# EXPERIMENTAL TECHNIQUES & ERROR ANALYSIS

Assignment - 3, Group - 4

Question → Find all the values of $(-1-i)^{1/5}$ & also find the product of all the values.

Solution →

Let us assume $z = (-1-i)^{1/5}$ ——(1)

Here z has 5 distinct roots. To evaluate these values we have to express z in Cis θ form (i.e. cos θ + i sin θ form). In order to do that let us take some assumption & proceed as below.

Take $r\cos\theta = -1$ ——(2)

$r\sin\theta = -1$ ——(3)

Squaring & adding eqn (2) & (3) we will get

$$r^2\cos^2\theta + r^2\sin^2\theta = 1+1$$

$$\Rightarrow r^2 = 2$$

$$\Rightarrow r = \sqrt{2} \text{ ——(4)}$$

Dividing eqn (3) by eqn (2) we will also get

$$\tan\theta = 1$$

$$\Rightarrow \theta = \tan^{-1}(1) = \frac{\pi}{4} \text{ ——(5)}$$

Now by using eqn (4) & (5) we can be able to express z is Cisθ form.

$$z = (r\cos\theta + i\, r\sin\theta)^{1/5} \quad (\because \text{considering eq}^n \text{ (1), (2) \& (3)})$$

$$= \left(\sqrt{2}\cos\frac{\pi}{4} + \sqrt{2}\, i \sin\frac{\pi}{4}\right)^{1/5}$$

$$= \left\{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right\}^{1/5}$$

$$= \left(\sqrt{2}\ \mathrm{Cis}\frac{\pi}{4}\right)^{1/5} = (\sqrt{2})^{1/5} \cdot \mathrm{Cis}\, \frac{1}{5}\left(2n\pi + \frac{\pi}{4}\right) \text{ ——(6)}$$

where; $n = 0, 1, 2, 3, 4$

Now substituting the individual values of $n$ in eqn (6) we can get each root of $z$.

If $n=0$, $z_0 = (\sqrt{2})^{\frac{1}{5}} \text{Cis} \frac{\pi}{20}$

If $n=1$, $z_1 = (\sqrt{2})^{\frac{1}{5}} \text{Cis} \frac{1}{5}(2\pi + \frac{\pi}{4}) = (\sqrt{2})^{\frac{1}{5}} \text{Cis} \frac{9\pi}{20}$

If $n=2$, $z_2 = (\sqrt{2})^{\frac{1}{5}} \text{Cis} \frac{1}{5}(4\pi + \frac{\pi}{4}) = (\sqrt{2})^{\frac{1}{5}} \text{Cis} \frac{17\pi}{20}$

If $n=3$, $z_3 = (\sqrt{2})^{\frac{1}{5}} \text{Cis} \frac{1}{5}(6\pi + \frac{\pi}{4}) = (\sqrt{2})^{\frac{1}{5}} \text{Cis} \frac{25\pi}{20}$

If $n=4$, $z_4 = (\sqrt{2})^{\frac{1}{5}} \text{Cis} \frac{1}{5}(8\pi + \frac{\pi}{4}) = (\sqrt{2})^{\frac{1}{5}} \text{Cis} \frac{33\pi}{20}$

$\therefore$ All the values of $(-1-i)^{1/5}$ are given by $z_0, z_1, z_2, z_3$ & $z_4$ respectively.

Now $z_0 \cdot z_1 \cdot z_2 \cdot z_3 \cdot z_4$

$= (\sqrt{2})^{\frac{1}{5} \times 5} \cdot \text{Cis} \frac{\pi}{20} \cdot \text{Cis} \frac{9\pi}{20} \cdot \text{Cis} \frac{17\pi}{20} \cdot \text{Cis} \frac{25\pi}{20} \cdot \text{Cis} \frac{33\pi}{20}$

$= \sqrt{2} \ \text{Cis} \left( \frac{\pi}{20} + \frac{9\pi}{20} + \frac{17\pi}{20} + \frac{25\pi}{20} + \frac{33\pi}{20} \right)$

$= \sqrt{2} \ \text{Cis} \left( \frac{\cancel{85}^{17} \pi}{\cancel{20}_{4}} \right)$ [$\because$ corrolary of De Moivre's Theorem i.e. $\text{Cis}\,\theta_1 \cdot \text{Cis}\,\theta_2 \cdot \text{Cis}\,\theta_3 = \text{Cis}(\theta_1 + \theta_2 + \theta_3)$]

