) weget

$$\frac{\sin 6\theta}{\cos\theta} = 6(1-\sin^2\theta)^2 \sin\theta - 20(1-\sin^2\theta)\sin^3\theta + 6\sin^5\theta$$

$$\frac{\sin 6\theta}{\cos\theta} = 6(1+\sin^4\theta - 2\sin^2\theta)\sin\theta - 20(\sin^3\theta - \sin^5\theta) + 6\sin^5\theta$$

$$\frac{\sin 6\theta}{\cos\theta} = (6 + 6\sin^4\theta - 12\sin^2\theta)\sin\theta - 20\sin^3\theta + 20\sin^5\theta + 6\sin^5\theta$$

$$\frac{\sin 6\theta}{\cos\theta} = 6\sin\theta + 6\sin^5\theta - 12\sin^3\theta - 20\sin^3\theta + 20\sin^5\theta + 6\sin^5\theta$$

$$\frac{\sin 6\theta}{\cos\theta} = 32\sin^5\theta - 32\sin^3\theta + 6\sin\theta$$

Problem-3 Assignment - 4 Group - 3

7 Prove that $\sin 7\theta = 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta$

Sol^n :- $\cos 7\theta + i\sin 7\theta = (\cos\theta + i\sin\theta)^7$

$$= \cos^7\theta + {}^7C_1\cos^6\theta\,(i\sin\theta) + {}^7C_2\cos^5\theta\,(i\sin\theta)^2$$
$$+ {}^7C_3\cos^4\theta\,(i\sin\theta)^3 + {}^7C_4\cos^3\theta\,(i\sin\theta)^4 + {}^7C_5\cos^2\theta\,(i\sin\theta)^5$$
$$+ {}^7C_6\cos\theta\,(i\sin\theta)^6 + (i\sin\theta)^7$$

Comparing imaginary parts on both sides, we get

$$\sin 7\theta = {}^7C_1\cos^6\theta\sin\theta - {}^7C_3\cos^4\theta\sin^3\theta + {}^7C_5\cos^2\theta\,,\sin^5\theta$$
$$+ (-1)\sin^7\theta$$

$$= 7(1-\sin^2\theta)^3\sin\theta - 35(1-\sin^2\theta)^2\sin^3\theta + 21(1-\sin^2\theta)\sin^5\theta$$
$$- \sin^7\theta$$

$$= 7(1-\sin^6\theta - 3\sin^2\theta + 3\sin^4\theta)\sin\theta - 35(1+\sin^4\theta - 2\sin^2\theta)\sin$$
$$+ 21\sin^5\theta - 21\sin^7\theta - \sin^7\theta$$

$$= 7\sin\theta - 7\sin^7\theta - 21\sin^3\theta + 21\sin^5\theta - 35\sin^3\theta - 35\sin^7\theta$$
$$+ 70\sin^5\theta + 21\sin^5\theta - 22\sin^7\theta$$

$$\boxed{\sin 7\theta = 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta}$$

Hence proved

Assignment-4 (Group-4)

4.Q. Prove that: $\dfrac{1+\cos 7\theta}{1+\cos\theta} = (x^3-x^2-2x+1)^2$; where $x = 2\cos\theta$

Proof: First lets calculate $1+\cos 7\theta$ in terms of x.

Now: $1+\cos 7\theta$

$$= 1+\cos^7\theta - {}^7C_2 \cdot \cos^5\theta \cdot \sin^2\theta + {}^7C_4 \cdot \cos^3\theta \cdot \sin^4\theta - {}^7C_6 \cdot \cos\theta \cdot \sin^6\theta$$

As $x = 2\cos\theta$

$\Rightarrow \cos\theta = \frac{x}{2}$

So $1+\cos 7\theta$

$$= 1+\frac{x^7}{128} - 21\cdot\frac{x^5}{32}\cdot\left(1-\frac{x^2}{4}\right) + 35\cdot\frac{x^3}{8}\cdot\left(1-\frac{x^2}{4}\right)^2 - 7\cdot\frac{x}{2}\cdot\left(1-\frac{x^2}{4}\right)^3$$

$$= 1+\frac{x^7}{128} - \frac{21}{32}x^5 + \frac{21}{128}x^7 + \frac{35}{8}x^3\left(1+\frac{x^4}{16}-\frac{x^2}{2}\right) - \frac{7}{2}x\left(1-\frac{3}{4}x^2 + \frac{3}{16}x^4 - \frac{x^6}{64}\right)$$

