Assignment - 2

① Prove that $$\frac{(\cos 5\theta - i\sin 5\theta)^2 \; (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 \; (\cos\theta + i\sin\theta)^5} = 1$$

② Prove that $$\left[\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right]^4 = \cos 8\theta + i\sin 8\theta$$

③ If $p = \text{cis}\,\theta$ and $q = \text{cis}\,\phi$. show that

(i) $$\frac{p-q}{p+q} = i\tan\frac{\theta-\phi}{2}$$

(ii) $$\frac{(p+q)(pq-1)}{(p-q)(pq+1)} = \frac{\sin\theta + \sin\phi}{\sin\theta - \sin\phi}$$

④ If $a = \text{cis}\,2\alpha$, $b = \text{cis}\,2\beta$, $c = \text{cis}\,2\gamma$ and $d = \text{cis}\,2\delta$,

prove that

(i) $$\sqrt{\frac{ab}{c}} + \sqrt{\frac{c}{ab}} = 2\cos(\alpha+\beta-\gamma)$$

(ii) $$\sqrt{\frac{ab}{cd}} + \sqrt{\frac{cd}{ab}} = 2\cos(\alpha+\beta-\gamma-\delta)$$

⑤ Simplify $$[\cos\alpha - \cos\beta + i(\sin\alpha - \sin\beta)]^n + [\cos\alpha - \cos\beta - i(\sin\alpha - \sin\beta)]^n$$

⑥ Prove that $$(1+\sin\theta + i\cos\theta)^n + (1+\sin\theta - i\cos\theta)^n = 2^{n+1}\cos^n\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{n\pi}{4}-\frac{n\theta}{2}\right)$$

⑦ Prove that $$\left[\frac{1+\sin\alpha + i\cos\alpha}{1+\sin\alpha - i\cos\alpha}\right]^n = \cos\left(\frac{n\pi}{2}-n\alpha\right) + i\sin\left(\frac{n\pi}{2}-n\alpha\right)$$

(8) If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, show that one of the values of

(i) $x^m y^n + \frac{1}{x^m y^n}$ is $2\cos(m\theta + n\phi)$

(ii) $\frac{x^m}{y^n} + \frac{y^n}{x^m}$ is $2\cos(m\theta - n\phi)$

G-1	6
G-2	4(ii)
G-3	1
G-4	2
G-5	8(ii)
G-6	3(i)
G-7	4(i)
G-8	3(ii)
G-9	5
G-10	7
G-11	8(i)

Submit by 16-8-2012