$= \sqrt{2} \ \text{Cis} \frac{17\pi}{4}$

$\therefore$ Product of all the roots of $(-1-i)^{\frac{1}{5}}$ is $\sqrt{2} \ \text{Cis} \frac{17\pi}{4}$

$= \sqrt{2} \left( \cos \frac{17\pi}{4} + i \sin \frac{17\pi}{4} \right)$

$= \sqrt{2} \left\{ \cos \left(4\pi + \frac{\pi}{4}\right) + i \sin \left(4\pi + \frac{\pi}{4}\right) \right\}$

$= \sqrt{2} \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$

$= \sqrt{2} \left( \frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = 1 + i$

Ans

Mathematical ~~Me~~ & Numerical Methods   Group-6

(Assignment - 3) (Complex Numbers)

Q. 2 (ii) Find all values of

$$(-1+i\sqrt{3})^{3/2}$$

Solⁿ– First we will change the complex number $(-1+i\sqrt{3})$ in to polar form by using

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\underbrace{-1 = r\cos\theta, \quad \sqrt{3} = r\sin\theta}$$

$$\Downarrow$$

$$\tan\theta = -\sqrt{3} \Rightarrow \theta = -\pi/3 = \frac{5\pi}{3}$$
$$r^2 = 4 \Rightarrow r = 2$$

So we can write

$$(-1+i\sqrt{3}) = 2\left[\cos(5\pi/3) + i\sin(5\pi/3)\right]$$

$$(-1+i\sqrt{3})^{3/2} = \left[2\,\mathrm{cis}\left(\frac{5\pi}{3}\right)\right]^{3/2}$$

$$= \left[2\,\mathrm{cis}\left(2n\pi + \frac{5\pi}{3}\right)\right]^{3/2}$$

$$= \left[2\,\mathrm{cis}\left(\frac{6n\pi + 5\pi}{3}\right)\right]^{3/2}$$

$$= (2)^{3/2}\,\mathrm{cis}\left[\frac{3}{2}\cdot\left(\frac{6n\pi+5\pi}{3}\right)\right] \qquad (\because (\mathrm{cis}\,\theta)^n = \mathrm{cis}\,n\theta)$$

$$= (2)^{3/2}\,\mathrm{cis}\,\frac{(6n\pi+5\pi)}{2}$$

The roots of complex no can be found by substituting $n=0,1$. respectively

$$(2)^{3/2}\,\mathrm{cis}\,\frac{5\pi}{2}, \quad (2)^{3/2}\,\mathrm{cis}\,\frac{11\pi}{2}$$

$\Rightarrow$ ~~$2\sqrt{2}\,(\cos\frac{5\pi}{2}),\ 2\sqrt{2}$~~

$$\Rightarrow 2\sqrt{2}\left\{\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right\}, \quad 2\sqrt{2}\left\{\cos\frac{11\pi}{2} + i\sin\frac{11\pi}{2}\right\}$$

$$\Rightarrow 2\sqrt{2}\,(0+i), \quad 2\sqrt{2}\,(0+i(-1))$$

Assignment - 3

Gr - 1

Vikas Chand & Newton Nath

2(iii) Find all the values of $\longrightarrow$

$$(1+i\sqrt{3})^{3/4} + (1-i\sqrt{3})^{3/4}$$

Sol$^n$/ $= (1+i\sqrt{3})^{3/4} + (1-i\sqrt{3})^{3/4}$

Deviding & multipling each term by 2 we get $\longrightarrow$

$$= \left\{2\left(\tfrac{1}{2}+i\tfrac{\sqrt{3}}{2}\right)\right\}^{3/4} + \left\{2\left(\tfrac{1}{2}-i\tfrac{\sqrt{3}}{2}\right)\right\}^{3/4}$$

$$= 2^{3/4}\left[\left(\cos\tfrac{\pi}{3}+i\sin\tfrac{\pi}{3}\right)^{3/4} + \left(\cos\tfrac{\pi}{3}-i\sin\tfrac{\pi}{3}\right)^{3/4}\right].$$

$$= 2^{3/4}\left[\left(\mathrm{cis}\,\tfrac{\pi}{3}\right)^{3/4} + \left(\mathrm{cis}\left(-\tfrac{\pi}{3}\right)\right)^{3/4}\right]$$