$$= 1+\frac{x^7}{128} - \frac{21}{32}x^5 + \frac{21}{128}x^7 + \frac{35}{8}x^3 + \frac{35}{128}x^7 - \frac{35}{16}x^5 - \frac{7}{2}x + \frac{21}{8}x^3 - \frac{21}{32}x^5 + \frac{7}{128}x^7$$

$$= 1+\left(\frac{1}{128}+\frac{21}{128}+\frac{35}{128}+\frac{7}{128}\right)x^7 - \left(\frac{21}{32}+\frac{35}{16}+\frac{21}{32}\right)x^5 + \left(\frac{35}{8}+\frac{21}{8}\right)x^3 - \frac{7}{2}x$$

$$= 1+\frac{\cancel{64}}{\cancel{128}_{\,2}}x^7 - \frac{\cancel{112}^{\,7}}{\cancel{32}_{\,2}}x^5 + \frac{\cancel{56}^{\,7}}{\cancel{8}}x^3 - \frac{7}{2}x$$

$$= \frac{x^7}{2} - \frac{7}{2}x^5 + 7x^3 - \frac{7}{2}x + 1$$

$$= \frac{x^7}{2} + x^6 - x^6 - 2x^5 - \frac{3}{2}x^5 - 3x^4 + 3x^4 + 6x^3 + x^3 + 2x^2 - 2x^2 - 4x + \frac{x}{2} + 1$$

$$= x^6\left(\frac{x}{2}+1\right) - 2x^5\left(\frac{x}{2}+1\right) - 3x^4\left(\frac{x}{2}+1\right) + 6x^3\left(\frac{x}{2}+1\right) + 2x^2\left(\frac{x}{2}+1\right) - 4x\left(\frac{x}{2}+1\right) + 1\left(\frac{x}{2}+1\right)$$

$$= \left(x^6 - 2x^5 - 3x^4 + 6x^3 + 2x^2 - 4x + 1\right)\left(\frac{x}{2}+1\right) \quad \text{——— (1)}$$

$$L.H.S. = \frac{1+\cos 7\theta}{1+\cos\theta}$$

$$= \frac{1+\cos 7\theta}{\left(1+\frac{x}{2}\right)} \qquad \left(\because \cos\theta = \frac{x}{2}\right)$$

$$= x^6 - 2x^5 - 3x^4 + 6x^3 + 2x^2 - 4x + 1$$ $\left[\because\right.$ substituting value of $1+\cos 7\theta$ from eqn (1)

$$= x^6 - 2x^5 + x^4 - 4x^4 + 2x^3 + 4x^3$$
$$+ 4x^2 - 2x^2 - 4x + 1$$

$$= x^6 + x^4 - 2x^5 + 4x^2 + 1 - 4x - 4x^4 + 2x^3 + 4x^3 - 2x^2$$

$$= (x^3 - x^2)^2 + (2x-1)^2 - 2\left(2x^4 - x^3 - 2x^3 + x^2\right)$$

$$= (x^3 - x^2)^2 + (2x \mp 1)^2 - 2\left\{x^3(2x-1) - x^2(2x-1)\right\}$$

$$= (x^3 - x^2)^2 + (2x \mp 1)^2 - 2(x^3 - x^2)(2x-1)$$

$$= \left[(x^3 - x^2) - (2x-1)\right]^2$$

$$= (x^3 - x^2 - 2x + 1)^2$$

$= R.H.S.$ Proved

Assignment 4

Group 5: Apurv & Sanjay

Question

5. Expand $\cos^8\theta$ in a series of cosines of mulltiples of θ

Solution

Let

$$z = \cos\theta + i\sin\theta \tag{0.0.1}$$

So

$$z + \overline{z} = 2\cos\theta \tag{0.0.2}$$

We observe that

$$z\overline{z} = 1 \tag{0.0.3}$$

and

$$(z + \overline{z})^n = 2\cos n\theta \tag{0.0.4}$$

This we will use in the subsequent steps for simplification. Now raising both the sides of Eq. 0.0.2 to the 8^{th} power, we have

$$\begin{aligned}
\cos^8\theta &= \frac{1}{2^8}\left[z+\overline{z}\right]^8 \qquad (0.0.5)\\
&= \frac{1}{2^8}\left[z^8+z^7\overline{z}+28z^6\overline{z}^2+56z^5\overline{z}^3+70z^4\overline{z}^4+56z^3\overline{z}^5+28z^2\overline{z}^6+8z\overline{z}^7+\overline{z}^8\right]\\
&= \frac{1}{2^8}\left[(z^8+\overline{z}^8)+8(z^6+\overline{z}^6)+28(z^4+\overline{z}^4)+56(z^2+\overline{z}^2)+70\right]\\
&= \frac{1}{2^8}\left[2\cos 8\theta+8\times 2\cos 6\theta+28\times 2\cos 4\theta+56\times 2\cos 2\theta+70\right]\\
&= \frac{1}{2^7}\left[\cos 8\theta+8\cos 6\theta+28\cos 4\theta+56\cos 2\theta+35\right]
\end{aligned}$$

Mathematical & Numerical Methods

Group-6

(Assignment – 4) (Complex No.)

