MATHEMATICAL AND NUMERICAL METHODS

ASSIGNMENT - 1 (Sem-I)

Submitted by

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Group - 4

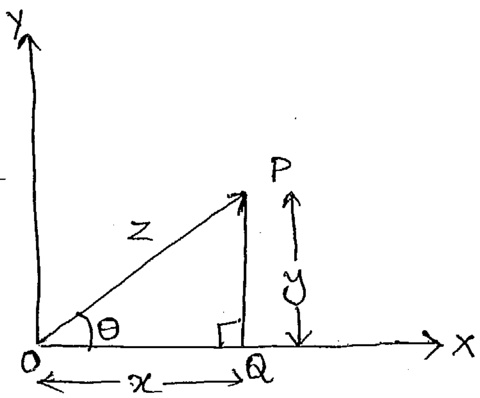
- Question -

1. Find the modulus & argument of

$$\frac{(3-\sqrt{2}i)^2}{1+2i}$$

- Answer -

Let us consider any complex number z which is expressed as $z = x+iy$. It is represented as below.



(Fig-1)

In Fig-1,

$\triangle PQO$ is a right angle triangle with

$OQ = x$ unit

$PQ = y$ unit

$\angle PQO = 90^\circ$, $\angle POQ = \theta$

$OP = |z|$

$\therefore$ According to pythagoras's theorem

$$|OP|^2 = |OQ|^2 + |PQ|^2$$

$$\Rightarrow |z|^2 = |x|^2 + |y|^2$$

$$\Rightarrow \boxed{|z| = \sqrt{x^2+y^2}} \quad \text{——— (1)}$$

Here $|z|$ is called the modulus of z.

$\therefore$ Again in the right angle triangle PQO,

$$\tan \angle POQ = \frac{PQ}{OQ}$$

$$\Rightarrow \tan\theta = \frac{y}{x}$$

$$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)} \quad \text{——— (2)}$$

Here θ is called the argument of z.

In this assignment our objective is to find the modulus & argument of

$$z = \frac{(3-\sqrt{2}i)^2}{1+2i}$$

Since the numerator of z is a square term, so let us solve it at first by using certain algebra rule, which is as follows.

$$z = \frac{(3-\sqrt{2}i)^2}{1+2i}$$

$$= \frac{(3)^2 + (\sqrt{2}i)^2 - 2\cdot 3\cdot \sqrt{2}i}{1+2i}$$

$$= \frac{9-2-6\sqrt{2}i}{1+2i}$$

$$= \frac{7-6\sqrt{2}i}{1+2i}$$

Now z is expressed in $\frac{a+ib}{c+id}$ form. To find its modulus & argument we have to convert it to $(x+iy)$ form. To do this we will follow the normalization technique. Hence we have to multiply & divide $c-id$ both in numerator & denominator & proceed as below,

$$z = \frac{7-6\sqrt{2}i}{1+2i}$$

$$= \frac{(7-6\sqrt{2}i)(1-2i)}{(1+2i)(1-2i)}$$

$$= \frac{7(1-2i) - 6\sqrt{2}i\,(1-2i)}{(1)^2-(2i)^2}$$

$$= \frac{7-14i-6\sqrt{2}i-12\sqrt{2}}{1+4}$$

$$= \left(\frac{7-12\sqrt{2}}{5}\right) + i\left(\frac{-14-6\sqrt{2}}{5}\right)$$

Now z has been expressed in $x+iy$ form.