$\because$ $\boxed{\mathrm{cis}(\pm\theta) = (\cos\theta \pm i\sin\theta)}$

$$= 2^{3/4}\left[(\mathrm{cis}\,\pi)^{1/4} + (\mathrm{cis}(-\pi))^{1/4}\right] \longrightarrow (1)$$

using De Moivre's theorem.

i.e. $(\mathrm{cis}\,\theta)^n = \mathrm{cis}\,n\theta$

(1) becomes

$$= 2^{3/4}\left[\mathrm{cis}\,\frac{(2n+1)\pi}{4} + \mathrm{cis}\left(\frac{2n-1}{4}\right)\pi\right].$$

$\longrightarrow (2)$

where, $n = 0, 1, 2, 3$

since given prob. has '4' roots.

for, $n=0$, from (2) we have.

$$= 2^{3/4}\left[\text{cis}\frac{\pi}{4} + \text{cis}\left(-\frac{\pi}{4}\right)\right]$$

$$= 2^{3/4}\left[\cos\frac{\pi}{4} + i\cancel{\sin\frac{\pi}{4}} + \cos\frac{\pi}{4} - i\cancel{\sin\frac{\pi}{4}}\right]$$

$$= 2^{3/4} \cdot 2\cos\frac{\pi}{4} = 2^{3/4} \cdot 2 \cdot \frac{1}{\sqrt{2}} = 2^{5/2}$$

$n=1$, $$= 2^{3/4}\left[\text{cis}\frac{3\pi}{4} + \text{cis}\frac{\pi}{4}\right]$$

$$= 2^{3/4}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} + \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$$

$$= 2^{3/4}\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{2}+\frac{\pi}{4}\right) + \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]$$

$$= 2^{3/4}\left[-\sin\frac{\pi}{4} + i\cos\frac{\pi}{4} + \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]$$

$$= 2^{3/4}\left[-\cancel{\frac{1}{\sqrt{2}}} + i\frac{1}{\sqrt{2}} + \cancel{\frac{1}{\sqrt{2}}} + i\frac{1}{\sqrt{2}}\right]$$

$$= i\,2^{3/4} \cdot 2\frac{1}{\sqrt{2}} = i\,2^{5/2}.$$

$n=2$, $$= 2^{3/4}\left[\text{cis}\frac{5\pi}{4} + \text{cis}\left(\frac{3\pi}{4}\right)\right]$$

$$= 2^{3/4}\left[\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} + \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$$

$$= 2^{3/4}\left[\cos\left(\pi+\frac{\pi}{4}\right) + i\sin\left(\pi+\frac{\pi}{4}\right) + \cos\left(\pi-\frac{\pi}{4}\right) + i\sin\left(\pi-\frac{\pi}{4}\right)\right]$$

$$= 2^{3/4}\left[-\cos\frac{\pi}{4} + i\cancel{\sin\frac{\pi}{4}} - \cos\frac{\pi}{4} + i\cancel{\sin\frac{\pi}{4}}\right].$$

$$= -2^{3/4} \cdot 2 \cdot \frac{1}{\sqrt{2}} \qquad \left[\because \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right.$$

$$= -2^{5/2}$$

$n=3$, $$= 2^{3/4}\left[\text{cis}\frac{7\pi}{4} + \text{cis}\frac{5\pi}{4}\right]$$

$$= 2^{3/4}\left[\cos\left(2\pi-\frac{\pi}{4}\right) + i\sin\left(2\pi-\frac{\pi}{4}\right) + \cos\left(\pi+\frac{\pi}{4}\right) + i\sin\left(\pi+\frac{\pi}{4}\right)\right]$$

$$= 2^{3/4}\left[\cancel{\cos\pi/4} - i\sin\pi/4 - \cancel{\cos\pi/4} - i\sin\pi/4\right]$$

$$= -i2^{3/4}\, 2 \cdot \frac{1}{\sqrt{2}} = -i\,2^{5/2}.$$

Ans: The values are $\pm 2^{5/2}$ & $\pm i2^{5/2}$

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Group - 8      assignment - 3

Find out all values of $(-1)^{1/6}$.

