Assignment - 2 Group - 3

Problem (1):— prove that $\dfrac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5} = 1$

Solⁿ:— L.H.S

$$\frac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5}$$

$$= \frac{[cis(-5\theta)]^2 \; [cis\, 7\theta]^{-3}}{[cis(-4\theta)]^9 \; [cis\,\theta]^5} \quad \left\{ \because \begin{array}{l} cis\,\theta = \cos\theta + i\sin\theta \\ cis(-\theta) = \cos\theta - i\sin\theta \end{array} \right\}$$

$$= \frac{cis(-10\theta)\; cis(-21\theta)}{cis(-36\theta)\; cis(5\theta)} \quad \left\{ \because [cis\,\theta]^n = cis\, n\theta \right\}$$

$$= \frac{cis(-10\theta - 21\theta)}{cis(-36\theta + 5\theta)} \quad \left\{ \because cis\,\theta_1 \cdot cis\,\theta_2 = cis(\theta_1 + \theta_2) \right\}$$

$$= \frac{cis(-31\theta)}{cis(-31\theta)}$$

$$= cis(-31\theta + 31\theta) \quad \left\{ \because \frac{cis\,\theta}{cis(-\theta)} = cis(\theta)\cdot cis(\theta) = cis(\theta + \theta) = cis\, 2\theta \right.$$

$$= cis(0)$$

$$= \cos 0 + i\sin 0$$

$$= 1 + i\cdot 0$$

$$= 1$$

$= R.H.S$

Hence proved.

EXPERIMENTAL TECHNIQUES & ERROR ANALYSIS

Assignment – 2 , Group – 4

Question → Prove that : $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4 = \cos 8\theta + i\sin 8\theta$

Solution →

Lets consider a complex number z of the form,

$$z = \cos\theta + i\sin\theta \quad \text{——— (1)}$$

then $\frac{1}{z} = \frac{1}{\cos\theta + i\sin\theta}$

We have to normalize $\frac{1}{z}$ as follows by multiplying and dividing $(\cos\theta - i\sin\theta)$ both in numerator & denominator

$$\text{So } \frac{1}{z} = \frac{\cos\theta - i\sin\theta}{(\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)}$$

$$= \frac{\cos\theta - i\sin\theta}{(\cos\theta)^2 - (i\sin\theta)^2}$$

$$= \frac{\cos\theta - i\sin\theta}{\cos^2\theta + \sin^2\theta}$$

$$\Rightarrow \frac{1}{z} = \cos\theta - i\sin\theta$$

$$\Rightarrow \frac{i}{z} = i(\cos\theta - i\sin\theta)$$

$$\Rightarrow \frac{i}{z} = \sin\theta + i\cos\theta \quad \text{——— (2)}$$

$$\text{L.H.S.} = \left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4$$

$$= \left(\frac{z}{i/z}\right)^4 \qquad [\because \text{Substituting from eq}^n \text{ (1) \& (2)}]$$

$$= \left(\frac{z^2}{i}\right)^4$$

$$= \frac{z^8}{i^4}$$

$$= z^8 = (\cos\theta + i\sin\theta)^8 \qquad [\because \text{by De Moiver's Theorem}]$$

$$= \cos 8\theta + i\sin 8\theta = \text{R.H.S.}$$

Proved

Group - 6
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Mathematical & Numerical Methods

भौतिक अनुसंधान प्रयोगशाला
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Assignment # 2

Question If $p = cis\theta$ and $q = cis\phi$ show that

(i) $\dfrac{p-q}{p+q} = i\tan\dfrac{\theta-\phi}{2}$

L.H.S. $= \dfrac{p-q}{p+q} = \dfrac{cis\theta - cis\phi}{cis\theta + cis\phi} = \dfrac{\cos\theta + i\sin\theta - (\cos\phi + i\sin\phi)}{\cos\theta + i\sin\theta + \cos\phi + i\sin\phi}$

$$= \frac{\cos\theta - \cos\phi + i(\sin\theta - \sin\phi)}{\cos\theta + \cos\phi + i(\sin\theta + \sin\phi)}$$

$$= \frac{\cos\theta - \cos\phi + i(\sin\theta - \sin\phi)}{(\cos\theta + \cos\phi)^2 + (\sin\theta + \sin\phi)^2} \times (\cos\theta + \cos\phi - i(\sin\theta + \sin\phi)) \quad \text{--- (1)}$$