Here $x = \dfrac{7-12\sqrt{2}}{5}$ &

$y = \dfrac{-14-6\sqrt{2}}{5}$

$\therefore$ modulus of $z = \sqrt{x^2+y^2}$ [$\because$ from eqⁿ (1)]

$$\Rightarrow |z| = \sqrt{\left(\frac{7-12\sqrt{2}}{5}\right)^2 + \left(\frac{-14-6\sqrt{2}}{5}\right)^2}$$

$$= \sqrt{\frac{49+288-\cancel{168\sqrt{2}}+196+72+\cancel{168\sqrt{2}}}{25}}$$

$$= \sqrt{\frac{605}{25}} = \sqrt{\frac{121\times 5}{25}} = \frac{11\sqrt{5}}{5}$$

$$\Rightarrow \boxed{|z| = \frac{11\sqrt{5}}{5}} \quad \text{———} (3)$$

$\therefore$ argument of $z = \theta = \tan^{-1}\left(\frac{y}{x}\right)$ [$\because$ from eqⁿ (2)]

$$= \tan^{-1}\left(\frac{-14-6\sqrt{2}/5}{7-12\sqrt{2}/5}\right)$$

$$= \tan^{-1}\left(\frac{-14-6\sqrt{2}}{7-12\sqrt{2}}\right)$$

$$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{14+6\sqrt{2}}{12\sqrt{2}-7}\right)} \quad \text{———} (4)$$

So the modulus & argument of $\dfrac{(3-\sqrt{2}i)^2}{1+2i}$ is calculated to be $\dfrac{11\sqrt{5}}{5}$ & $\tan^{-1}\left(14+6\sqrt{2}/12\sqrt{2}-7\right)$ respectively.

MATHEMATICAL & NUMERICAL METHODS

ASSIGNMENT - I (Complex Numbers)

(August 9, 2012)

Group - G6
(I) Lalit Kr. Shukla
(II) Rahul Yadav

Problem-

Convert $\dfrac{(2-\sqrt{3}\,i)}{(1+i)}$ into $(x+iy)$ form.

Solution- We have the complex number

$$\frac{2-\sqrt{3}\,i}{1+i}$$

to simplify this we can always ~~do~~ write

$$\frac{2-\sqrt{3}\,i}{1+i} \times \frac{1-i}{1-i}$$

which is equal to

$$\frac{(2-\sqrt{3}i)\cdot(1-i)}{(1)^2-(i)^2} = \frac{(2-\sqrt{3}i)(1-i)}{1-(-1)}$$

$$= \frac{2\times 1 - 2i - \sqrt{3}i + \sqrt{3}i^2}{2}$$

$$= \frac{(2-\sqrt{3}) - i(2+\sqrt{3})}{2}$$

$$= \frac{(2-\sqrt{3})}{2} + i\left(-\frac{(2+\sqrt{3})}{2}\right)$$

which is required form.

ie.

$$\frac{(2-\sqrt{3}\,i)}{(1+i)} = \frac{(2-\sqrt{3})}{2} + i\left(-\frac{(2+\sqrt{3})}{2}\right)$$