Now,

$$-1 = e^{i\pi} = e^{i(2n+1)\pi}$$

So,

$$(-1)^{1/6} = e^{i(2n+1)\pi/6} \quad \text{where } n = 0, 1, \ldots, 5$$

$$= e^{i\pi/6},\ e^{i(3\pi)/6},\ e^{i5\pi/6},\ e^{i7\pi/6},\ e^{i9\pi/6},\ e^{i11\pi/6}$$

# ASSIGNMENT - 03

## G-07

To find the roots of

$$x^7 + x^4 + x^3 + 1 = 0$$

$$x^4(x^3+1) + 1(x^3+1) = 0$$

$$(x^3+1)(x^4+1) = 0$$

$$x^3 = -1, \quad x^4 = -1$$

$$x^3 = \text{cis}(\pi), \quad x^4 = \text{cis}(\pi)$$

Roots are

$$\text{cis}\left(\frac{\pi}{3}\right), \quad \text{cis}\left(\frac{3\pi}{3}\right), \quad \text{cis}\left(\frac{5\pi}{3}\right)$$

$$\text{cis}\left(\frac{\pi}{4}\right), \quad \text{cis}\left(\frac{3\pi}{4}\right), \quad \text{cis}\left(\frac{5\pi}{4}\right), \quad \text{cis}\left(\frac{7\pi}{4}\right)$$

G - 11 Assignment - 3

(1) Venkatesh chinni
(2) Durga prasad



(Q) Solve $x^8 - x^5 + x^3 - 1 = 0$

Sol

$$x^8 - x^5 + x^3 - 1 = 0$$

$$\Rightarrow x^5(x^3-1) + x^3 - 1 = 0$$

$$\Rightarrow (x^3-1)(x^5+1) = 0.$$

$\Rightarrow x^3 - 1 = 0$ and $x^5 + 1 = 0$

$x^5 = (-1)$

$\Rightarrow x^3 = 1$

$x = (-1)^{1/5}$

$\Rightarrow x = (1)^{1/3}$

$x = (\text{cis}\,\pi)^{1/5}$

$x = [\text{cis}(0)]^{1/3}$

$= \text{cis}(2n\pi + \pi)^{1/5}$

$x = [\text{cis}\,(2n\pi + 0)]^{1/3}$

$= [\text{cis}\ (2n+1)\pi]^{1/5}$

$x = \text{cis}\left(\frac{2n\pi}{3}\right)$

$x = \text{cis}\,\frac{(2n+1)\pi}{5}$

put $n = 0, 1, 2$

put $n = 0, 1, 2, 3, 4$

$x = \text{cis}\,0, \text{cis}\left(\frac{2\pi}{3}\right), \text{cis}\left(\frac{4\pi}{3}\right)$

$x = \text{cis}\left(\frac{\pi}{5}\right), \text{cis}\left(\frac{3\pi}{5}\right), \text{cis}\left(\frac{5\pi}{5}\right)$
$\text{cis}\left(\frac{7\pi}{5}\right), \text{cis}\left(\frac{9\pi}{5}\right)$

$\therefore x = 1, \text{cis}\left(\frac{2\pi}{3}\right), \text{cis}\left(\frac{4\pi}{3}\right), \text{cis}\left(\frac{\pi}{5}\right), \text{cis}\left(\frac{3\pi}{5}\right)$

$\text{cis}(\pi), \text{cis}\left(\frac{7\pi}{5}\right), \text{cis}\left(\frac{9\pi}{5}\right)$

These are the roots ~~of~~ required

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Assignment - 3

(Biven Geo George and Ritwik) Group 10

Q. 6

Solve $(x-1)^5 + (x)^5 = 0$.

Ans

$$(x-1)^5 + x^5 = 0$$

$$\Rightarrow \left(1-\frac{1}{x}\right)^5 + 1 = 0 \quad \text{(dividing by } x^5\text{)}$$

$$\Rightarrow \left(\frac{1}{x}-1\right)^5 = 1 = e^{2\pi i n}$$

$$\Rightarrow \frac{1}{x} - 1 = e^{2\pi i n/5}$$

For $n=0$, $\frac{1}{x} - 1 = 1 \Rightarrow x = 1/2$

For $n=1$, $\frac{1}{x} - 1 = e^{2\pi i/5} \Rightarrow x = \frac{1}{1+e^{2\pi i/5}}$

For $n=2$, $\frac{1}{x} - 1 = e^{4\pi i/5} \Rightarrow x = \frac{1}{1+e^{4\pi i/5}}$

For $n=3$, $\frac{1}{x} - 1 = e^{6\pi i/5} \Rightarrow x = \frac{1}{1+e^{6\pi i/5}}$

For $n=4$, $\frac{1}{x} - 1 = e^{8\pi i/5} \Rightarrow x = \frac{1}{1+e^{8\pi i/5}}$

so, There are five roots and that's it.

