Assignment - 12

Group1 Newton & Vikas

Q.1 Sol Given

$v(x,y)$: Imaginary part $= \dfrac{\sinh 2x}{\cosh 2x + \cos 2y}$

To find real part, we use Cauchy Riemann's conditions

i.e. $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ & $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial}{\partial y}\left(\frac{\sinh 2x}{\cosh 2x + \cos 2y}\right)$$

$$= \frac{(\cosh 2x + \cos 2y)(0) + 2\sinh 2x \sin 2y}{(\cosh 2x + \cos 2y)^2}$$

$$= \frac{2\sinh 2x \sin 2y}{(\cosh 2x + \cos 2y)^2}$$

$$\Rightarrow u = \int \frac{2\sinh 2x \sin 2y}{(\cosh 2x + \cos 2y)^2}\, dx + f(y)$$

$$= \frac{2\sin 2y}{2} \int \frac{\sinh 2x\, dx}{(\cosh 2x + \cos 2y)^2} + f(y)$$

$$u = \frac{-2\sin 2y}{2(\cosh 2x + \cos 2y)} + f(y)$$

Now

using this u in $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$$\Rightarrow \frac{\partial}{\partial y}\left(\frac{-2\sin 2y}{(\cosh 2x + \cos 2y)} + f(y)\right) = -\frac{\partial}{\partial x}\left(\frac{\sinh 2x}{\cosh 2x + \cos 2y}\right)$$

$$\frac{-2(\cosh 2x + \cos 2y)\cos 2y \times 2 + 2\sin 2y \sin 2y}{(\cosh 2x + \cos 2y)^2} + f'(y) = \frac{2(\cosh 2x + \cos 2y)\cosh 2x - 2\sinh 2x \sinh 2x}{(\cosh 2x + \cos 2y)^2}$$

$$\frac{-2\cosh 2x \cos 2y - 2\cos^2 2y + 2\sin^2 2y}{(\cosh 2x + \cos 2y)^2} = \frac{2(\cosh^2 x - \sinh^2 x) + 2\cos 2y \cosh 2x}{(\cosh 2x + \cos 2y)^2}$$

$$\frac{-2(\cosh 2x + \cos 2y + 1)}{(\cosh 2x + \cos 2y)^2} + f'(y)^2 = \frac{2(1 + \cos 2y \cosh 2x)}{(\cosh 2x + \cos 2y)^2}$$

$$\Rightarrow f'(y) = 0$$

$$\Rightarrow f(y) = C = \text{constant of } y$$

$$\therefore \boxed{u = \frac{-\sin 2y}{(\cosh 2x + \cos 2y)} + C}$$