[$\because$ Rationalized]

solving denominator from (1)

$$(\cos\theta + \cos\phi)^2 + (\sin\theta + \sin\phi)^2 = \cos^2\theta + \cos^2\phi + 2\cos\theta\cos\phi + \sin^2\theta + \sin^2\phi + 2\sin\theta\sin\phi$$

$$= 2[1 + \cos\theta\cos\phi + \sin\theta\sin\phi]$$

$$= 2[1 + \cos(\theta - \phi)]$$

$$= 2 \times 2\cos^2\left(\frac{\theta-\phi}{2}\right) \quad \text{--- (2)}$$

simplifying numerator from eqn (1)

$$[\cos\theta - \cos\phi + i(\sin\theta - \sin\phi)] \times [\cos\theta + \cos\phi - i(\sin\theta + \sin\phi)]$$

$$= \cos^2\theta - \cos^2\phi + i(\sin\theta - \sin\phi)(\cos\theta + \cos\phi) + \sin^2\theta - \sin^2\phi - i(\cos\theta - \cos\phi)(\sin\theta + \sin\phi)$$

$$= i[\cancel{\sin\theta\cos\theta} - \sin\phi\cos\theta + \cos\phi\sin\theta - \cancel{\sin\phi\cos\phi}] - i[\cancel{\cos\theta\sin\theta} + \cos\theta\sin\phi - \cos\phi\sin\theta - \cancel{\cos\phi\sin\phi}]$$

$$= 2i[\sin\theta\cos\phi - \sin\phi\cos\theta]$$

$$= 2i\,[\sin\theta\cos\phi - \sin\phi\cos\theta] \qquad \text{——— (3)}$$

Substituting denominator & numerator from (2) & (3) in equation (1), we get

$$\frac{p-q}{p+q} = \frac{2i[\sin\theta\cos\phi - \sin\phi\cos\theta]}{2\times 2\cos^2\left(\frac{\theta-\phi}{2}\right)}$$

$$= \frac{2i\sin(\theta-\phi)}{4\cos^2\left(\frac{\theta-\phi}{2}\right)}$$

$$= \frac{4i\sin\left(\frac{\theta-\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)}{4\cos^2\left(\frac{\theta-\phi}{2}\right)}$$

$$\boxed{\frac{p-q}{p+q} = i\tan\left(\frac{\theta-\phi}{2}\right)}$$

Group - 8

assignment - 2

③ ii) $p = cis\theta = e^{i\theta} = \cos\theta + i\sin\theta$

$\frac{1}{p} = \frac{1}{cis\theta} = e^{-i\theta} = \cos\theta - i\sin\theta$

So, $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

$q = cis\phi = e^{i\phi} = \cos\phi + i\sin\phi$. So $\frac{1}{q} = e^{-i\phi}$

Similarly, $\sin\phi = \frac{e^{i\phi} - e^{-i\phi}}{2i}$

Now, $R.H.S = \frac{\sin\theta + \sin\phi}{\sin\theta - \sin\phi} = \frac{\left(\frac{e^{i\theta} - e^{-i\theta}}{2i}\right) + \left(\frac{e^{i\phi} - e^{-i\phi}}{2i}\right)}{\left(\frac{e^{i\theta} - e^{-i\theta}}{2i}\right) - \left(\frac{e^{i\phi} - e^{-i\phi}}{2i}\right)}$

$= \frac{e^{i\theta} - e^{-i\theta} + e^{i\phi} - e^{-i\phi}}{e^{i\theta} - e^{-i\theta} - e^{i\phi} + e^{-i\phi}}$

$= \frac{p - \frac{1}{p} + q - \frac{1}{q}}{p - \frac{1}{p} - q + \frac{1}{q}}$ [using $p = e^{i\theta}$ and $\frac{1}{p} = e^{-i\theta}$, $q = e^{i\phi}$, $\frac{1}{q} = e^{-i\phi}$]

