Assignment 9

1. Find the eigen values and eigen vectors for the following matrices.

(i) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

2. If λ is an eigen value of a non-singular matrix A, find out eigen value of matrix adj A.

3. For a symmetrical square matrix, show that eigen vectors corresponding to two unequal eigen values are orthogonal.

4. Using Cayley-Hamilton theorem, find the inverse of

$$\begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

5. Reduce the matrix $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to diagonal form.

6. Show that an $n \times n$ matrix A is singular if and only if zero is one of its eigenvalues. Give an example of a singular 3×3 matrix and find out its eigen values and eigen vectors.

7. For a given matrix A, find out eigen values and eigen vectors of A and A', both. Comment on the result. Take 3×3 matrix.

8. Prove that an $n \times n$ matrix A is diagonalizable by a ~~similarity~~ similarity transformation if and only if it has a complete set of n linearly independent eigen vectors.

9. Given $A = \begin{bmatrix} 0 & 0 & i \\ -i & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$, show that $A^n = 1$ (with proper choice of n, $n \neq 0$)

(10) A rotation $\phi_1 + \phi_2$ about the z-axis is carried out as two successive rotations ϕ and ϕ_2, each about the z-axis. Use the matrix representation of the rotations to derive the trigonometric identities:

$$\cos(\phi_1 + \phi_2) = \cos\phi_1 \cos\phi_2 - \sin\phi_1 \sin\phi_2$$

$$\sin(\phi_1 + \phi_2) = \sin\phi_1 \cos\phi_2 + \cos\phi_1 \sin\phi_2$$

(11) Prove that if $A^k = 0$ for some positive integer k (such a matrix is said to be nilpotent), then all of the eigen values of A are zero.

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(12) Use a software to compute the eigen values and vectors of the matrix

$$A = \begin{bmatrix} -149 & -50 & -154 \\ 537 & 180 & 546 \\ -27 & -9 & -25 \end{bmatrix}.$$

Repeat the excercise for $a_{22} = 180.01$ and 179.99. Try to draw conclusion about the conditioning of the eigen values of A.

(13) The matrix exponential function of an $n \times n$ matrix A is defined by the infinite series

$$\exp(A) = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

(a) Write a program to evaluate exp(A) using the series definition

(b) An alternate way to compute the matrix exponential uses the eigen value-eigenvector decomposition

$$A = U \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n) U^{-1}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigen values of A and U is a matrix whose columns are corresponding eigen vectors. Then the matrix exponential is given by

$$\exp(A) = U \,\mathrm{diag}\,(e^{\lambda_1}, e^{\lambda_2}, \ldots, e^{\lambda_n})\, U^{-1}$$

Write a program to evaluate $\exp(A)$ using eigen value- eigen vector decomposition.

Test both the methods for the following matrices

$$A = \begin{bmatrix} 113 & -114 \\ 152 & -153 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

G-1	2
G-2	1
G-3	5
G-4	4
G-5	3
G-6	9
G-7	8
G-8	10
G-9	6
G-10	11
G-11	7

Problem no. 12 and 13 are to be attempted by all the groups using C or Fortran. The results are to be checked using higher level software like MATLAB. Copy of program and verification, if any, are to be submitted.

Submit by 7-9-2012, except problem no. 12 and 13. Submit problem no. 12 and 13 by 20-9-2012.