

## Assignment 7

① For what values of  $x$ , the matrix  $\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$  is singular?

② Express  $\begin{bmatrix} 2 & 5 & -7 \\ -9 & 12 & 4 \\ 15 & -13 & 6 \end{bmatrix}$  as the sum of

→ a lower triangular matrix and

→ an upper triangular matrix with zero leading diagonal.

③ Evaluate  $\begin{bmatrix} 2 & 1 & -1 \\ 4 & -5 & 6 \\ -3 & 7 & 3 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -6 & 4 \\ -2 & 5 \end{bmatrix} \times \begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix}$

④ If  $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$  and  $I$  is the unit

matrix of order 3, evaluate  $A^2 - 3A + 9I$ .

⑤ Factorize the matrix  $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$

into the form  $LU$ , where  $L$  is lower triangular and  $U$  is upper triangular matrix.

⑥ Express  $\begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix}$  as the sum of a symmetric

and a skew-symmetric matrix.

⑦ Find the inverse of a matrix  $\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 0 \\ 1 & 4 & 1 \end{bmatrix}$

from the adjoint.

⑧ Show that

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}^{-1}$$

⑨ If  $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

verify that  $(AB)' = B'A'$ , and  $(AB)^{-1} = B^{-1}A^{-1}$ .

⑩ If  $D = \text{diag}[d_1, d_2, \dots, d_n]$ ,  $d_1, d_2, \dots, d_n \neq 0$ ,  
prove that  $D^{-1} = \text{diag}[d_1^{-1}, d_2^{-1}, \dots, d_n^{-1}]$ .

⑪ If  $A$  and  $B$  are two  $n \times n$  non-singular matrices, show that  $\text{adj}(AB) = \text{adj} B \times \text{adj} A$ .

G-1 - 4

G-2 - 5

G-3 - 1

G-4 - 6

G-5 - 11

G-6 - 2

G-7 - 3

G-8 - 9

G-9 - 10

G-10 - 7

G-11 - 8

Submit by

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