Assignment 3

Date: 11th Aug.'1[illegible]
Group-9

Problem 7:

Find the roots common to the equations $x^4+1=0$ and $x^6-i=0$.

$\Rightarrow$ Equation (1)

$$x^4+1=0$$
$$\text{or, } x^4=-1$$
$$\text{or, } x^4=\text{Cis}((2n+1)\pi) \quad (\text{where } n=0,1,2,\dots)$$
$$\text{or, } x=\{\text{Cis}((2n+1)\pi)\}^{1/4}$$
$$=\text{Cis}\left((2n+1)\frac{\pi}{4}\right) \quad \{\text{from De' Moivre's theorem}\}.$$
$$=\text{Cis}\,\pi/4,\ \text{Cis}\,3\pi/4,\ \text{Cis}\,5\pi/4,\ \text{Cis}\,7\pi/4$$

So, the roots of the equation (1). $x^4+1=0$ are:

$$\text{Cis}\,\pi/4,\ \text{Cis}\,3\pi/4,\ \text{Cis}\,5\pi/4,\ \text{Cis}\,7\pi/4$$
$$\Rightarrow \tfrac{1}{\sqrt2}(1+i),\ \tfrac{1}{\sqrt2}(-1+i),\ -\tfrac{1}{\sqrt2}(1+i),\ \tfrac{1}{\sqrt2}(1-i)$$

Equation (2)

$$x^6-i=0$$
$$\text{or, } x^6=i$$
$$\text{or, } x^6=\text{Cis}[(2n+1)\pi/2] \quad [\text{where } n=0,2,4,\dots]$$
$$\therefore x=\{\text{Cis}[(2n+1)\pi/2]\}^{1/6}$$
$$=\text{Cis}\left[(2n+1)\frac{\pi}{12}\right] \quad \{\text{from De' Moivre's theorem}\}$$
$$=\text{Cis}\,\pi/12,\ \text{Cis}\,5\pi/12,\ \text{Cis}\,9\pi/12,\ \text{Cis}\,13\pi/12,\ \text{Cis}\,17\pi/12,\ \text{Cis}\,21\pi/12.$$
$$=\text{Cis}\,\pi/12,\ \text{Cis}\,5\pi/12,\ \text{Cis}\,3\pi/4,\ \text{Cis}\,13\pi/12,\ \text{Cis}\,17\pi/12,\ \text{Cis}\,7\pi/4$$

So, the roots of the eq$^n$ (2) $x^6-i=0$ are:

$\operatorname{Cis}\frac{\pi}{12}, \operatorname{Cis}\frac{5\pi}{12}, \operatorname{Cis}\frac{3\pi}{4}, \operatorname{Cis}\frac{13\pi}{12}, \operatorname{Cis}\frac{17\pi}{12}, \operatorname{Cis}\frac{7\pi}{4}$

So, the common roots of the two equations are:

$\operatorname{Cis}\frac{3\pi}{4}$ and $\operatorname{Cis}\frac{7\pi}{4}$

$= \frac{1}{\sqrt{2}}(-1+i)$ and $\frac{1}{\sqrt{2}}(1-i)$ Ans

ASSIGNMENT NO 3

GROUP-2
CHANDANA
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PROBLEM 8:

Solve $x^{12}-1=0$, and find which of its roots satisfy the equation: $x^4+x^2+1=0$

Solution:

$$x^{12}-1=0$$

$$x^{12}=1$$

$$x=(1)^{1/12}$$

$$x=(\text{cis } 2n\pi)^{1/12}$$

$$x=\text{cis}\frac{n\pi}{6}=\cos\frac{n\pi}{6}+i\sin\frac{n\pi}{6}$$

$$x=0,1,2,3,4,5$$

Roots $= 1, \frac{\sqrt{3}}{2}+i\frac{1}{2}, \frac{1}{2}+\frac{i\sqrt{3}}{2}, 0+i, -\frac{1}{2}+\frac{i\sqrt{3}}{2},$