(6) Prove that

$$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$$

Sol^n. Let $x = \cos\theta + i\sin\theta$, $\frac{1}{x} = \cos\theta - i\sin\theta$

So $\left(x + \frac{1}{x}\right) = 2\cos\theta$

So

$$(2\cos\theta)^6 = \left(x + \frac{1}{x}\right)^6$$

$$64\cos^6\theta = x^6 + {}^6C_1 x^4 + {}^6C_2 x^2 + {}^6C_3 + {}^6C_4 \frac{1}{x^2} + {}^6C_5 \frac{1}{x^4} + \frac{1}{x^6}$$

$$= \left(x^6 + \frac{1}{x^6}\right) + 6\left(x^4 + \frac{1}{x^4}\right) + 15\left(x^2 + \frac{1}{x^2}\right) + 20$$

Using De Moivre's theorem

$$x^n = (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

$$\frac{1}{x^n} = (\cos\theta - i\sin\theta)^n = (\cos n\theta - i\sin n\theta)$$

So

$$64\cos^6\theta = (2\cos 6\theta) + 6(2\cos 4\theta) + 15(2\cos 2\theta) + 20$$

$$\Rightarrow \boxed{32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10}$$

Proved.

ASSIGNMENT-04
GROUP-07

To Prove that

$$2^6 \sin^7\theta = 35\sin\theta - 21\sin 3\theta + 7\sin 5\theta - \sin 7\theta \quad \text{--- (\#)}$$

Use binomial expansion of cis θ to expand sin nθ

$$\sin 7\theta = {}^7C_1 \cos^6\theta \sin\theta - {}^7C_3 \cos^4\theta \sin^3\theta + {}^7C_5 \cos^2\theta \sin^5\theta - {}^7C_7 \sin^7\theta$$

$$\sin 5\theta = {}^5C_1 \cos^4\theta \sin\theta - {}^5C_3 \cos^2\theta \sin^3\theta + {}^5C_5 \sin^5\theta$$

and $$\sin 3\theta = {}^3C_1 \cos^2\theta \sin\theta - {}^3C_3 \sin^3\theta$$

Use the expansion of sin7θ, sin5θ and sin3θ in equation (#) R.H.S. of (#)

$$35\sin\theta - 21\left[3\cos^2\theta\sin\theta - \sin^3\theta\right]$$
$$+7\left[5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta\right]$$
$$-\left[7\cos^6\theta\sin\theta - 70\cos^4\theta\sin^3\theta + 21\cos^2\theta\sin^5\theta - \sin^7\theta\right]$$

RESULT FOLLOWS.

Group -8 assignment-4

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Prove that, $\sin^8\theta = 2^{-7}[\cos 8\theta - 8\cos 6\theta + 28\cos 4\theta - 56\cos 2\theta + 35]$

Now, $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

So, $\sin^8\theta = \left(\dfrac{e^{i\theta} - e^{-i\theta}}{2i}\right)^8$

$= \dfrac{1}{2^8}\left[(e^{i\theta})^8 + {}^8C_1 (e^{i\theta})^7(-e^{-i\theta}) + {}^8C_2 (e^{i\theta})^6(-e^{-i\theta})^2 + {}^8C_3 (e^{i\theta})^5(-e^{-i\theta})^3 + {}^8C_4 (e^{i\theta})^4(-e^{-i\theta})^4 + {}^8C_5 (e^{i\theta})^3(-e^{-i\theta})^5 + {}^8C_6 (e^{i\theta})^2(-e^{-i\theta})^6 + {}^8C_7 e^{i\theta}(-e^{-i\theta})^7 + (e^{-i\theta})^8\right]$

$= \dfrac{1}{2^8}\left[\{e^{i8\theta} + e^{-i8\theta}\} - {}^8C_1\{e^{i6\theta} + e^{-i6\theta}\} + {}^8C_2\{e^{i4\theta} + e^{-i4\theta}\} - {}^8C_3\{e^{i2\theta} + e^{-i2\theta}\} + {}^8C_4\right]$

[since, ${}^8C_1 = {}^8C_7 = 8$
${}^8C_2 = {}^8C_6 = 28$
${}^8C_3 = {}^8C_5 = 56$