$= \frac{p^2q - q + pq^2 - p}{p^2q - q - pq^2 + p}$

$= \frac{p(pq-1) + q(pq-1)}{p(pq+1) - q(pq+1)} = \frac{(p+q)(pq-1)}{(p-q)(pq+1)}$

$= R.H.S$ (proved)

ASSIGNMENT - 02

GROUP - 7

Given that $a = cis(2\alpha)$, $b = cis(2\beta)$, $c = cis(2\gamma)$

We have to prove that

$$\sqrt{\frac{ab}{c}} + \sqrt{\frac{c}{ab}} = 2\cos(\alpha+\beta-\gamma) \quad — (\#)$$

Game begins by moving from cis to exponential representation of complex equality / condn given in the problem, and reverting back to its $\cos\theta + i\sin\theta$ form at the end

We have

$a = Exp(i2\alpha)$, $b = Exp(i2\beta)$, $c = Exp(i2\gamma)$

From L.H.S. of (#)

$$\{Exp[i2(\alpha+\beta-\gamma)]\}^{1/2} + \{Exp[i2(-\alpha-\beta+\gamma)]\}^{1/2}$$

$$= \cos(\alpha+\beta-\gamma) + i\sin(\alpha+\beta-\gamma) + \cos(-\alpha-\beta+\gamma) + i\sin(-\alpha-\beta+\gamma)$$

Exploiting even and odd prop. of cosine and sine function respectively

$$= 2\cos(\alpha+\beta-\gamma) = 2\cos(-\alpha-\beta+\gamma)$$

Proved

ASSIGNMENT : 2

GROUP : 2
CHANDANA
JINIA SIKDAR

Q. If $a = \text{cis } 2\alpha$, $b = \text{cis } 2\beta$, $c = \text{cis } 2\gamma$ & $d = \text{cis } 2\delta$, prove that:

$$\sqrt{\frac{ab}{cd}} + \sqrt{\frac{cd}{ab}} = 2\cos(\alpha + \beta - \gamma - \delta)$$

Sol^n:

$$L.H.S = \sqrt{\frac{ab}{cd}} + \sqrt{\frac{cd}{ab}}$$

$$= \sqrt{\frac{(\text{cis } 2\alpha).(\text{cis } 2\beta)}{(\text{cis } 2\gamma).(\text{cis } 2\delta)}} + \sqrt{\frac{(\text{cis } 2\gamma).(\text{cis } 2\delta)}{(\text{cis } 2\alpha).(\text{cis } 2\beta)}}$$

$$= \sqrt{\frac{(\text{cis}\alpha)^2.(\text{cis}\beta)^2}{(\text{cis}\gamma)^2.(\text{cis}\delta)^2}} + \sqrt{\frac{(\text{cis}\gamma)^2.(\text{cis}\delta)^2}{(\text{cis}\alpha)^2.(\text{cis}\beta)^2}}$$

$$= \frac{(\text{cis}\alpha)(\text{cis}\beta)}{(\text{cis}\gamma)(\text{cis}\delta)} + \frac{(\text{cis}\gamma)(\text{cis}\delta)}{(\text{cis}\alpha)(\text{cis}\beta)}$$

$$= \frac{(\text{cis}^2\alpha)(\text{cis}^2\beta) + (\text{cis}^2\gamma)(\text{cis}^2\delta)}{(\text{cis}\gamma)(\text{cis}\delta)(\text{cis}\alpha)(\text{cis}\beta)}$$

$$= \frac{(\text{cis } 2\alpha)(\text{cis } 2\beta) + (\text{cis } 2\gamma)(\text{cis } 2\delta)}{\text{cis}(\alpha + \beta + \gamma + \delta)}$$

$$= \frac{\text{cis}(2\alpha + 2\beta) + \text{cis}(2\gamma + 2\delta)}{\text{cis}(\alpha + \beta + \gamma + \delta)} \longrightarrow (1)$$