QED

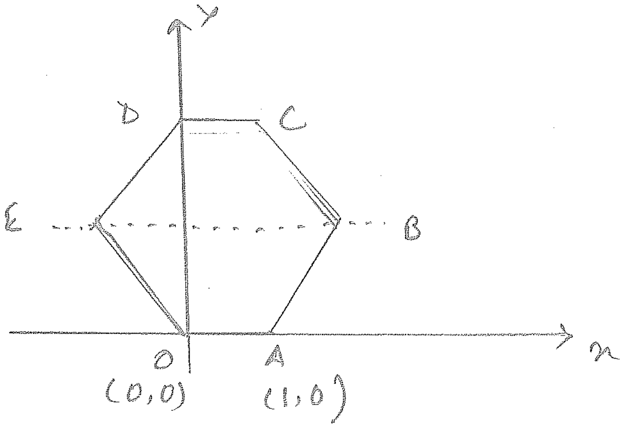
MATHEMATICAL & NUMERICAL METHODS

ASSIGNMENT: 1

Group 2
Chandana
Jinia Sikdar

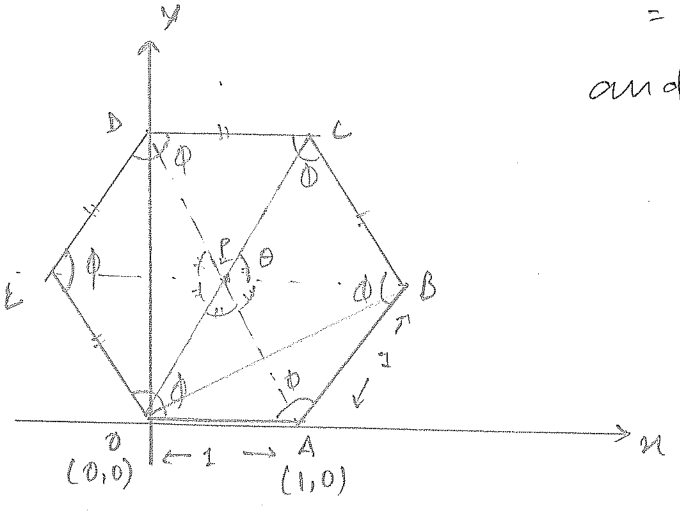
Problem 3:

One of the vertices of a regular hexagon is A(1,0) and the origin is at O(0,0). $\bar{OA}$ is also one of the sides as shown in the figure. Find out remaining vertices.



Sol^n: Since the given fig. is that of a regular hexagon, hence it has equal sides. Here OA = 1 unit. $\therefore$ OA = AB = BC = DC = DE = EO = 1 unit

and $\theta = 60°$; $\phi = 120°$



In $\triangle OAB$, OA = 1 unit, AB = 1 unit, OB = ?

$\angle OAB = 120°$, $\angle AOB = 30°$, $\angle OBA = 30°$.

Applying law of Sines, we get: $\frac{OA}{\sin ABO} = \frac{AB}{\sin AOB} = \frac{BO}{\sin OAB}$

$$\Rightarrow \frac{1}{\sin 30°} = \frac{BO}{\sin 120°}$$

$$\therefore BO = \frac{\sin 120°}{\sin 30°}$$

$\therefore \ BO = \frac{\sqrt{3}}{2} \div \frac{1}{2} = \sqrt{3}$

Now, for vertex B, $r = \sqrt{3}$, $\theta = 30^\circ$

$\therefore \ x = \sqrt{3} \cos 30^\circ$

$y = \sqrt{3} \sin 30^\circ$

$\therefore \ z = x + iy = \sqrt{3}\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \frac{3}{2} + \frac{i\sqrt{3}}{2} \longrightarrow (1)$.

In $\triangle OBC$, $\angle OBC = 120 - 30 = 90^\circ$

$OB = \sqrt{3}$, $BC = 1$

$\therefore \ OC = \sqrt{3+1} = 2. \longrightarrow$ Since its right-angled triangle, hence pythagoras theorem is applicable.

For vertex C, $r = 2$, $\theta = 60^\circ$

$x = 2\cos 60^\circ$

$y = 2 \sin 60^\circ$

$\therefore \ z = x + iy = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 1 + i\sqrt{3} \longrightarrow (2)$

$\triangle ODC$ is a right angled $\triangle$, $OC = 2$, $CD = 1$

$\therefore \ OD = \sqrt{4-1} = \sqrt{3}$

For vertex D, $r = \sqrt{3}$, $\theta = 90^\circ$

$\therefore \ z = x + iy = \sqrt{3}\left(\cos 90^\circ + i \sin 90^\circ\right)$

$= \sqrt{3}(i \cdot 1) = i\sqrt{3} \longrightarrow (3)$

For vertex E, $OE = 1$, $\theta = 120^\circ$

$\therefore \ z = x + iy = 1\left(\cos 120^\circ + i \sin 120^\circ\right)$

$= 1\left[\cos(90+30) + i \sin(90+30)\right]$

$= 1\left[-\sin 30 + i\cos 30\right]$

$= 1\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right] = -\frac{1}{2} + i\frac{\sqrt{3}}{2}.$

$\longrightarrow (4)$.

Hence coordinates of B, C, D, E are:

$\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$, $(1, \sqrt{3})$, $(0, \sqrt{3})$, $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ respectively.