$= \dfrac{1}{2^8}\left[2\cos 8\theta - (8\times 2\cos 6\theta) + (28\times 2\times\cos 4\theta) - (56\times 2\cos 2\theta) + 35\right]$

$= 2^{-7}[\cos 8\theta - 8\cos 6\theta + 28\cos 4\theta - 56\cos 2\theta + 35]$ (proved)

Assignment 4 — Group 9

Problem : 9

Prove that: $32\sin^4\theta\cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$

$\Rightarrow$ Let $x = \cos\theta + i\sin\theta = \text{Cis}\,\theta$ $\qquad \frac{1}{x} = \cos\theta - i\sin\theta = \text{Cis}(-\theta)$.

$\therefore \quad \cos\theta = \frac{1}{2}(x + 1/x)$

$\sin\theta = \frac{1}{2i}(x - 1/x)$.

So, $\sin^4\theta = \frac{1}{(2i)^4}(x - 1/x)^4$

$= \frac{1}{2^4}\left[x^4 + {}^4C_1\, x^3.(-1/x) + {}^4C_2\, x^2.(-1/x)^2 + {}^4C_3\, x.(-1/x)^3 + {}^4C_4\left(-\frac{1}{x}\right)^4\right]$

$= \frac{1}{2^4}\left[x^4 - 4x^2 + 6 - 4/x^2 + 1/x^4\right]$.

and $\cos^2 x = \frac{1}{2^2}\left(x + \frac{1}{x}\right)^2 = \frac{1}{2^2}\left(x^2 + 2.x.\frac{1}{x} + \frac{1}{x^2}\right)$

$= \frac{1}{2^2}\left(x^2 + \frac{1}{x^2} + 2\right)$.

So, $32\sin^4\theta\cos^2\theta = 2^5 \times \frac{1}{2^4}\left[x^4 - 4x^2 + 6 - 4/x^2 + 1/x^4\right] \times \frac{1}{2^2}\left(x^2 + 2 + 1/x^2\right)$

$= \frac{1}{2}\left[x^6 + 2x^4 + x^2 - 4x^4 - 8x^2 - 4 + 6x^2 + 12 + 6/x^2 - 4 - 8/x^2 - 4/x^4 + 1/x^2 + 2/x^4 + 1/x^6\right]$

$= \frac{1}{2}\left[\left(x^6 + 1/x^6\right) - 2\left(x^4 + 1/x^4\right) - \left(x^2 + 1/x^2\right) + 4\right]$

$= \frac{1}{2}\left[\text{Cis}(6\theta) + \text{Cis}(-6\theta) - 2\left\{\text{Cis}(4\theta) + \text{Cis}(-4\theta)\right\} - \left\{\text{Cis}(2\theta) + \text{Cis}(-2\theta)\right\} + 4\right]$.

$= \frac{1}{2}\left[2\cos 6\theta - (2\times 2)\cos 4\theta - (2\times 2)\cos 2\theta + 4\right]$

$= \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$

$\therefore$ L.H.S. = R.H.S. (proved)

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Assignment 4.

Group 10.

Q. $\sin^5\theta\cos^2\theta = \frac{1}{64}\left[\sin 7\theta - 3\sin 5\theta + \sin 3\theta + 5\sin\theta\right]$

Soln: $\left(x-\frac{1}{x}\right)^5 = \left(2i\sin\theta\right)^5$ | From $x = \cancel{(\sin\theta + i\cos\theta)}$ $\cos\theta + i\sin\theta$

$\frac{1}{x} = \cancel{\sin}(\cos\theta - i\sin\theta)$

$$x^5 - 5C_1 x^4\frac{1}{x} + 5C_2 x^3\frac{1}{x^2} - 5C_3 x^2\cdot\frac{1}{x^3} + 5C_4 x\cdot\frac{1}{x^4} - 5C_5\frac{1}{x^5} = (2i\sin\theta)^5$$

$$x^5 - 5x^3 + 10x - 10\times\frac{1}{x} + 5\times\frac{1}{x^3} - \frac{1}{x^5} = (2i\sin\theta)^5$$

$$\left(x^5 - \frac{1}{x^5}\right) - 5\left(x^3 - \frac{1}{x^3}\right) + 10\left(x - \frac{1}{x}\right) = (2i\sin\theta)^5$$

$$32i\sin^5\theta = 2i\sin 5\theta - 5(2i\sin 3\theta) + 10\cdot 2i\sin\theta$$

$$32\sin^5\theta = 2\sin 5\theta - 10\sin 3\theta + 20\sin\theta \quad \text{—— (1)}$$