Multiplying eq^n (1) with $\text{cis}(-2\gamma - 2\delta)$ and dividing it with the same value, we get :-

$$L.H.S = \frac{\text{cis}(2\alpha + 2\beta) + \text{cis}(2\gamma + 2\delta)}{\text{cis}(\alpha + \beta + \gamma + \delta)} \times \frac{\text{cis}(-2\gamma - 2\delta)}{\text{cis}(-2\gamma - 2\delta)}$$

$$= \frac{\text{cis}(2\alpha + 2\beta)\,\text{cis}(-2\gamma - 2\delta) + 1}{\text{cis}(\alpha + \beta - \gamma - \delta)}$$

$$= \frac{\left(\text{Cis}\,[2(\alpha+\beta-\gamma-\delta)]+1\right)}{\text{Cis}\,(\alpha+\beta-\gamma-\delta)}$$

$$= \left\{\text{Cis}\,[2(\alpha+\beta-\gamma-\delta)]+1\right\}\cdot\left\{\text{Cis}\,(-\alpha-\beta+\delta+\gamma)\right\}$$

$$= \left\{\text{Cis}\,2(\alpha+\beta-\gamma-\delta)\cdot\text{Cis}\,(-\alpha-\beta+\delta+\gamma)\right\} + \left\{\text{Cis}\,(-\alpha-\beta+\delta+\gamma)\right\}$$

$$= \left\{\text{Cis}\,(\alpha+\beta-\gamma-\delta) + \text{Cis}\,(-\alpha-\beta+\gamma+\delta)\right\}$$

Let $\alpha+\beta-\gamma-\delta = \theta$, then

$$L.H.S = \text{Cis}\,\theta + \text{Cis}\,(-\theta)$$

$$= (\cos\theta + i\sin\theta) + (\cos\theta - i\sin\theta)$$

$$= 2\cos\theta$$

$$= 2\cos(\alpha+\beta-\gamma-\delta)$$

Hence Proved.

Assignment - 2

Date: 10. 08. 2012
Group - 9

Problem 5:

Simplify $[\cos\alpha - \cos\beta + i(\sin\alpha - \sin\beta)]^n + [\cos\alpha - \cos\beta - i(\sin\alpha - \sin\beta)]^n$

$$\Rightarrow z = [\cos\alpha - \cos\beta + i(\sin\alpha - \sin\beta)]^n + [\cos\alpha - \cos\beta - i(\sin\alpha - \sin\beta)]^n$$

Let $\cos\alpha - \cos\beta = r\cos\theta$

$\sin\alpha - \sin\beta = r\sin\theta$.

So that
$$r^2 = (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$$
$$= \cos^2\alpha + \cos^2\beta - 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta - 2\sin\alpha\sin\beta.$$

$$= 1 + 1 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta).$$

$$= 2 - 2\cos(\alpha - \beta). \qquad [\because \cos A\cos B + \sin A\sin B = \cos(A-B)]$$

$$\therefore r = [2(1 - \cos(\alpha - \beta))]^{1/2} \qquad [\because 1 - \cos\phi = 2\sin^2\phi/2$$

$$= \left[2 \cdot 2\sin^2\left(\frac{\alpha-\beta}{2}\right)\right]^{1/2}$$

$$= 2\sin\left(\frac{\alpha-\beta}{2}\right).$$

and
$$\theta = \tan^{-1}\left(\frac{\sin\alpha - \sin\beta}{\cos\alpha - \cos\beta}\right).$$

$$= \tan^{-1}\left\{-\left[\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha}\right]\right\}$$

$[\because \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$ and $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$$= \tan^{-1}\left\{-\frac{2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}{2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}\right\}$$

$$= \tan^{-1}\left(-\cot\frac{\alpha+\beta}{2}\right) = \pi/2 + \cot^{-1}\left(\cot\left(\frac{\alpha+\beta}{2}\right)\right) \quad [\because \tan^{-1}\phi = \pi/2 - \cot^{-1}\phi$$

$$= \pi/2 + \frac{\alpha+\beta}{2} = \frac{\pi + \alpha + \beta}{2}$$

So, $z = [r\cos\theta + ir\sin\theta]^n + [r\cos\theta - ir\sin\theta]^n$

$= r^n[(\cos\theta + i\sin\theta)^n + (\cos\theta - i\sin\theta)^n]$

$= r^n[\cos n\theta + \cancel{i\sin n\theta} + \cos n\theta - \cancel{i\sin n\theta}]$ {Using De'Moivre's theorem}