Assignment 1

Group 5: Apurv & Sanjay

Question 4

1. Determine region in the z plane represented by $R(z) > 3$.

Solution

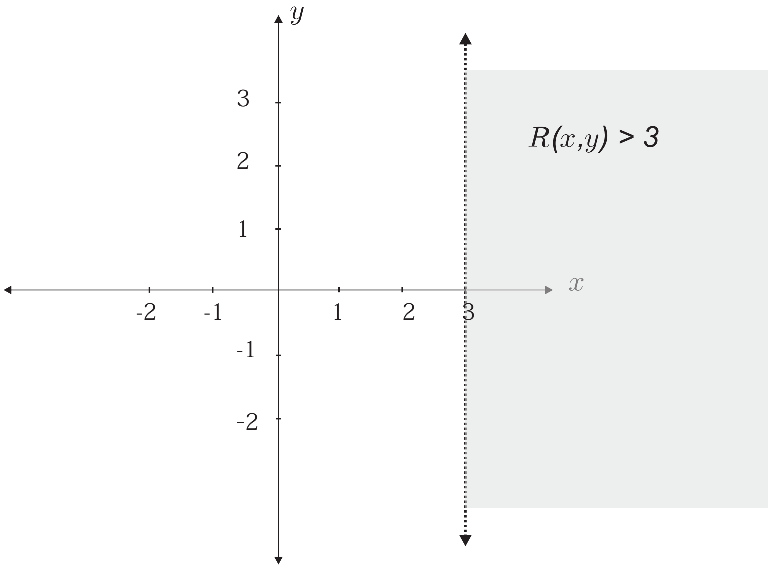


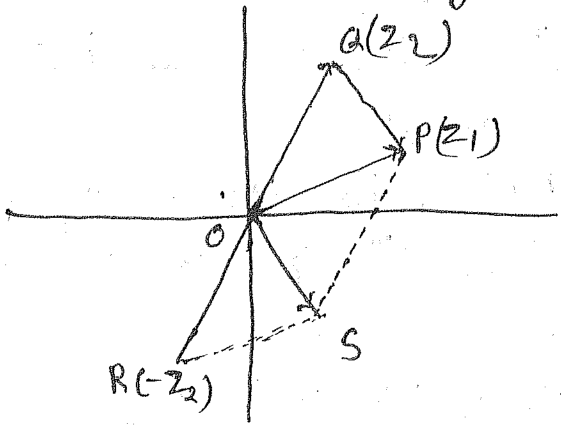
Figure 1: Region in the complex plane showing $R(z) > 3$, the dotted line is not included in the region.

Group G-3

Problem No. 5.

Q. Prove that $|z_1 - z_2| \geq |z_1| - |z_2|$ if z_1 and z_2 are complex Number.

Solⁿ :- Let P and Q represent the two complex Number z_1 and z_2 in the argand diagram.



$z_1 - z_2$ means addition of z_1 and $-z_2$.

$-z_2$ represented by OR formed by extending OQ to OR such that OQ = OR

On Completing the parallelogram OPSR, the sum of z_1 and $-z_2$ is represented by OS

Now, any side of traingle is greater than the difference between other two sides. Thus

In $\triangle OPS$ $\quad OS > OP - PS \Rightarrow OS > OP - OQ$

$\Rightarrow |z_1 - z_2| > |z_1| - |z_2|$

However, if O,P,S are collinear then

$|z_1 - z_1| = |z_1| - |z_2|$

Thus $\quad |z_1 - z_2| \geq |z_1| - |z_2|$ Proved.

Another way

$|z_1| = |(z_1 - z_2) + z_2| \leq |(z_1 - z_2)| + |z_2|$ $\quad (\because |z_1 + z_2| \leq |z_1| + |z_2|)$

$|z_1| - |z_2| \leq |z_1 - z_2|$

OR $|z_1 - z_2| \geq |z_1| - |z_2|$ Proved.

Assignment 1

Newton Nath
Vikas Chand] Group 1.