$$\left(x+\frac{1}{x}\right)^2 = (2\cos\theta)^2$$

$$x^2 + \frac{1}{x^2} + 2 = 4\cos^2\theta$$

$$2\cos 2\theta + 2 = 4\cos^2\theta$$

$$4\cos^2\theta = 2\cos 2\theta + 2 \quad \text{—— (2)}$$

$$2\cos^2\theta = \cos 2\theta + 1$$

① × ② ⇒

$\cancel{32\cos^2\theta \cdot 2\cos^2\theta}$

$$32\sin^5\theta \cdot 2\cos^2\theta = \left[2\sin 5\theta - 10\sin 3\theta + 20\sin\theta\right]\left[\cos 2\theta + 1\right]$$

$$= \left[2\sin 5\theta\cos 2\theta + 2\sin 5\theta - 10\sin 3\theta\cos 2\theta - 10\sin 3\theta + 20\sin\theta\cos 2\theta + 20\sin\theta\right]$$

$$= \left[\sin 7\theta + \sin 3\theta + 2\sin 5\theta - 5\sin 5\theta - 5\sin\theta - 10\sin 3\theta + 10\sin 3\theta - 10\sin\theta + 20\sin\theta\right]$$

From $2\sin A\cos B = \sin(A+B) + \sin(A-B)$.

$$64\sin^5\theta \cdot \cos^2\theta = \left[\sin 7\theta + \sin 3\theta - 3\sin 5\theta + 5\sin\theta\right]$$

$$\sin^5\theta \cdot \cos^2\theta = \frac{1}{64}\left[\sin 7\theta + \sin 3\theta - 3\sin 5\theta + 5\sin\theta\right].$$

Proved.

G-11

Assignment-4

(1) Venkatesh Chinni
(2) Durga prasad

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1Q) P.T $2^8\cos^4\theta\sin^5\theta = \sin 9\theta - \sin 7\theta - 4\sin 5\theta + 4\sin 3\theta + 6\sin\theta$.

Sol: Let us consider, $x = \cos\theta + i\sin\theta$ then $\frac{1}{x} = \cos\theta - i\sin\theta$.

$\Rightarrow \quad x + \frac{1}{x} = 2\cos\theta$ and $x - \frac{1}{x} = 2i\sin\theta$

So, $\sin^5\theta = \frac{1}{(2i)^5}\left(x - \frac{1}{x}\right)^5$

$$\sin^5\theta = \frac{1}{32i}\left[x^5 - 5x^3 + 10x - \frac{10}{x} + \frac{5}{x^3} - \frac{1}{x^5}\right] \text{ and}$$

$$\cos^4\theta = \frac{1}{2^4}\left(x + \frac{1}{x}\right)^4 = \frac{1}{16}\left[x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}\right]$$

Now take L.H.S,

$$2^8\cos^4\theta\sin^5\theta = 2^8 \times \frac{1}{2^4}\left[x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}\right]\frac{1}{2^5 i}\left[x^5 - 5x^3 + 10x - \frac{10}{x} + \frac{5}{x^3} - \frac{1}{x^5}\right]$$

$$= \frac{2^8}{2^9 i}\Big[x^9 + 4x^7 + 6x^5 + 4x^3 + x - 5x^7 - 20x^5 - 30x^3 - 20x - \frac{5}{x} + 10x^5$$
$$+ 40x^3 + 60x + \frac{40}{x} + \frac{10}{x^3} - 10x^3 - 40x - \frac{60}{x} - \frac{40}{x^3} - \frac{10}{x^5} + 5x$$
$$+ \frac{20}{x} + \frac{30}{x^3} + \frac{20}{x^5} + \frac{5}{x^7} - \frac{1}{x} - \frac{4}{x^3} - \frac{6}{x^5} - \frac{4}{x^7} - \frac{1}{x^9}\Big]$$

By simplifying and rearranging the terms we have

$$= \frac{1}{2i}\left[\left(x^9 - \frac{1}{x^9}\right) - \left(x^7 - \frac{1}{x^7}\right) - 4\left(x^5 - \frac{1}{x^5}\right) + 4\left(x^3 - \frac{1}{x^3}\right) + 6\left(x - \frac{1}{x}\right)\right]$$

$$= \frac{1}{2i}\left[2i\sin 9\theta - 2i\sin 7\theta - 4\times 2i\sin 5\theta + 4\times 2i\sin 3\theta + 6\sin\theta \times 2i\right]$$

$$= \sin 9\theta - \sin 7\theta - 4\sin 5\theta + 4\sin 3\theta + 6\sin\theta \quad \text{R.H.S}$$

Hence, proved