$= 2r^n\cos n\theta.$

$= 2.\left\{2\sin\left(\frac{\alpha-\beta}{2}\right)\right\}^n \cos\left\{n\left(\frac{\pi}{2} + \frac{\alpha+\beta}{2}\right)\right\}$

$\cancel{= 2^{n+1}\sin}$

$= 2^{n+1}\sin^n\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{n\pi}{2} + \frac{n}{2}(\alpha+\beta)\right)$

$= 2^{n+1}\sin^n\left(\frac{\alpha-\beta}{2}\right)\left[\cos\left(\frac{n\pi}{2}\right)\cos\left\{\frac{n}{2}(\alpha+\beta)\right\} - \sin\left(\frac{n\pi}{2}\right)\sin\left\{\frac{n}{2}(\alpha+\beta)\right\}\right]$

$$= \begin{cases} -2^{n+1}\sin^n\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{n\pi}{2}\right)\sin\left\{\frac{n}{2}(\alpha+\beta)\right\} & \text{for odd } n \\ 2^{n+1}\sin^n\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{n\pi}{2}\right)\cos\left\{\frac{n}{2}(\alpha+\beta)\right\} & \text{for even } n \end{cases}$$

Ans

Assignment - 2
Q-1
Vikas Chand & Newton Nath.

(6) Prove that $(1+\sin\theta+i\cos\theta)^n+(1+\sin\theta-i\cos\theta)^n$
$= 2^{n+1}\cos^n\left(\frac{\pi}{4}-\theta/2\right)\cos\left(\frac{n\pi}{4}-\frac{n\theta}{2}\right)$

Sol^n/ L.H.S, $(1+\sin\theta+i\cos\theta)^n+(1+\sin\theta-i\cos\theta)^n \longrightarrow (1)$

Let, $1+\sin\theta = r\cos\alpha$
$\cos\theta = r\sin\alpha.$

$\therefore \quad r^2 = \cos^2\theta + (1+\sin\theta)^2$
$= 2+2\sin\theta = 2(1+\sin\theta)$
$r = [2(1+\sin\theta)]^{1/2}$

& $\tan\alpha = \dfrac{\cos\theta}{1+\sin\theta} = \dfrac{\cos^2\theta/2-\sin^2\theta/2}{\cos^2\theta/2+\sin^2\theta/2+2\cos\theta/2\sin\theta/2}$

$= \dfrac{\cos^2\theta/2-\sin^2\theta/2}{(\cos\theta/2+\sin\theta/2)^2}$

$= \dfrac{(\cos\theta/2-\sin\theta/2)\cancel{(\cos\theta/2+\sin\theta/2)}}{(\cos\theta/2+\sin\theta/2)^{\cancel{2}}}$

$= \dfrac{\cos\theta/2-\sin\theta/2}{\cos\theta/2+\sin\theta/2}$

$= \dfrac{1-\tan\theta/2}{1+\tan\theta/2}$ [Deviding N & D by $\cos\theta/2$.

$= \tan\left(\frac{\pi}{4}-\theta/2\right)$

$\therefore \alpha = \left(\frac{n\pi}{4} - n\theta/2\right)$ $\left[\because \tan(a-b) = \frac{\tan a - \tan b}{\tan a + \tan b}\right]$

(1) becomes $\rightarrow$

$(r\cos\alpha + i r\sin\alpha)^n + (r\cos\alpha - i r\sin\alpha)^n$

$= r^n\left[(\cos\alpha + i\sin\alpha)^n + (\cos\alpha - i\sin\alpha)^n\right]$

$= r^n\left[(\cos n\alpha + i\sin n\alpha) + (\cos n\alpha - i\sin n\alpha)\right]$

using De'Moivre's theorem i.e. $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$= r^n \, 2\cos n\alpha$. $\rightarrow$ (2)

$\because r = [2(1+\sin\theta)]^{1/2}$.

Taking, $(1+\sin\theta) = \sin^2\theta/2 + \cos^2\theta/2 + 2\sin\theta/2\cos\theta/2$.