Q:6 If $P_1(z_1)$, $P_2(z_2)$ & $P_3(z_3)$ be any three points

Prove that $\text{Arg}\left(\frac{z_3 - z_2}{z_1 - z_2}\right) = \angle P_1 P_2 P_3$
↑ Angle

Show the points in Argand diagram

Solution z_1 & z_2 are two complex no.s $z_1 = x_1 + iy_1$
$z_2 = x_2 + iy_2$

then in polar form

$z_1 = r_1 e^{i\theta_1}$ $\quad r_1 = \sqrt{x_1^2 + y_1^2}$ $\theta_1 = \text{Arg}\, z_1 = \tan^{-1}(y_1/x_1)$

$z_2 = r_2 e^{i\theta_2}$ $\quad r_2 = \sqrt{x_2^2 + y_2^2}$ $\theta_2 = \text{Arg}\, z_2 = \tan^{-1}\left(\frac{y_2}{x_2}\right)$

$$\Rightarrow \text{Arg}\left(\frac{z_1}{z_2}\right) = \text{Arg}\left(\frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}}\right) = \text{Arg}\left[\frac{r_1}{r_2} e^{i\underbrace{(\theta_1 - \theta_2)}_{\text{Arg}}}\right]$$

$$= (\theta_1 - \theta_2)$$

$$\boxed{\text{Arg}\left(\frac{z_1}{z_2}\right) = \text{Arg}\, z_1 - \text{Arg}\, z_2} \quad \text{—(i)}$$

Using this

$$\text{Arg}\left[\frac{(z_3 - z_2)}{(z_1 - z_2)}\right] = \text{Arg}(z_3 - z_2) - \text{Arg}(z_1 - z_2)$$

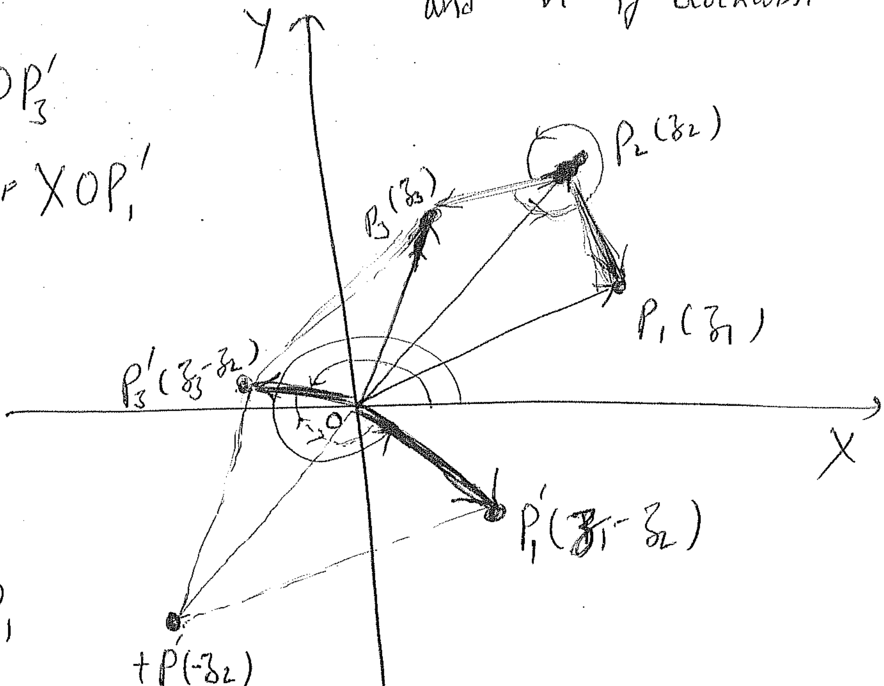
From Argand diagram

Angles are +ve if taken in counterclockwise and −ve if clockwise

$$\operatorname{Arg}\left(\frac{z_3 - z_2}{z_1 - z_2}\right) = \text{Angle } XOP_3' - \text{Angle } XOP_1'$$