$-\frac{\sqrt{3}}{2}+\frac{i1}{2}$

Roots for the quadratic equation:

$$x^4+x^2+1=0$$

let $x^2=a$

$$a^2+a+1=0$$

$$a=\frac{-1\pm\sqrt{3}i}{2}$$

$$x_1=\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{1/2} \quad x_2=\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{1/2}$$

We know that $x + iy = r\cos\theta + ir\sin\theta$

$r = \sqrt{x^2+y^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}}$

$r = 1$

$\tan\theta = \frac{+\sqrt{3}/2}{-1/2}$

$\tan\theta = -\sqrt{3}$

$\theta = 120^\circ$

$x + iy = \cos 120^\circ + i\sin 120^\circ$

for root $x_1 = (\cos 120^\circ + i\sin 120^\circ)^{1/2}$

$= \cos 60 + i\sin 60$

$= \frac{1}{2} + i\frac{\sqrt{3}}{2}$

for root $x_2$ ~~cos 3~~ $r = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}$

$r = 1$

$\tan\theta = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$

$\theta = 60^\circ$

$\therefore \quad x + iy = (\cos 60^\circ + i\sin 60)^{1/2}$

$= \cos 30^\circ + i\sin 30^\circ$

$= \frac{\sqrt{3}}{2} + i\frac{1}{2}$

Hence, there are only 2 roots of the quadratic eqn which satisfies $x^2 - 1 = 0$

# Assignment 3

Group 5: Apurv & Sanjay

## Question

9. Prove that the $n^{th}$ root of unity form a GP. Also show that the sum of these $n$ roots is zero and their product is $-1^{n-1}$

## Solution

Let

$$z^n = 1 = e^{i(0+2\pi m)}, \qquad m \in \mathbb{Z}$$

Taking $n^{th}$ root on both sides,

$$z = e^{\frac{i2\pi m}{n}}, \qquad m = 0, 1, \ldots, (n-1).$$

So the $n$ roots are

$$\begin{aligned} z_0 &= \left(e^{\frac{i2\pi m}{n}}\right)^0 \\ z_1 &= \left(e^{\frac{i2\pi}{n}}\right)^1 \\ &\vdots \\ z_{n-1} &= \left(e^{\frac{i2\pi}{n}}\right)^{n-1} \end{aligned}$$

This may easily be identified as a GP with common ratio $e^{\frac{i2\pi}{n}}$. Now sum of the $n$ roots are given by

$$\begin{aligned}
\sum_{k=0}^{n-1} z_k &= 1 + \left(e^{\frac{i2\pi}{n}}\right)^1 + \left(e^{\frac{i2\pi}{n}}\right)^2 + \ldots + \left(e^{\frac{i2\pi}{n}}\right)^{n-1} \\
&= \frac{1 - \left(e^{\frac{i2\pi}{n}}\right)^{n-1+1}}{1 - e^{\frac{i2\pi}{n}}} \\
&= \frac{1 - \cancelto{1}{e^{i2\pi}}}{1 - e^{\frac{i2\pi}{n}}} = 0 \qquad Q.E.D
\end{aligned}$$

The product of the $n$ roots is given by

$$\begin{aligned}
\prod_{k=0}^{n-1} z_k &= e^0 \times e^{\frac{i2\pi}{n}} \times \left(e^{\frac{i2\pi}{n}}\right)^2 \times \ldots \times \left(e^{\frac{i2\pi}{n}}\right)^{n-1} \\
&= exp\left(0 + \frac{i2\pi}{n} + \frac{i2\pi}{n}2 + \ldots + \frac{i2\pi}{n}(n-1)\right) \\
&= exp\left(\frac{i2\pi}{n}[0 + 1 + 2 + \ldots + (n-1)]\right) \qquad (0.0.1)
\end{aligned}$$

We observe that what we have in the argument is an A.P. So from the equation of the sum of $n$ terms of an A.P the argument get reduced as

$$\begin{aligned}
\sum_{k=0}^{m-1} z_k = exp\left(\frac{i2\pi}{n}(n-1)\frac{n}{2}\right) = exp\left(i\pi(n-1)\right) &= exp(i\pi)^{n-1} \\
&= -1^{n-1} \qquad Q.E.D
\end{aligned}$$