$= (\sin\theta/2 + \cos\theta/2)^2$

Taking sq. root on both side $\rightarrow$

$(1+\sin\theta)^{1/2} = (\sin\theta/2 + \cos\theta/2)$

$\Rightarrow \frac{1}{\sqrt{2}}(1+\sin\theta/2)^{1/2} = \frac{1}{\sqrt{2}}(\sin\theta/2 + \cos\theta/2)$ [Deviding by $\sqrt{2}$ on both side

$= \sin\frac{\pi}{4}\sin\theta/2 + \cos\pi/4\cos\theta/2$

$= \cos\left(\frac{\pi}{4} - \theta/2\right)$ $\left[\because \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}\right.$

$\Rightarrow (1+\sin\theta/2)^{1/2} = \sqrt{2}\cos\left(\frac{\pi}{4} - \theta/2\right)$ & $[\cos(A-B) = \cos A\cos B + \sin A\sin B)$ $\rightarrow$ (3)

i. $r = 2^{1/2}(1+\sin\theta)^{1/2}$

$r^n = 2^{n/2}(1+\sin\theta)^{n/2} = 2^{n/2}.2^{n/2}\cos^n\left(\frac{\pi}{4} - \theta/2\right)$ $\rightarrow$ (4) using (3)

Now using eqn (4) & value of α in eqn (2)

we get, $\rightarrow$

$= 2.2^n\cos^n\left(\frac{\pi}{4} - \theta/2\right)\cos\left(\frac{n\pi}{4} - n\theta/2\right)$, R.H.S

$= 2^{n+1}\cos^n(\pi/4 - \theta/2)\cos\left(\frac{n\pi}{4} - n\theta/2\right)$ proved

Assignment 2.

Group - 10.

Q. 7. Prove that $\left[\dfrac{1+\sin\alpha + i\cos\alpha}{1+\sin\alpha - i\cos\alpha}\right]^n = \cos\left(\frac{n\pi}{2} - n\alpha\right) + i\sin\left(\frac{n\pi}{2} - n\alpha\right)$

Solution:

Let us consider,

$1+\sin\alpha = r\cos\theta$ —— (1)

$\cos\alpha = r\sin\theta$ —— (2)

(1)+(2) squaring and adding (1) + (2).

$\Rightarrow r^2 = (1+\sin\alpha)^2 + \cos^2\alpha.$

$= 1 + \sin^2\alpha + 2\sin\alpha + \cos^2\alpha.$

$= 1 + 1 + 2\sin\alpha$ (since, $\sin^2\alpha + \cos^2\alpha = 1$)

$= 2 + 2\sin\alpha.$

$= 2(1+\sin\alpha)$

$r^2 = 2(1+\cos(\pi/2 - \alpha))$ ($\sin \pi/2 - \theta = \cos\theta$) (or $\cos(\pi/2 - \theta) = \sin\theta$)

$r^2 = 2(2\cos^2(\pi/4 - \alpha/2))$ ($1 + \cos\theta = 2\cos^2(\theta/2)$)

$r = 2\cos(\pi/4 - \alpha/2)$

$\frac{(2)}{(1)}$ ~~$\tan\theta =$~~ $\Rightarrow \dfrac{r\sin\theta}{r\cos\theta} = \dfrac{\cos\alpha}{1+\sin\alpha}.$

$\tan\theta = \dfrac{\cos\alpha}{1+\sin\alpha}$

$= \dfrac{\sin(\pi/2 - \alpha)}{1+\cos(\pi/2 - \alpha)}$ | $\sin(\pi/2 - \theta) = \cos\theta.$

$$\Rightarrow \tan\theta = \frac{2\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cancel{\tfrac{1}{2}}\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$1+\cos 2\theta = 2\cos^2\theta$.

$$\tan\theta = \tan\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$$

$$\theta = \left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$$

$$\Rightarrow \left[\frac{1+\sin\alpha+i\cos\alpha}{1+\sin\alpha-i\cos\alpha}\right]^n = \left[\frac{(\cos\theta+i\sin\theta)\cancel{r}}{(\cos\theta-i\sin\theta)\cancel{r}}\right]^n$$

$$= \left[\frac{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)-i\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}\right]^n$$

Multiplying numerator and denominator using conjugate of denominator

$$\Rightarrow \left[\frac{\left[\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\right]^2}{1}\right]^n.$$

$$= \left[\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\right]^{2n}.$$

$$= \left[\cos\left(\frac{\pi}{2}-\alpha\right)n+i\sin\left(\frac{\pi}{2}-\alpha\right)n\right].$$

$$= \left[\cos\left(n\frac{\pi}{2}-n\alpha\right)+i\sin\left(n\frac{\pi}{2}-n\alpha\right)\right]$$

Proved.

