Assignment-6

① If $\sin(A+iB) = x+iy$, prove that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$

and $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$.

② If $\sin(\theta+i\phi) = \rho(\cos\alpha + i\sin\alpha)$, prove that

(i) $\rho^2 = \frac{1}{2}[\cosh 2\phi - \cos 2\theta]$

(ii) $\tan\alpha = \tanh\phi \cot\theta$.

③ If $\cos(\theta+i\phi) = \cos\alpha + i\sin\alpha$, prove that

(i) $\sin^2\theta = \pm \sin\alpha$

(ii) $\cos 2\theta + \cosh 2\phi = 2$.

④ If $\tan(A+iB) = x+iy$, prove that

(i) $x^2 + y^2 + 2x \cot 2A = 1$

(ii) $x^2 + y^2 - 2y \coth 2B + 1 = 0$

⑤ If $\tan(x+iy) = \sin(u+iv)$, prove that

$$\frac{\sin 2x}{\sinh 2y} = \frac{\tan u}{\tanh v}$$

⑥ If $x = 2\cos\alpha \cosh\beta$,

$y = 2\sin\alpha \sinh\beta$, prove that

$$\sec(\alpha+i\beta) + \sec(\alpha-i\beta) = \frac{4x}{x^2+y^2}.$$

(7) If $a+ib = \tanh\left(\theta + i\frac{\pi}{4}\right)$, prove that

$$a^2 + b^2 = 1.$$

(8) Separate $\cos^{-1}[\cos\theta + i\sin\theta]$ into real and imaginary parts, where θ is a positive acute angle.

G-1	(8)	G-6	3(ii)
G-2	(1)	G-7	4(i)
G-3	3(i)	G-8	(5)
G-4	2(ii)	G-9	(6)
G-5	2(i)	G-10	(7)
		G-11	4(ii)

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