$$= -\text{Angle } P_3'OP_1'$$

$$= \text{Angle } P_1'OP_3'$$



from diagram $OP_1' \parallel P_2P_1$

& $OP_3' \parallel P_2P_3$

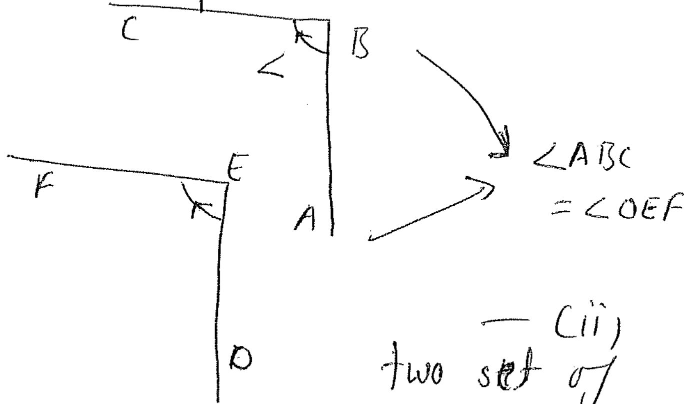
So using (ii)

$$\operatorname{Arg}\left(\frac{z_3 - z_2}{z_1 - z_2}\right) = \text{Angle } P_1'OP_3'$$

$$= \text{Angle } P_1P_2P_3$$

$$= \angle P_1P_2P_3$$

Hence proved.



— (ii) two set of $\parallel$ lines have same central angle

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Group - 8

Assignment - 1

(7) Let z_1 and z_2 are two complex number.

Then we can write, $z_1 = x_1 + iy_1$

and $z_2 = x_2 + iy_2$

So, $z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$

$= x_1 x_2 + i x_1 y_2 + i x_2 y_1 + i^2 y_1 y_2$

$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$

$\Rightarrow$ another complex ~~no~~ number with real part $(x_1 x_2 - y_1 y_2)$ and imaginary part $(x_1 y_2 + x_2 y_1)$.

Assignment I

Group - 10

Q.8. Convert the complex number $z = 3+4i$ into the polar form.

$z = 3+4i$.

z can be represented in polar form as $r(\cos\theta + i\sin\theta)$.

$r = \sqrt{x^2+y^2}$ $x = 3$, $y = 4$.

$= \sqrt{3^2+4^2}$

$= \sqrt{25} = \underline{\underline{5}}$.

$\theta = \tan^{-1}(y/x)$

$= \tan^{-1}(4/3) = 51.34^0$.

$\therefore$ z in polar form is $5(\cos(51.34^\circ) + i\sin(51.34^\circ))$.

Assignment - 1
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G-11

PROBLEM - 09

(a) Convert the complex number $2e^{i\pi/4}$ in to the rectangular form?

Sol

Given $2e^{i\pi/4}$, let $z = 2e^{i\pi/4}$

We know that $e^{i\theta} = \cos\theta + i\sin\theta$

There fore,

$$2e^{i\pi/4} = 2\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right]$$

$$\Rightarrow \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \quad \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 2\left[\frac{1}{\sqrt{2}} + i\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$\Rightarrow \left(\frac{2}{\sqrt{2}} + i\frac{2}{\sqrt{2}}\right)$$

$$z = \left(\sqrt{2} + i\sqrt{2}\right)$$

This is of the form $x+iy$

Real part of $(z) = \sqrt{2}$ and

imaginary part of $(z) = \sqrt{2}$

Mathematical Physics 07.08.2012

Assignment - 1 Group - 9

Problem 10 Find out the real and imaginary parts of the following quantity : $\frac{(1-i).(2+i)}{(3+4i)}$

$\Rightarrow$ Let,

$$z = \frac{(1-i)(2+i)}{(3+4i)} = \frac{2+i-2i-i^2}{(3+4i)}$$

$$= \frac{(2+1)-i}{(3+4i)} = \frac{(3-i)}{(3+4i)} = \cancel{(3+i)(3+i)}$$

$$= \frac{(3-i)(3-4i)}{(3+4i)(3-4i)}$$

$$= \frac{9-4-15i}{9+16} = \frac{5-15i}{25}$$

$$= 5/25 - i\,\frac{15}{25}$$

$$= 1/5 - i.3/5$$

$$= (1/5) + i(-3/5)$$

So, the real and imaginary parts of the given quantity of z are:

$$\left.\begin{array}{l} Re(z) = 1/5 \\ Im(z) = -3/5 \end{array}\right\} \text{Ans}$$