Q-11

Assignment - 2

(1) venkatesh chinni
(2) Durga prasad

(Q) If $2\cos\theta = x+\frac{1}{x}$ and $2\cos\phi = y+\frac{1}{y}$. show that one of the values of $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\theta+n\phi)$

Sol

Given that $2\cos\theta = x+\frac{1}{x}$ and $2\cos\phi = y+\frac{1}{y}$

$\Rightarrow x^2 - 2x\cos\theta + 1 = 0$ $\Rightarrow y^2 - 2y\cos\phi + 1 = 0$

$\Rightarrow x = \cos\theta \pm i\sin\theta$ $\Rightarrow y = \cos\phi \pm i\sin\phi$

Take L.H.S :

$$\Rightarrow x^m y^n + \frac{1}{x^m y^n} = (\cos m\theta \pm i\sin m\theta)(\cos n\phi \pm i\sin n\phi) + (\cos m\theta \mp i\sin m\theta)(\cos n\phi \mp i\sin n\phi)$$

$$\Rightarrow x^m y^n + \frac{1}{x^m y^n} = (\cos m\theta\cos n\phi \pm i\cos m\theta\sin n\phi \pm i\sin m\theta\cos n\phi - \sin m\theta\sin n\phi) + (\cos m\theta\cos n\phi \mp i\cos m\theta\sin n\phi \mp i\sin m\theta\cos n\phi - \sin m\theta\sin n\phi)$$

$$= \cos(m\theta+n\phi) + \cos(m\theta+n\phi)$$

$$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\theta+n\phi) \quad \text{R.H.S}$$

Hence, it is proved.

Assignment 2

Group 5: Apurv & Sanjay

Question

8 (ii). If $2\cos\theta = x + 1/x$ and $2\cos\phi = y + 1/y$, Show that one of the values of

$$\frac{x^m}{y^n} + \frac{y^n}{x^m} \qquad is \qquad 2\cos(m\theta - n\phi)$$

Solution

Given

$$2\cos\theta = x + \frac{1}{x}$$

Simplifying we get

$$x^2 - 2x\cos\theta + 1 = 0$$

Solving for x from the above quadratic equation we get,

$$\begin{aligned} x &= \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2} \\ &= \cos\theta \pm i\sin\theta \end{aligned}$$

Similarly from question if we solve for y we get,

$$y = \cos\phi \pm i\sin\phi$$

Now

$$\begin{aligned}\frac{x^m}{y^n}+\frac{y^n}{x^m} &= x^m y^{-n}+x^{-m}y^n \\ &= (\cos m\theta \pm i\sin m\theta)(\cos n\phi \mp i\sin n\phi)+(\cos m\theta \mp i\sin m\theta)(\cos n\phi \pm i\sin n\phi)\end{aligned}$$

Let $z = (\cos m\theta \pm i\sin m\theta)(\cos n\phi \mp i\sin n\phi)$, then the above equation is in the form $z+\overline{z}$. We know that $z+\overline{z} = 2\mathrm{Re}(z) \qquad \forall z \in \mathbb{C}$. So

$$\begin{aligned}\frac{x^m}{y^n}+\frac{y^n}{x^m} &= 2\mathrm{Re}\,(\cos m\theta \pm i\sin m\theta)(\cos n\phi \mp i\sin n\phi) \\ &= 2\left(\cos m\theta\cos n\phi(\pm\times\mp\times i^2)\sin m\theta\sin n\phi\right) \\ &= 2\left(\cos m\theta\cos n\phi+\sin m\theta\sin n\phi\right) \\ &= 2\cos(m\theta-n\phi)\end{aligned}$$

This is what we wanted to show.