ASSIGNMENT 01

GROUP-7

We have to simplify following complex no.

$$\frac{(4-2i)(2+i)}{(3-2i)}$$

and represent the no. so obtained on Argand Diagram

Part a)

$$\frac{(4-2i)(2+i)}{(3-2i)} \times \frac{(3+2i)}{(3+2i)} \quad [\text{Rationalisation}]$$

$$= \frac{(4-2i)[6+7i-2]}{13}$$

$$= \frac{(4-2i)(4+7i)}{13}$$

$$= \frac{16+20i+14}{13}$$

$$= \frac{30}{13} + \frac{20}{13}i$$

Part (a) stands completed

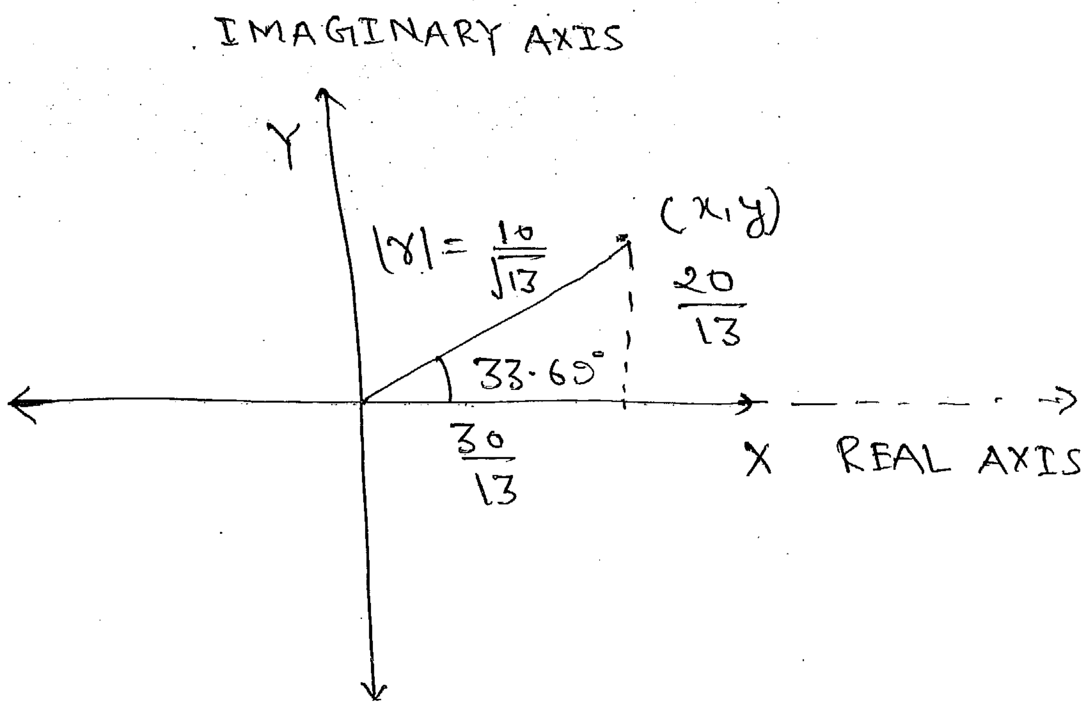
No. so obtained is $\frac{30}{13} + i\frac{20}{13}$

Part (b) $|r| = \sqrt{x^2+y^2} = \frac{10}{\sqrt{13}}$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(2/3)$$

$$\Rightarrow \theta = 33.69^\circ \text{ [using calculator]}$$

P.T.O.


IMAGINARY AXIS
Y
$|r| = \frac{10}{\sqrt{13}}$
(x, y)
$\frac{20}{13}$
$33.69°$
$\frac{30}{13}$
X
REAL AXIS